The C'T testsuite with *MuPAD* 1.2.2

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Abstract

In C'T 7 (1995), pp. 194-208, the capabilities of the Windows-versions of Derive, Mathplus, Macsyma, Maple and Mathematica were compared.

The authors Stefan Braun and Harald Häuser (Visual Analysis, München, Fax-No.: +49-89-6258033) designed a testsuite which consists of 38 problems from different areas of mathematics. Derive was able to solve 20 problems, Mathplus could solve 8 problems, Macsyma 33 problems, Maple 36 problems and Mathematica suceeded in 32 problems.

In this note it is demonstrated how *MuPAD* 1.2.2 solves 22 problems of the testsuite and which problems *MuPAD* can't solve.

1 Linear Algebra

Initialize this part by loading the `linalg` library into *MuPAD* :

```
>>   loadlib("linalg"):
```

Problem 1: Compute the invert of the 30×30 Hilbert matrix:

```
>>   MS:= SquareMatrix( 30,Rational ):
     H:= MS( func(1/(i+j-1),i,j) ):

>>   iH:= H^(-1):

>>   iH[30,30];

      53338390370510951379337308012614400
```

Problem 2: Compute the eigenvalues of the matrix

$$\begin{pmatrix} a & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

```
>>   MExpr:= Matrix( ExpressionField(normal) ):
     A:= MExpr( [[a,1,0],[1,1,1],[1,1,2]] );

      +-         -+
      | a, 1, 0 |
      |          |
      | 1, 1, 1 |
      |          |
      | 1, 1, 2 |
      +-         -+

>>   le:= linalg::eigenValues( A ):
```

The following outputs were cutted due to the long results:

```
>>   le[1];


      /                 /    2    3
      |           2     |   a    a
      | - 3 a + a   + 9 | - -- + -- + ...
      \                 \   6    27


             1/2                2        3      4      1/2        \ 1/3 \
            3    (108 a - 90 a  + 32 a  - 5 a  - 81)              |     |
      ... + ------------------------------------------ + 1/2 |     |
                                   18                             /     /


>>   le[2];

      /
      |        2            1/2             1/2           2  1/2
      | 3 a - a  + (- 9 I) 3    + (3 I) a 3    + (- I) a  3    + ...
      \

             1/2                2        3      4      1/2        \ 1/3 \
            3    (108 a - 90 a  + 32 a  - 5 a  - 81)              |     |
      ... + ------------------------------------------ + 1/2 |     |
                                   18                             /     /


>>   le[3];

      /
      |        2          1/2                1/2          2  1/2
      | 3 a - a  + (9 I) 3    + (- 3 I) a 3    + (I) a  3    + ...
      \
```

```
         1/2                2       3      4      1/2         \ 1/3 \
        3    (108 a - 90 a  + 32 a  - 5 a  - 81)              |     |
... + ----------------------------------------- + 1/2 |     |
                           18                                 /     /
```

2 Simplify Expressions

Problem 3: Simplify the expression $\sqrt{9+4\sqrt{5}}$:

```
>>   radsimp( sqrt( 9+4*sqrt(5) ) );

      1/2
     5    + 2
```

Problem 4: Simplify the expression $\frac{\sin(2x)-\sin(x)cos(x)}{cos(x)(1+\tan^2(x))}$:

```
>>   e:= (sin(2*x) - sin(x)*cos(x))/(cos(x)*(1+tan(x)^2)):
     combine( normal(expand(e)), sincos );

     sin(x)   sin(3 x)
     ------ + --------
       4         4
```

Problem 5: Simplify the expression $\frac{e^x-1}{e^{\frac{x}{2}}+1}$:

```
>>   simplify( (exp(x) - 1)/(exp(x/2)+1),exp );

        / x \
     exp| - | - 1
        \ 2 /
```

3 Solve Equations

Problem 6: Solve the equation $x^4 + ax^2 + 1 = 0$:

```
>>   solve( x^4 + a*x^2 + 1 = 0, x);

     --  1/2           2      1/2 1/2      1/2           2      1/2 1/2
     |  2    (- a + (a  - 4)    )         2    (- a + (a  - 4)    )
     |  ---------------------------, - ---------------------------,
     --              2                              2

         1/2           2      1/2 1/2      1/2           2      1/2 1/2 --
```

```
2    (- a - (a  - 4)   )       2    (- a - (a  - 4)   )     |
---------------------------, - ---------------------------  |
             2                              2              --
```

Problem 7: Solve the equation $2\sin(x)+5\cos(x)=3$:

```
>>   solve( 2*sin(x) + 5*cos(x) = 3, x);

      solve(5 cos(x) + 2 sin(x) - 3 = 0, x)
```

MuPAD is unable yet to solve equations containing trigonometric and exp-log functions. Therefore the next test fails too:

Problem 8: Solve the equation $e^{2x}+e^{x}-1=0$:

```
>>   solve(exp(2*x) + exp(x) - 1 = 0, x);

      solve(exp(2*x) + exp(x) - 1 = 0, x);
```

Problem 9: Solve the system

$$\begin{gathered} xy^2+2c+xz=1 \\ xy-x^2+2z=0 \\ xy-z=0 \end{gathered}$$

```
>>   solve( {x*y^2+2*c+x*z=1, x*y-x^2+2*z=0, x*y-z=0}, {x,y,z} );

      -- --             2                                     -- --
      |  |      x      x                         3             |  |
      |  |  y = -, z = --, x = RootOf(18 c + 4 x  - 9, x)  |  |
      -- --     3      3                                      -- --
```

4 Series and Summations

Problem 10: Compute the Taylor series of $\frac{\cosh(x)}{\sinh(x)-\frac{1}{x}}$ at $x=0$:

```
>>   s:= series( cosh(x)/sinh(x)-1/x,x,5 );

      x      3
      - + O(x )
      3

>>   testtype( s,Type::Series( Taylor ) );

      TRUE
```

Problem 11: Compute the Fourier series of x^3 with period 4 in $(2,-2)$:

Unable to do in MuPAD 1.2.2

Problem 12: Compute $\sum_{k=0}^{\infty} \frac{1}{k^2+a^2}$:

```
>>   sum(1/(k^2+a^2), k=1..infinity);

     /
     |         2 1/2                    2 1/2                 2 1/2
     | psi((- a )    + 1) - psi(- (- a )    + 1) + 2 (- a )
     |
     \

            /                2 1/2                   2 1/2
            | - psi(k + (- a )    ) + psi(k - (- a )    )
       limit| ----------------------------------------,
            |                            2 1/2
            \                  2 (- a )

                      \ \
                      | |          2 1/2
       k = infinity | | / (2 (- a )    )
                      | |
                      / /
```

5 Limit Computation and Differentation

Problem 13: Compute $\lim_{u\to\infty}(\operatorname{asinh}(\frac{u+a}{b}) - \operatorname{asinh}(\frac{u-a}{b}))$ $(a > 0, b > 0$):

```
>>   limit( asinh((u+a)/b) - asinh((u-a)/b), u=infinity );

     0
```

Problem 14: Compute $\lim_{x\to 0} \frac{e^{\frac{1}{x^2}}}{e^{\frac{1}{x^2}}+e^{\frac{1}{x^4}}}$:

```
>>   limit( exp(1/x^2)/(exp(1/x^2)+exp(1/x^4)), x=0);

     0
```

Problem 15: Compute the derivative of

$$\frac{-\frac{1}{5}x^2 + \frac{1}{5}x^2\tan(\frac{1}{2}\ln(x))^2 + \frac{4}{5}x^2\tan(\frac{1}{2}\ln(x))}{1+\tan(\frac{1}{2}\ln(x))^2}$$

```
>>   diff(
       (-1/5*x^2 + 1/5*x^2*tan(1/2*ln(x))^2 +
       4/5*x^2*tan(1/2*ln(x))) / (1 + tan(1/2*ln(x))^2
      ,x);

      /                / ln(x) \          / ln(x) \2
      |         8 x tan| ----- |   2 x tan| ----- |
      |   2 x          \   2   /          \   2   /
      | - --- + ---------------- + ----------------- + ...
      |    5           5                   5
      |
      \

          /           2    / ln(x) \    2    / ln(x) \2 \ \
          |    2   4 x  tan| ----- |   x  tan| ----- |  | |
          |   x            \   2   /         \   2   /  | |
      ... | - -- + ----------------- + ---------------- | | /
          \   5            5                   5        / /


         /      / ln(x) \2 /    / ln(x) \2     \2 \
         | x cos| ----- |  | tan| ----- |  + 1 |  |
         \      \   2   /  \    \   2   /      /  /
```

The output was cutted due to the long result.

Problem 16: Compute y'' of

$$\sqrt{x^3 + y(x)^3} + e^{\sin(y(x))}$$

Unable to do in MuPAD 1.2.2

6 Integration

Problem 17: Compute $\int \frac{2x^4+1}{(x^5+x)\sqrt{x^4+1}} dx$:

```
>>   int( (2*x^4 + 1)/((x^5+x)*sqrt(x^4+1)), x);

         /          4                \
         |       2 x  + 1            |
      int| --------------------, x |
         |       5    4     1/2      |
         \ (x + x ) (x  + 1)         /
```

As one can see, the integrator of *MuPAD* is not able yet to handle certain algebraic dependences of functions, but we hope that this will be implemented in a next *MuPAD* release.

Problem 18: Compute $\int e^{-x}\sin(x)\cos(x)dx$:

```
>>   int( exp(-x)*sin(x)*cos(x),x );

        cos(2 x)   sin(2 x)
     - -------- - ---------
        5 exp(x)   10 exp(x)
```

Problem 19: Compute $\int_0^{\infty}\frac{x^3}{e^x+1}dx$:

Unable to do in MuPAD 1.2.2

Problem 20: Compute $\int_0^1 \frac{\ln(x)^2}{\sqrt{1-x^2}}dx$

Unable to do in MuPAD 1.2.2

The integrator of *MuPAD* can not generally compute definite integrations, thus both tests fail.

7 Laplace and Fourier Transformation

MuPAD does not provide a package for Laplace and Fourier transformations, thus the following problems can't be solved with the current version.

Problem 21: Compute the Laplace transformation of $\sin(t)$:

Unable to do in MuPAD 1.2.2

Problem 22: Compute the inverse Laplace transformation of $\sin(t)$:

Unable to do in MuPAD 1.2.2

Problem 23: Compute the Laplace transformation of heaviside$(t-2)\sin(t)$:

Unable to do in MuPAD 1.2.2

Problem 24: Compute the inverse Laplace transformation of heaviside$(t-2)\sin(t)$:

Unable to do in MuPAD 1.2.2

Problem 25: Compute the Fourier transformation of e^{-t^2} :

Unable to do in MuPAD 1.2.2

Problem 26: Compute the inverse Fourier transformation of e^{-t^2} :

Unable to do in MuPAD 1.2.2

8 Differential Equations

Initialize this part by loading the `ode` library into *MuPAD* :

```
>>   loadlib("ode"):
```

Problem 27: Solve the linearly differential equation $x^2y' - xy = x\ln(x)$:

```
>>   solve( ode( x^2*diff(y(x),x)-x*y(x)=x*ln(x),y(x) ) );

     [C1 x - ln(x) - 1]
```

Problem 28: Solve the differential equation $y' - y^2 - 3y + 4 = 0$:

```
>>   solve( ode(diff(y(x),x) - y(x)^2 - 3*y(x) + 4 = 0, y(x)) );

     --              /             ln(y - 1)   ln(y + 4)        \ --
     |  -4, 1, solve| - C2 - x + --------- - --------- = 0, y |  |
     --              \                 5           5            / --
```

Problem 29: Solve the linearly differential equation $y'' - 4y = \sin(x)$:

```
>>   solve( ode( diff(y(x),x$2) - 4*y(x) = sin(x), y(x)) );

     --   sin(x)                              --
     |  - ------ + C3 exp(2 x) + C4 exp(- 2 x)  |
     --     5                                 --
```

Problem 30: Solve the differential equation $y'' - \frac{2}{x}y' + \frac{3}{x^2}y = 2x - 1$:

```
>>   solve( ode(
       diff(y(x),x$2) - 2/x*diff(y(x),x) + 3/x^2*y(x) = 2*x - 1, y(x)
     ) );
```

[]

MuPAD does not yet recognize this special kind of differential equations, that is Euler's equations.

Problem 31: Solve the equation $8y'' + 9y'^4 = 0$:

```
>>   solve( ode( 8*diff(y(x),x$2) + 9*diff(y(x),x)^4 = 0, y(x) ) );

      --          / /          9 y \1/2 /   16 C5   2 y \
      |  C6, solve| | - 2 C5 + --- |    | - ----- + --- | =
      --          \ \           4  /    \    27      3  /

                   \ --
         x + C7, y |  |
                   / --
```

Problem 32: Solve the differential equation

$$4y'''' - 12y''' + 11y'' - 3y' = 4\cos(x)$$

```
>>   solve( ode(
         4*diff(y(x),x$4) - 12*diff(y(x),x$3) +
         11*diff(y(x),x$2) - 3*diff(y(x),x) = 4*cos(x), y(x)
     ) );

      --      14 cos(x)   18 sin(x)                         / 3 x \
      |  C8 - --------- + --------- + C10 exp(x) + C9 exp| --- |
      --          65          65                            \  2  /

                  / x \ --
         + C11 exp| - |  |
                  \ 2 / --
```

Problem 33: Solve the system of differential equations of order 1

$$x' - x + y' + 2y = 1 + e^t$$
$$y' + 2y + z' + z = 2 + e^t$$
$$x' - x + z' + z = 3 + e^t$$

```
>>   solve( ode(
        {diff(x(t),t) - x(t) + diff(y(t),t) + 2*y(t) = 1+exp(t),
        diff(y(t),t) + 2*y(t) + diff(z(t),t) + z(t) = 2+exp(t),
        diff(x(t),t) - x(t) + diff(z(t),t) + z(t) = 3+exp(t)},
     {x(t), y(t), z(t)}) );
```

```
-- --         t exp(t)
|  |  x(t) = -------- + C12 exp(t) - 1,
|  |             2
-- --

       exp(t)     C13           exp(t)    C14        -- --
y(t) = ------ + -------, z(t) = ------ + ------ + 2  |  |
         6          2             4      exp(t)      |  |
                exp(t)                               -- --
```

9 Graphics

Initialize this part by loading the `plotlib` library into *MuPAD* and export all their functions:

```
>>  loadlib("plotlib"): export( plotlib ):
```

Problem 34: Plot the function $\frac{\cosh(x)}{\sinh(x)-\frac{1}{x}}$ for $x, y \in [0, 5]$.

In the current version of *MuPAD* the plot command can not handle singularities, therefore a plot of this function would only be possible for the interval $(0, 5]$.

Problem 35: Plot the implicit function $T(x, y) := xe^{y} - ye^{-x} = 1$ for $x, y \in [-4, 4]$:

```
>>  implicitplot(
        fun(args(1)*exp(args(2))-args(2)*exp(-args(1)) - 1)
    ,-4..4, -4..4, 7 );
```

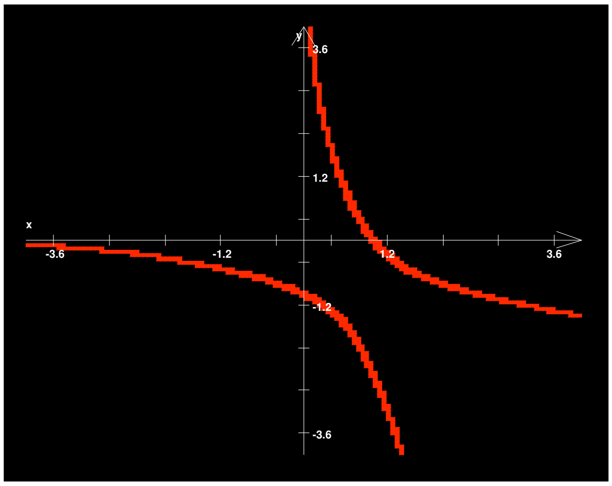

Figure 1: Graph of problem 35

Problem 36: Plot the function $x^2 \cos(y) + \sin(x)$ for $x, y \in [-4, 4]$:

```
>>   plot3d(
         Scaling=UnConstrained,
         [ Mode=Surface,[u, v, u^2*cos(v) + sin(u)],
           u=[-4.0,4.0],v=[-4.0,4.0],
           Color=[Height,[0.996109,0.164050,0.0],
                  [0.992203,0.996109,0.0]],
           Style=[ColorPatches,AndMesh]
         ]
     );
```

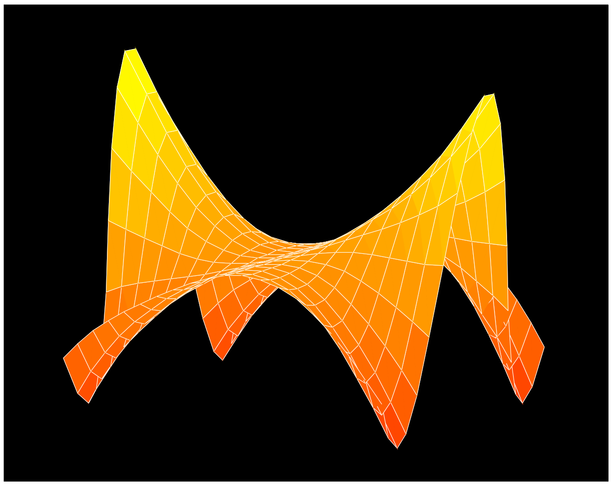

Figure 2: Graph of problem 36

Problem 37: Plot the high surfaces of $z(x, y) := x^2 \cos(y) + \sin(x)$ for $x, y \in [-4, 4]$:

Unable to do in MuPAD 1.2.2

Problem 38: Plot the vector field of $(-y^2, x^2)$ for $x, y \in [-4, 4]$:

```
>>   fieldplot(
         Axes = Origin,
         [ [-y^2, x^2], x = [-4, 4], y = [-4, 4],
           Grid = [30, 30]
         ]
     );
```

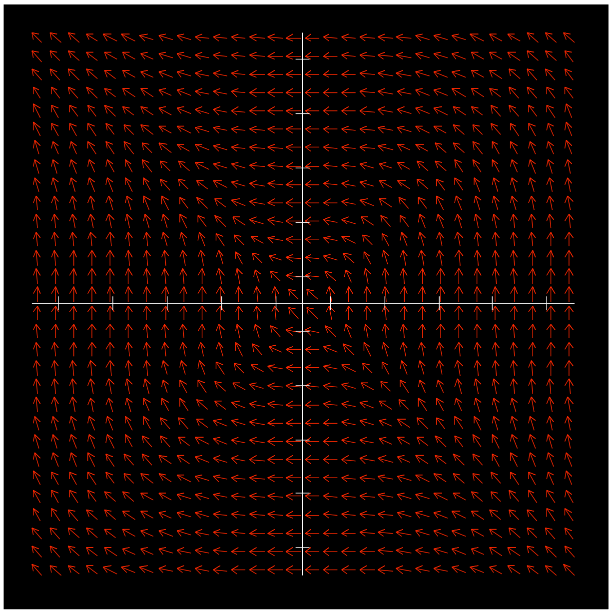

Figure 3: Graph of problem 38