

How to use the findSegments function to fit a piecewise constant curve

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October 13, 2005

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1 Introduction

The problem of segmenting a series of numbers into piecewise constant segments occurs in multiple application areas. Two examples are

- arrayCGH data, where the segments correspond to regions of copy number gain, loss, or no change.
- tiling microarray data for transcriptomics, where the segments correspond to transcripts. Here we assume that the probe effects (which lead to different fluorescence intensities even for the same mRNA abundance) have been normalized away, so that all probes for one transcript have the same fluorescence (in expectation).

To demonstrate and verify the correctness of the algorithm, let's generate simulated data:

```
> genData = function(lenx, nrCP, stddev = 0.1) {  
+   x = numeric(lenx)  
+   cp = c(1, sort(sample(1:floor(lenx/15), nrCP - 1) * 15),  
+         lenx + 1)
```

```

+     s = 0
+     for (j in 2:length(cp)) {
+         sel = cp[j - 1]:(cp[j] - 1)
+         s = (0.5 + runif(1)) * sign(rnorm(1)) + s
+         x[sel] <- rnorm(length(sel), mean = s, sd = stddev)
+     }
+     return(list(x = x, cp = cp[-1]))
+ }

> lenx = 1000
> nrcp = 10
> gd = genData(lenx, nrcp)
> plot(gd$x, pch = ".")
> abline(v = gd$cp, col = "red")

```

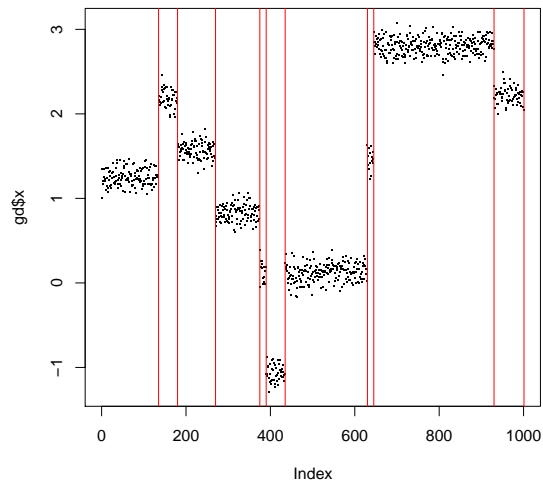


Figure 1: A simulated data example with 10 change points, shown with red vertical lines

The result is shown in Figure 1. We can use the function *findSegments* to reconstruct the changepoints from the x data alone.

```

> library(tilingArray)

```

```

> library(tilingArray)
> maxk = 500
> maxcp = 12
> seg = findSegments(gd$x, maxk = maxk, maxcp = maxcp)

Assessing arguments...
Computing cost matrix for segmentation...
Running Picard's segmentation algorithm...

> seg

$J
[1]          -Inf -0.00723881  1.77873685  2.01904877  2.43848184  2.72562236
[7]  3.07738266  3.59285445  4.11671549  4.60142171  4.61195439  4.62227622

$th
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
[1,] 1001    0    0    0    0    0    0    0    0    0    0    0
[2,]  501 1001    0    0    0    0    0    0    0    0    0    0
[3,]  375  630 1001    0    0    0    0    0    0    0    0    0
[4,]  375  435  630 1001    0    0    0    0    0    0    0    0
[5,]  270  390  435  630 1001    0    0    0    0    0    0    0
[6,]  270  390  435  630  645 1001    0    0    0    0    0    0
[7,]  270  390  435  630  645  930 1001    0    0    0    0    0
[8,]  135  270  390  435  630  645  930 1001    0    0    0    0
[9,]  135  180  270  390  435  630  645  930 1001    0    0    0
[10,] 135  180  270  375  390  435  630  645  930 1001    0    0
[11,] 135  180  270  375  390  435  532  630  645  930 1001    0
[12,] 135  180  270  375  390  435  441  476  630  645  930 1001

$dat
[1]  1.350387069  1.006429699  1.169911863  1.210065320  1.250245747
[6]  1.346140881  1.232914154  1.211613210  1.209731416  1.356134682
[11]  1.100545691  1.148856138  1.242704810  1.074938969  1.117124794
[16]  1.355234723  1.213109332  1.317901212  1.147521802  1.267476776
[21]  1.318493672  1.122059596  1.159125242  1.236859600  1.297420243
[26]  1.418051516  1.309806873  1.310617774  1.258752203  1.182837280
[31]  1.191722622  1.167200081  1.203486282  1.264500880  1.360757630
[36]  1.419737910  1.458658312  1.217523928  1.166270677  1.119781238
[41]  1.276589208  1.245373473  1.453095398  1.171207005  1.141073827

```

[46]	1.215983246	1.306367845	1.268363928	1.236796852	1.088809725
[51]	1.293327224	1.385529277	1.360158281	1.443935317	1.079049719
[56]	1.265300895	1.220740035	1.410546341	1.408471162	1.234073017
[61]	1.227686339	1.246799620	1.220875899	1.196505290	1.468639063
[66]	1.317536099	1.287479570	1.080940784	1.284115335	1.266985471
[71]	1.163648725	1.211503330	1.148158670	1.401723085	1.155441622
[76]	1.414873093	1.213933753	1.261240107	1.300611855	1.126962852
[81]	1.061208175	1.225644388	1.241958243	1.341128441	1.321329304
[86]	1.224054520	1.324908640	1.217118027	1.269436568	1.231416983
[91]	1.197136125	1.354560591	1.322482877	1.201727740	1.351208948
[96]	1.107588773	1.144859829	1.353899358	1.175635730	1.201293299
[101]	1.209292455	1.168629994	1.372734694	1.438369245	1.234108084
[106]	1.451394439	1.355760638	1.053737038	1.282515851	1.167627628
[111]	1.236866806	1.331697200	1.242814715	1.369323407	1.167775831
[116]	1.160990438	1.171073525	1.405534287	1.174800264	1.099336451
[121]	1.223041492	1.244457075	1.244736243	1.394749986	1.134267919
[126]	1.381995290	1.245576225	1.277628894	1.175924995	1.377653948
[131]	1.254637037	1.280185267	1.250593573	1.262369707	2.169563367
[136]	2.151054787	2.216028303	2.103637576	2.144452374	2.129050721
[141]	2.205522532	2.312807131	2.279721270	2.455118592	2.259086744
[146]	2.140373476	2.289932784	2.344201411	2.157612522	2.309578339
[151]	2.085158139	2.064474045	2.323781211	2.180413525	2.314682478
[156]	2.207366148	2.148208129	2.109875035	2.193319770	2.178124115
[161]	1.989118603	2.093876563	1.963738326	2.105848793	2.312363522
[166]	2.173134333	2.283763270	2.169806874	2.218990873	2.286091218
[171]	2.002163905	2.228818719	2.060074944	1.960795219	2.254899990
[176]	2.255550436	2.156103342	2.191380621	2.134202481	1.638623185
[181]	1.606764686	1.633445493	1.455004817	1.606579310	1.549920014
[186]	1.500050776	1.587311286	1.555356658	1.635334734	1.601174256
[191]	1.767791179	1.455798201	1.581489225	1.646953395	1.669212493
[196]	1.456091796	1.617301678	1.538849638	1.570535197	1.635108329
[201]	1.655705630	1.549135897	1.519741465	1.601084627	1.414011554
[206]	1.544589689	1.528045871	1.578844287	1.506297574	1.488331921
[211]	1.521226498	1.583467810	1.630248244	1.640770684	1.675857450
[216]	1.788790845	1.422293164	1.486543926	1.604854111	1.620187347
[221]	1.698748741	1.684697171	1.682535084	1.534909137	1.481545463
[226]	1.511616655	1.624417123	1.459800041	1.298747268	1.644254699
[231]	1.469081640	1.448681457	1.549917937	1.732680314	1.442253793
[236]	1.702647674	1.625642386	1.693356708	1.655885962	1.553251795
[241]	1.563612888	1.473508577	1.346365077	1.663398768	1.823051307

[246]	1.550330995	1.540998067	1.731158686	1.462078818	1.524013804
[251]	1.636912521	1.483630859	1.584089210	1.651086476	1.605930616
[256]	1.645239151	1.579314340	1.555583028	1.524036683	1.603813719
[261]	1.385458228	1.487882884	1.665058364	1.446144952	1.616377638
[266]	1.639099956	1.500738365	1.648003341	1.483247053	0.676657075
[271]	0.726307234	0.790724927	0.892996656	0.752414920	0.856825323
[276]	0.899376242	0.756598110	0.825223823	0.775504683	0.715978504
[281]	0.917083497	0.831985974	0.709441757	0.794445557	0.914818789
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[301]	0.775410639	0.705127050	0.750701451	0.848753021	0.840761683
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[321]	0.698079015	0.868548266	0.978010098	0.883242026	1.059718994
[326]	0.879977043	0.847855379	0.880652696	0.694607286	0.644176348
[331]	0.736024018	0.856576146	0.824042900	0.897007794	0.759774900
[336]	0.760891996	0.930800754	1.020940645	0.690868965	0.808717953
[341]	0.859174317	0.816655807	0.918413889	0.986294327	0.727821403
[346]	0.804737388	0.771584072	1.063069067	0.878638844	1.008692529
[351]	1.009994232	0.913888192	0.736901590	0.788571486	0.778069030
[356]	0.817054960	0.873532866	0.878366382	0.636365830	0.899486364
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[366]	0.889459762	0.664871033	0.790103974	0.902946366	0.728136953
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[381]	0.229066750	-0.009151682	0.058320137	0.067362697	0.011272393
[386]	0.188367391	-0.011951659	0.146703048	0.188190050	-1.080362170
[391]	-1.072117449	-0.876361364	-1.101793777	-0.977511763	-0.996417226
[396]	-1.285164181	-1.122489163	-1.170938779	-0.960206904	-1.082801984
[401]	-1.164352358	-1.190876190	-1.012932583	-0.914338524	-1.144855998
[406]	-0.903318636	-1.035978679	-1.143349822	-0.953070329	-1.235575515
[411]	-0.968481205	-1.190287627	-1.000846870	-1.101590443	-1.086529285
[416]	-1.164675810	-1.060947350	-1.206755697	-0.912179648	-1.130922851
[421]	-0.953452919	-1.027917655	-0.906964650	-1.033852266	-1.051505047
[426]	-1.099175618	-1.013369117	-1.112045650	-1.114146017	-1.048750394
[431]	-0.968225900	-1.086708731	-1.163590047	-1.073108465	0.236721413
[436]	0.166352196	0.188713145	0.340080311	0.141739418	0.210704109
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[446]	0.078134908	0.190571839	-0.017065693	0.089129561	-0.137288952
[451]	0.044824520	0.356315712	0.048154129	0.080863754	0.191508454
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[466]	0.188150799	0.132757975	0.085153369	-0.042783972	0.041419287
[471]	0.150679338	0.020605315	0.010421405	0.119707751	-0.048783506
[476]	0.149445756	0.304813365	0.092180167	-0.043689280	0.144590870
[481]	0.198388362	0.152303783	0.136393123	0.012663442	0.139200856
[486]	0.285542423	0.208464938	0.059523648	0.124649420	-0.028812459
[491]	-0.050349157	-0.026527737	0.272145421	0.048095558	0.171609487
[496]	0.081130799	0.201421641	-0.137214133	0.101682181	0.034301805
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[526]	0.091615837	0.143072431	0.217091759	0.130506175	0.141262333
[531]	-0.085318383	0.186924645	0.216922738	0.119121150	0.277071393
[536]	0.179775440	0.102186021	0.015309557	0.297417303	0.206982510
[541]	0.129715776	0.147777177	0.138742058	0.170384257	0.271335863
[546]	0.239290761	0.389344489	0.208812762	0.163078228	-0.034203927
[551]	0.205011698	0.025719100	0.112351177	0.194845642	0.198780387
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[566]	-0.097882447	0.161077523	0.187384245	-0.049532362	0.197468421
[571]	0.253713009	0.160842504	0.304594109	-0.016032149	0.154925409
[576]	0.244605396	0.148816951	0.153662666	0.049189855	0.321047356
[581]	0.010934052	0.112502567	0.072649774	0.168947523	0.258526436
[586]	0.233515294	0.037380192	0.032273404	0.127133293	0.104108346
[591]	0.290511299	0.112526303	0.001708249	0.114708039	0.133760239
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[606]	0.281156726	0.225499470	0.005344421	0.168357962	0.168165562
[611]	0.004646601	0.070452146	0.245223267	0.051900624	0.108458744
[616]	0.130748801	0.046363821	0.007866534	0.206018747	0.230012366
[621]	0.208561240	0.044449296	0.322493726	0.280636955	0.197389715
[626]	0.163763230	0.103041177	0.025629320	0.165672905	1.628325350
[631]	1.498057924	1.390431483	1.350432961	1.592203533	1.534021114
[636]	1.413666640	1.234072534	1.268111959	1.615034247	1.489533410
[641]	1.436709663	1.423936478	1.466062856	1.553349793	2.854911288

[646]	2.968264232	2.798272804	2.848249186	2.711959897	2.916530944
[651]	2.827753849	2.884224655	2.789320550	2.850920865	2.817953920
[656]	2.895917454	2.766457088	2.941253943	2.706534094	2.872265251
[661]	2.880627178	2.674182343	2.758316976	2.747285493	2.988867119
[666]	2.804931916	2.713147200	2.896833386	2.947571928	2.686149580
[671]	2.658645275	2.757553488	2.757836254	2.662465911	2.967755138
[676]	2.597701186	2.619947482	2.822785512	2.843884505	2.716409447
[681]	2.816916943	2.810946929	2.723955016	2.923111498	2.915373365
[686]	2.751592070	2.877924108	2.922961282	2.817467514	2.597401047
[691]	2.765512777	2.664963956	2.750876504	2.932978188	2.822290537
[696]	2.853342565	2.683141556	2.655108774	2.850391031	2.705267555
[701]	3.073459940	2.654968741	2.911319963	2.844329255	2.694057795
[706]	2.803658177	2.859957864	2.948389179	2.780457614	2.868064350
[711]	2.681740816	2.751454647	2.852078214	2.805456906	2.675989518
[716]	2.839326526	2.708222064	2.676064731	2.691125879	2.603143956
[721]	2.814809972	2.762042631	2.931061421	2.916693214	2.751182733
[726]	2.728547850	2.883300517	2.801516049	2.790785524	3.038566336
[731]	2.959193250	2.683446215	2.823061506	2.846741218	2.751804215
[736]	2.817134702	2.859319025	2.758785143	2.907079929	2.857415963
[741]	2.654026281	2.771777136	2.862062757	2.661724307	2.725878009
[746]	2.890053931	2.804878813	2.789983352	2.789417173	2.934786831
[751]	2.779949555	2.913309721	2.798915510	2.769202435	2.741474717
[756]	2.859175266	2.831746293	2.790305509	2.731596424	2.608669497
[761]	2.953629002	2.695983966	2.710353660	2.666955452	2.855568130
[766]	2.974778080	2.789216283	3.017553568	2.837488662	2.687072791
[771]	2.856948751	2.697816935	2.688862848	2.805063223	2.826379974
[776]	2.767268893	2.906466981	2.836429349	2.873461716	2.686938813
[781]	2.868917634	2.879443259	2.774072249	2.884841843	2.758346460
[786]	3.027661645	2.818888411	2.755770265	2.687132379	2.822743635
[791]	2.790629279	2.806648736	2.765208732	2.705822455	2.740598518
[796]	2.951159601	2.830570292	2.830804437	2.826568365	2.695156033
[801]	2.861099113	2.871208849	2.817586049	2.788987413	2.838617369
[806]	2.950860030	2.961725823	2.458079902	2.646710044	2.615674730
[811]	2.866206574	2.908342235	2.924045918	2.604747457	2.780234348
[816]	2.827041206	2.654433023	2.884161171	2.869098890	2.624181862
[821]	2.915018700	2.706960950	2.871257652	2.765099298	2.724030419
[826]	2.818510793	2.692473942	2.878379941	2.957236070	2.885646474
[831]	2.683076166	3.028755383	2.761921571	2.919193800	2.703498514
[836]	2.669060776	2.649645953	2.861431778	2.922968618	2.783142151
[841]	2.817782380	2.731160948	2.802682180	2.843474593	2.806097910

[846]	2.968097477	2.833381076	2.709610225	2.922950011	2.769140968
[851]	2.875855816	2.918342156	2.964006753	2.649783481	2.810009955
[856]	2.678835378	2.738286149	2.679808050	2.847787922	2.816452622
[861]	2.834055181	2.759410724	2.878820673	2.686701482	2.797005989
[866]	2.684330053	2.812223998	2.814010450	2.893430118	2.804669407
[871]	2.775994529	2.870277620	2.616424690	2.884266143	2.859949800
[876]	2.749404814	2.838492070	2.807218311	2.928102443	2.722971625
[881]	2.882326923	2.854936341	2.762134047	2.893095132	2.695803610
[886]	2.823097250	2.739757742	2.763823190	2.769392012	2.711522855
[891]	2.773063715	2.859383369	2.968080861	2.660802744	2.714280690
[896]	2.832495519	2.964530427	2.735111522	2.842658369	2.692804476
[901]	2.855394258	2.900900488	2.714221467	2.923850302	2.880981504
[906]	2.859107100	2.922427933	2.848523538	2.621908433	2.930996679
[911]	2.668126639	2.890412474	2.970267596	2.821683600	2.833643733
[916]	2.959474199	2.789214665	2.870550222	2.778762524	2.814385143
[921]	2.966214784	2.802170824	2.822429627	2.960006750	2.860730629
[926]	2.779182558	2.864632220	2.823777564	2.769645310	2.331795464
[931]	2.193598310	2.071681434	2.177839256	2.227119988	2.293469136
[936]	2.230433363	2.252742059	2.119964304	2.000583472	2.116591805
[941]	2.253356733	2.259342498	2.260302330	2.264208568	2.234572162
[946]	2.304640931	2.239333519	2.193347059	2.195222843	2.146546206
[951]	2.494228911	2.233818578	2.368808608	2.411453531	2.268082138
[956]	2.280988733	2.364406951	2.065863957	2.045455139	2.163631497
[961]	2.248675009	2.299283013	2.126348237	2.140867765	2.150227538
[966]	2.213800181	2.332037983	2.407525394	2.167434598	2.308382712
[971]	2.301143801	2.284316971	2.154381377	2.110777274	2.179830380
[976]	2.175624672	2.195347707	2.367812823	2.223707702	2.215444227
[981]	2.227540265	2.305136431	2.280808741	2.108473282	2.174576436
[986]	2.344338397	2.244354876	2.143889712	2.236625496	2.192291428
[991]	2.128180119	2.262238298	2.227349133	2.092068425	2.259029334
[996]	2.212399628	2.272311468	2.084891847	2.253193239	2.040250922

\$call

findSegments(x = gd\$x, maxcp = maxcp, maxk = maxk)

attr(,"class")

[1] "segmentation" "list"

> gd\$cp

[1] 135 180 270 375 390 435 630 645 930 1001

We see that the 10-th row of the matrix `seg$th` exactly reconstructs the change points `gd$cp` that were used in the simulation.

The parameters `maxcp` and `maxk` are the maximum number of segments and the maximum length per segment. The algorithm finds for each value of k from 1 to `maxcp` the best segmentation under the restriction that no individual segment be longer than `maxk`. In the original paper of Picard et al. [1] and in their software, `maxk` is implicitly set to the number of data points `length(x)`. I have introduced this parameter to reduce the algorithm's complexity. The complexity of Picard's software is `length(x)*length(x)` in memory and `length(x)*length(x)*maxcp` in time, the complexity of the *findSegments* function is `length(x)*maxk` in memory and `length(x)*maxk*maxcp` in time. As I am envisaging applications with `length(x) \approx 105` and `maxk \approx 250`, the difference can be substantial.

2 How to choose the number of segments?

Need to assess goodness of fit (contained in `gd$J`) together with model complexity (i.e. the number of change points). Details will follow ...

3 Some more testing

Here is a little for-loop that generates data using random parameters and checks whether *findSegments* can reconstruct them. The purpose of this is for checking the validity of the code.

```
> set.seed(4711)
> for (i in 1:20) {
+   gd = genData(lenx, nrCP)
+   seg = findSegments(gd$x, maxk = maxk, maxcp = maxcp)
+   stopifnot(seg$th[nrCP, 1:nrCP] == gd$cp)
+ }
```

Assessing arguments...

Computing cost matrix for segmentation...

Running Picard's segmentation algorithm...

Assessing arguments...

Computing cost matrix for segmentation...

Running Picard's segmentation algorithm...

Assessing arguments...

Computing cost matrix for segmentation...

[illegible]

Assessing arguments...
Computing cost matrix for segmentation...
Running Picard's segmentation algorithm...
Assessing arguments...
Computing cost matrix for segmentation...
Running Picard's segmentation algorithm...
Assessing arguments...
Computing cost matrix for segmentation...
Running Picard's segmentation algorithm...
Assessing arguments...
Computing cost matrix for segmentation...
Running Picard's segmentation algorithm...

References

- [1] A statistical approach for CGH microarray data analysis.
Franck Picard, Stephane Robin, Marc Lavielle, Christian Vaisse,
Gilles Celeux, Jean-Jacques Daudin. Rapport de recherche
No. 5139, Mars 2004, *Institut National de Recherche en In-*
formatique et en Automatique (INRIA), ISSN 0249-6399.
http://www.inapg.fr/ens_rech/mathinfo/recherche/mathematique/outil.html