

glucat
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1 Directory Hierarchy	1
1.1 Directories	1
2 Namespace Index	3
2.1 Namespace List	3
3 Hierarchical Index	5
3.1 Class Hierarchy	5
4 Class Index	7
4.1 Class List	7
5 File Index	9
5.1 File List	9
6 Directory Documentation	11
6.1 glucat Directory Reference	11
6.2 pyclical Directory Reference	12
6.3 test Directory Reference	12
7 Namespace Documentation	13
7.1 cga3 Namespace Reference	13
7.1.1 Detailed Description	13
7.1.2 Function Documentation	13
7.1.2.1 agc3()	13
7.1.2.2 cga3()	14
7.1.2.3 cga3std()	14
7.2 glucat Namespace Reference	14
7.2.1 Typedef Documentation	26
7.2.1.1 index_t	26
7.2.1.2 intfn	26
7.2.1.3 intintfn	27
7.2.1.4 set_value_t	27
7.2.1.5 tuning_fast	27
7.2.1.6 tuning_naive	27
7.2.1.7 tuning_slow	28
7.2.2 Function Documentation	28
7.2.2.1 _GLUCAT_CTAssert() [1/3]	28
7.2.2.2 _GLUCAT_CTAssert() [2/3]	28
7.2.2.3 _GLUCAT_CTAssert() [3/3]	28
7.2.2.4 abs()	29
7.2.2.5 acos() [1/2]	29
7.2.2.6 acos() [2/2]	29
7.2.2.7 acosh() [1/2]	30

7.2.2.8 <code>acosh()</code> [2/2]	30
7.2.2.9 <code>approx_equal()</code> [1/2]	30
7.2.2.10 <code>approx_equal()</code> [2/2]	31
7.2.2.11 <code>asin()</code> [1/2]	31
7.2.2.12 <code>asin()</code> [2/2]	31
7.2.2.13 <code>asinh()</code> [1/2]	32
7.2.2.14 <code>asinh()</code> [2/2]	32
7.2.2.15 <code>atan()</code> [1/2]	32
7.2.2.16 <code>atan()</code> [2/2]	33
7.2.2.17 <code>atanh()</code> [1/2]	33
7.2.2.18 <code>atanh()</code> [2/2]	33
7.2.2.19 <code>cascade_log()</code>	34
7.2.2.20 <code>check_complex()</code>	34
7.2.2.21 <code>clifford_exp()</code>	34
7.2.2.22 <code>compare()</code>	35
7.2.2.23 <code>complexifier()</code>	35
7.2.2.24 <code>conj()</code>	35
7.2.2.25 <code>cos()</code> [1/2]	36
7.2.2.26 <code>cos()</code> [2/2]	36
7.2.2.27 <code>cosh()</code>	36
7.2.2.28 <code>cr_sqrt()</code>	37
7.2.2.29 <code>crd_of_mult()</code> [1/2]	37
7.2.2.30 <code>crd_of_mult()</code> [2/2]	37
7.2.2.31 <code>db_sqrt()</code>	37
7.2.2.32 <code>db_step()</code>	38
7.2.2.33 <code>elliptic()</code>	38
7.2.2.34 <code>error_squared()</code>	38
7.2.2.35 <code>error_squared_tol()</code>	39
7.2.2.36 <code>even()</code>	39
7.2.2.37 <code>exp()</code> [1/2]	39
7.2.2.38 <code>exp()</code> [2/2]	39
7.2.2.39 <code>fast()</code>	40
7.2.2.40 <code>folded_dim()</code>	40
7.2.2.41 <code>imag()</code>	40
7.2.2.42 <code>inv()</code>	41
7.2.2.43 <code>inverse_gray()</code>	41
7.2.2.44 <code>inverse_reversed_gray()</code>	41
7.2.2.45 <code>involute()</code>	41
7.2.2.46 <code>log()</code> [1/4]	42
7.2.2.47 <code>log()</code> [2/4]	42
7.2.2.48 <code>log()</code> [3/4]	42
7.2.2.49 <code>log()</code> [4/4]	42

7.2.2.50 <code>log2()</code>	43
7.2.2.51 <code>matrix_log()</code>	43
7.2.2.52 <code>matrix_sqrt()</code>	43
7.2.2.53 <code>max_abs()</code>	44
7.2.2.54 <code>max_pos()</code>	44
7.2.2.55 <code>min_neg()</code>	44
7.2.2.56 <code>norm()</code>	44
7.2.2.57 <code>odd()</code>	45
7.2.2.58 <code>offset_level()</code>	45
7.2.2.59 <code>operator!==()</code> [1/3]	45
7.2.2.60 <code>operator!==()</code> [2/3]	45
7.2.2.61 <code>operator!==()</code> [3/3]	46
7.2.2.62 <code>operator%()</code> [1/3]	46
7.2.2.63 <code>operator%()</code> [2/3]	46
7.2.2.64 <code>operator%()</code> [3/3]	46
7.2.2.65 <code>operator&()</code> [1/4]	47
7.2.2.66 <code>operator&()</code> [2/4]	47
7.2.2.67 <code>operator&()</code> [3/4]	47
7.2.2.68 <code>operator&()</code> [4/4]	47
7.2.2.69 <code>operator*()</code> [1/6]	48
7.2.2.70 <code>operator*()</code> [2/6]	48
7.2.2.71 <code>operator*()</code> [3/6]	48
7.2.2.72 <code>operator*()</code> [4/6]	48
7.2.2.73 <code>operator*()</code> [5/6]	49
7.2.2.74 <code>operator*()</code> [6/6]	49
7.2.2.75 <code>operator+()</code> [1/3]	49
7.2.2.76 <code>operator+()</code> [2/3]	49
7.2.2.77 <code>operator+()</code> [3/3]	50
7.2.2.78 <code>operator-()</code> [1/3]	50
7.2.2.79 <code>operator-()</code> [2/3]	50
7.2.2.80 <code>operator-()</code> [3/3]	50
7.2.2.81 <code>operator/()</code> [1/5]	51
7.2.2.82 <code>operator/()</code> [2/5]	51
7.2.2.83 <code>operator/()</code> [3/5]	51
7.2.2.84 <code>operator/()</code> [4/5]	51
7.2.2.85 <code>operator/()</code> [5/5]	52
7.2.2.86 <code>operator<<()</code> [1/5]	52
7.2.2.87 <code>operator<<()</code> [2/5]	52
7.2.2.88 <code>operator<<()</code> [3/5]	52
7.2.2.89 <code>operator<<()</code> [4/5]	52
7.2.2.90 <code>operator<<()</code> [5/5]	53
7.2.2.91 <code>operator>>()</code> [1/3]	53

7.2.2.92 operator>>() [2/3]	53
7.2.2.93 operator>>() [3/3]	53
7.2.2.94 operator^() [1/4]	54
7.2.2.95 operator^() [2/4]	54
7.2.2.96 operator^() [3/4]	54
7.2.2.97 operator^() [4/4]	54
7.2.2.98 operator" () [1/4]	55
7.2.2.99 operator" () [2/4]	55
7.2.2.100 operator" () [3/4]	55
7.2.2.101 operator" () [4/4]	55
7.2.2.102 outer_pow()	56
7.2.2.103 pade_approx()	56
7.2.2.104 pade_log()	56
7.2.2.105 pos_mod()	57
7.2.2.106 pow() [1/2]	57
7.2.2.107 pow() [2/2]	57
7.2.2.108 pure()	58
7.2.2.109 quad()	58
7.2.2.110 real()	58
7.2.2.111 reframe()	59
7.2.2.112 reverse()	59
7.2.2.113 scalar()	59
7.2.2.114 sign_of_square()	60
7.2.2.115 sin() [1/2]	60
7.2.2.116 sin() [2/2]	60
7.2.2.117 sinh()	60
7.2.2.118 sqrt() [1/4]	61
7.2.2.119 sqrt() [2/4]	61
7.2.2.120 sqrt() [3/4]	61
7.2.2.121 sqrt() [4/4]	61
7.2.2.122 star() [1/3]	62
7.2.2.123 star() [2/3]	62
7.2.2.124 star() [3/3]	62
7.2.2.125 tan() [1/2]	62
7.2.2.126 tan() [2/2]	63
7.2.2.127 tanh()	63
7.2.2.128 to_demote()	63
7.2.2.129 to_promote()	64
7.2.2.130 try_catch() [1/2]	64
7.2.2.131 try_catch() [2/2]	64
7.2.2.132 vector_part()	64
7.2.3 Variable Documentation	65

7.2.3.1 BITS_PER_SET_VALUE	65
7.2.3.2 clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation	65
7.2.3.3 DEFAULT_HI	65
7.2.3.4 l_ln2	65
7.2.3.5 l_pi	65
7.2.3.6 MS_PER_S	66
7.2.3.7 Tuning_Fast_Basis_Max_Count	66
7.2.3.8 Tuning_Fast_CR_Sqrt_Max_Steps	66
7.2.3.9 Tuning_Fast_DB_Sqrt_Max_Steps	66
7.2.3.10 Tuning_Fast_Div_Max_Steps	66
7.2.3.11 Tuning_Fast_Fast_Size_Threshold	66
7.2.3.12 Tuning_Fast_Inv_Fast_Dim_Threshold	66
7.2.3.13 Tuning_Fast_Log_Max_Inner_Steps	67
7.2.3.14 Tuning_Fast_Log_Max_Outer_Steps	67
7.2.3.15 Tuning_Fast_Mult_Matrix_Threshold	67
7.2.3.16 Tuning_Fast_Products_Size_Threshold	67
7.2.3.17 Tuning_Int_Digits	67
7.2.3.18 Tuning_Max_Threshold	67
7.2.3.19 Tuning_Naive_Basis_Max_Count	67
7.2.3.20 Tuning_Naive_Fast_Size_Threshold	67
7.2.3.21 Tuning_Naive_Inv_Fast_Dim_Threshold	68
7.2.3.22 Tuning_Naive_Mult_Matrix_Threshold	68
7.2.3.23 Tuning_Slow_Basis_Max_Count	68
7.2.3.24 Tuning_Slow_Fast_Size_Threshold	68
7.2.3.25 Tuning_Slow_Inv_Fast_Dim_Threshold	68
7.2.3.26 Tuning_Slow_Mult_Matrix_Threshold	68
7.2.3.27 Tuning_Slow_Products_Size_Threshold	68
7.3 glucat::gen Namespace Reference	69
7.3.1 Typedef Documentation	69
7.3.1.1 signature_t	69
7.3.2 Variable Documentation	69
7.3.2.1 offset_to_super	69
7.4 glucat::matrix Namespace Reference	69
7.4.1 Typedef Documentation	71
7.4.1.1 eig_case_t	71
7.4.2 Function Documentation	71
7.4.2.1 classify_eigenvalues()	71
7.4.2.2 eigenvalues()	72
7.4.2.3 inner()	72
7.4.2.4 isinf()	72
7.4.2.5 isnan()	72
7.4.2.6 kron()	73

7.4.2.7 mono_kron()	73
7.4.2.8 mono_prod()	73
7.4.2.9 nnz()	74
7.4.2.10 nork()	74
7.4.2.11 nork_range()	74
7.4.2.12 norm_frob2()	75
7.4.2.13 prod()	75
7.4.2.14 signed_perm_nork()	75
7.4.2.15 sparse_prod()	75
7.4.2.16 to_blaze()	76
7.4.2.17 trace()	76
7.4.2.18 unit()	76
7.5 glucat::timing Namespace Reference	76
7.5.1 Function Documentation	77
7.5.1.1 elapsed()	77
7.5.2 Variable Documentation	77
7.5.2.1 EXTRA_TRIALS	77
7.5.2.2 MS_PER_CLOCK	77
7.5.2.3 MS_PER_SEC	77
7.6 pade Namespace Reference	78
7.6.1 Variable Documentation	79
7.6.1.1 pade_log_denom< dd_real >::denom	79
7.6.1.2 pade_log_denom< float >::denom	79
7.6.1.3 pade_log_denom< longdouble >::denom	79
7.6.1.4 pade_log_denom< qd_real >::denom	80
7.6.1.5 pade_log_denom< Scalar_T >::denom	80
7.6.1.6 pade_log_numer< dd_real >::numer	80
7.6.1.7 pade_log_numer< float >::numer	81
7.6.1.8 pade_log_numer< longdouble >::numer	81
7.6.1.9 pade_log_numer< qd_real >::numer	81
7.6.1.10 pade_log_numer< Scalar_T >::numer	82
7.6.1.11 pade_sqrt_denom< dd_real >::denom	82
7.6.1.12 pade_sqrt_denom< float >::denom	83
7.6.1.13 pade_sqrt_denom< longdouble >::denom	83
7.6.1.14 pade_sqrt_denom< qd_real >::denom	83
7.6.1.15 pade_sqrt_denom< Scalar_T >::denom	84
7.6.1.16 pade_sqrt_numer< dd_real >::numer	84
7.6.1.17 pade_sqrt_numer< float >::numer	84
7.6.1.18 pade_sqrt_numer< longdouble >::numer	84
7.6.1.19 pade_sqrt_numer< qd_real >::numer	85
7.6.1.20 pade_sqrt_numer< Scalar_T >::numer	85
7.7 PyClical Namespace Reference	85

7.7.1 Function Documentation	86
7.7.1.1 <code>_test()</code>	86
7.7.1.2 <code>clifford_hidden_doctests()</code>	86
7.7.1.3 <code>e()</code>	87
7.7.1.4 <code>index_set_hidden_doctests()</code>	88
7.7.1.5 <code>istpq()</code>	89
7.7.2 Variable Documentation	89
7.7.2.1 <code>__version__</code>	89
7.7.2.2 <code>cl</code>	89
7.7.2.3 <code>fill</code>	90
7.7.2.4 <code>i</code>	90
7.7.2.5 <code>ist</code>	90
7.7.2.6 <code>ixt</code>	90
7.7.2.7 <code>lhs</code>	90
7.7.2.8 <code>nbar3</code>	90
7.7.2.9 <code>ninf3</code>	90
7.7.2.10 <code>None</code>	90
7.7.2.11 <code>obj</code>	91
7.7.2.12 <code>pi</code>	91
7.7.2.13 <code>rhs</code>	91
7.7.2.14 <code>scalar_epsilon</code>	91
7.7.2.15 <code>tau</code>	91
7.7.2.16 <code>threshold</code>	91
7.7.2.17 <code>tol</code>	91
7.8 std Namespace Reference	91
8 Class Documentation	93
8.1 <code>glucat::basis_table< Scalar_T, LO, HI, Matrix_T ></code> Class Template Reference	93
8.1.1 Detailed Description	94
8.1.2 Constructor & Destructor Documentation	94
8.1.2.1 <code>basis_table()</code> [1/2]	94
8.1.2.2 <code>~basis_table()</code>	94
8.1.2.3 <code>basis_table()</code> [2/2]	94
8.1.3 Member Function Documentation	95
8.1.3.1 <code>basis()</code>	95
8.1.3.2 <code>operator=()</code>	95
8.1.4 Friends And Related Symbol Documentation	95
8.1.4.1 <code>friend_for_private_destructor</code>	95
8.2 <code>glucat::bool_to_type< truth_value ></code> Class Template Reference	95
8.2.1 Detailed Description	96
8.2.2 Member Enumeration Documentation	96
8.2.2.1 <code>anonymous enum</code>	96

8.3 PyClical.clifford Class Reference	96
8.3.1 Detailed Description	98
8.3.2 Member Function Documentation	98
8.3.2.1 __add__()	98
8.3.2.2 __and__()	99
8.3.2.3 __call__()	99
8.3.2.4 __cinit__()	99
8.3.2.5 __contains__()	100
8.3.2.6 __dealloc__()	100
8.3.2.7 __getitem__()	101
8.3.2.8 __iadd__()	101
8.3.2.9 __iand__()	101
8.3.2.10 __idiv__()	102
8.3.2.11 __imod__()	102
8.3.2.12 __imul__()	102
8.3.2.13 __ior__()	103
8.3.2.14 __isub__()	103
8.3.2.15 __iter__()	103
8.3.2.16 __ixor__()	103
8.3.2.17 __mod__()	104
8.3.2.18 __mul__()	104
8.3.2.19 __neg__()	104
8.3.2.20 __or__()	105
8.3.2.21 __pos__()	105
8.3.2.22 __pow__()	105
8.3.2.23 __radd__()	106
8.3.2.24 __rand__()	106
8.3.2.25 __repr__()	106
8.3.2.26 __richcmp__()	107
8.3.2.27 __rmod__()	107
8.3.2.28 __rmul__()	107
8.3.2.29 __rsub__()	108
8.3.2.30 __rtruediv__()	108
8.3.2.31 __rxor__()	108
8.3.2.32 __str__()	108
8.3.2.33 __sub__()	109
8.3.2.34 __truediv__()	109
8.3.2.35 __xor__()	109
8.3.2.36 abs()	110
8.3.2.37 conj()	110
8.3.2.38 even()	110
8.3.2.39 frame()	111

8.3.2.40	inv()	111
8.3.2.41	involute()	111
8.3.2.42	isinf()	112
8.3.2.43	isnan()	112
8.3.2.44	max_abs()	112
8.3.2.45	norm()	113
8.3.2.46	odd()	113
8.3.2.47	outer_pow()	113
8.3.2.48	pow()	114
8.3.2.49	pure()	114
8.3.2.50	quad()	114
8.3.2.51	reframe()	115
8.3.2.52	reverse()	115
8.3.2.53	scalar()	115
8.3.2.54	truncated()	116
8.3.2.55	vector_part()	116
8.3.3	Member Data Documentation	116
8.3.3.1	instance	116
8.4	glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T > Class Template Reference	117
8.4.1	Detailed Description	119
8.4.2	Member Typedef Documentation	119
8.4.2.1	index_set_t	119
8.4.2.2	multivector_t	120
8.4.2.3	pair_t	120
8.4.2.4	scalar_t	120
8.4.2.5	vector_t	120
8.4.3	Constructor & Destructor Documentation	120
8.4.3.1	~clifford_algebra()	120
8.4.4	Member Function Documentation	120
8.4.4.1	classname()	120
8.4.4.2	conj()	121
8.4.4.3	even()	121
8.4.4.4	frame()	121
8.4.4.5	grade()	121
8.4.4.6	inv()	122
8.4.4.7	involute()	122
8.4.4.8	isinf()	122
8.4.4.9	isnan()	122
8.4.4.10	max_abs()	123
8.4.4.11	norm()	123
8.4.4.12	odd()	123
8.4.4.13	operator%=()	123

8.4.4.14	operator&=()	123
8.4.4.15	operator()()	124
8.4.4.16	operator*=() [1/2]	124
8.4.4.17	operator*=() [2/2]	124
8.4.4.18	operator+=() [1/2]	124
8.4.4.19	operator+=() [2/2]	124
8.4.4.20	operator-()	124
8.4.4.21	operator-=() [1/2]	125
8.4.4.22	operator-=() [2/2]	125
8.4.4.23	operator/=() [1/2]	125
8.4.4.24	operator/=() [2/2]	125
8.4.4.25	operator==() [1/2]	125
8.4.4.26	operator==() [2/2]	125
8.4.4.27	operator[]()	126
8.4.4.28	operator^=()	126
8.4.4.29	operator" =()	126
8.4.4.30	outer_pow()	126
8.4.4.31	pow()	126
8.4.4.32	pure()	127
8.4.4.33	quad()	127
8.4.4.34	reverse()	127
8.4.4.35	scalar()	127
8.4.4.36	truncated()	128
8.4.4.37	vector_part() [1/2]	128
8.4.4.38	vector_part() [2/2]	128
8.4.4.39	write() [1/2]	128
8.4.4.40	write() [2/2]	129
8.4.5	Member Data Documentation	129
8.4.5.1	default_truncation	129
8.4.5.2	v_hi	129
8.4.5.3	v_lo	129
8.5	glucat::compare_types< LHS_T, RHS_T > Class Template Reference	129
8.5.1	Detailed Description	130
8.5.2	Member Enumeration Documentation	130
8.5.2.1	anonymous enum	130
8.6	glucat::compare_types< T, T > Class Template Reference	130
8.6.1	Detailed Description	130
8.6.2	Member Enumeration Documentation	131
8.6.2.1	anonymous enum	131
8.6.2.2	anonymous enum	131
8.7	glucat::control_t Class Reference	131
8.7.1	Detailed Description	132

8.7.2 Constructor & Destructor Documentation	132
8.7.2.1 control_t() [1/3]	132
8.7.2.2 control_t() [2/3]	133
8.7.2.3 ~control_t()	133
8.7.2.4 control_t() [3/3]	133
8.7.3 Member Function Documentation	133
8.7.3.1 call() [1/2]	133
8.7.3.2 call() [2/2]	133
8.7.3.3 catch_exceptions()	133
8.7.3.4 control()	134
8.7.3.5 operator=()	134
8.7.3.6 valid()	134
8.7.3.7 verbose()	134
8.7.4 Friends And Related Symbol Documentation	134
8.7.4.1 friend_for_private_destructor	134
8.7.5 Member Data Documentation	135
8.7.5.1 m_catch_exceptions	135
8.7.5.2 m_valid	135
8.7.5.3 m_verbose_output	135
8.8 glucat::CTAssertion< bool > Struct Template Reference	135
8.8.1 Detailed Description	135
8.9 glucat::CTAssertion< true > Struct Reference	136
8.9.1 Detailed Description	136
8.10 glucat::numeric_traits< Scalar_T >::demoted Struct Reference	136
8.10.1 Detailed Description	136
8.10.2 Member Typedef Documentation	136
8.10.2.1 type [1/2]	136
8.10.2.2 type [2/2]	137
8.11 glucat::matrix::eig_genus< Matrix_T > Struct Template Reference	137
8.11.1 Detailed Description	137
8.11.2 Member Typedef Documentation	137
8.11.2.1 Scalar_T	137
8.11.3 Member Data Documentation	138
8.11.3.1 m_eig_case	138
8.11.3.2 m_is_singular	138
8.11.3.3 m_safe_arg	138
8.12 glucat::error< Class_T > Class Template Reference	139
8.12.1 Detailed Description	140
8.12.2 Constructor & Destructor Documentation	140
8.12.2.1 error() [1/2]	140
8.12.2.2 error() [2/2]	140
8.12.3 Member Function Documentation	141

8.12.3.1	classname()	141
8.12.3.2	heading()	141
8.12.3.3	print_error_msg()	141
8.13	glucat::framed_multi< Scalar_T, LO, HI, Tune_P > Class Template Reference	142
8.13.1	Detailed Description	147
8.13.2	Member Typedef Documentation	147
8.13.2.1	const_iterator	147
8.13.2.2	error_t	147
8.13.2.3	framed_multi_t	147
8.13.2.4	framed_pair_t	148
8.13.2.5	index_set_t	148
8.13.2.6	iterator	148
8.13.2.7	map_t	148
8.13.2.8	matrix_multi_t	148
8.13.2.9	matrix_t	148
8.13.2.10	multivector_t	149
8.13.2.11	scalar_t	149
8.13.2.12	size_type	149
8.13.2.13	sorted_map_t	149
8.13.2.14	term_t	149
8.13.2.15	tune_p	149
8.13.2.16	var_term_t	150
8.13.2.17	vector_t	150
8.13.3	Constructor & Destructor Documentation	150
8.13.3.1	~framed_multi()	150
8.13.3.2	framed_multi() [1/15]	150
8.13.3.3	framed_multi() [2/15]	150
8.13.3.4	framed_multi() [3/15]	151
8.13.3.5	framed_multi() [4/15]	151
8.13.3.6	framed_multi() [5/15]	151
8.13.3.7	framed_multi() [6/15]	151
8.13.3.8	framed_multi() [7/15]	152
8.13.3.9	framed_multi() [8/15]	152
8.13.3.10	framed_multi() [9/15]	152
8.13.3.11	framed_multi() [10/15]	152
8.13.3.12	framed_multi() [11/15]	153
8.13.3.13	framed_multi() [12/15]	153
8.13.3.14	framed_multi() [13/15]	153
8.13.3.15	framed_multi() [14/15]	153
8.13.3.16	framed_multi() [15/15]	154
8.13.4	Member Function Documentation	154
8.13.4.1	centre_pm4_qp4()	154

8.13.4.2 centre_pp4_qm4()	154
8.13.4.3 centre_qp1_pm1()	155
8.13.4.4 classname()	155
8.13.4.5 divide()	155
8.13.4.6 fast()	155
8.13.4.7 fast_framed_multi()	156
8.13.4.8 fast_matrix_multi()	156
8.13.4.9 fold()	156
8.13.4.10 nbr_terms()	156
8.13.4.11 operator+=()	157
8.13.4.12 random()	157
8.13.4.13 unfold()	157
8.13.5 Friends And Related Symbol Documentation	157
8.13.5.1 exp	157
8.13.5.2 framed_multi	158
8.13.5.3 matrix_multi	158
8.13.5.4 operator%	158
8.13.5.5 operator&	158
8.13.5.6 operator*	158
8.13.5.7 operator/	159
8.13.5.8 operator<< [1/2]	159
8.13.5.9 operator<< [2/2]	159
8.13.5.10 operator>>	159
8.13.5.11 operator^	159
8.13.5.12 operator"	159
8.13.5.13 star	160
8.14 glucat::gen::generator_table< Matrix_T > Class Template Reference	160
8.14.1 Detailed Description	161
8.14.2 Constructor & Destructor Documentation	161
8.14.2.1 generator_table() [1/2]	161
8.14.2.2 ~generator_table()	162
8.14.2.3 generator_table() [2/2]	162
8.14.3 Member Function Documentation	162
8.14.3.1 gen_from_pm1_qm1()	162
8.14.3.2 gen_from_pm4_qp4()	162
8.14.3.3 gen_from_pp4_qm4()	163
8.14.3.4 gen_from_qp1_pm1()	163
8.14.3.5 gen_vector()	163
8.14.3.6 generator()	163
8.14.3.7 operator()()	164
8.14.3.8 operator=()	164
8.14.4 Friends And Related Symbol Documentation	164

8.14.4.1 friend_for_private_destructor	164
8.15 glucat::glucat_error Class Reference	165
8.15.1 Detailed Description	166
8.15.2 Constructor & Destructor Documentation	166
8.15.2.1 glucat_error()	166
8.15.2.2 ~glucat_error()	166
8.15.3 Member Function Documentation	166
8.15.3.1 classname()	166
8.15.3.2 heading()	166
8.15.3.3 print_error_msg()	167
8.15.4 Member Data Documentation	167
8.15.4.1 name	167
8.16 glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t Class Reference	167
8.16.1 Detailed Description	167
8.16.2 Constructor & Destructor Documentation	168
8.16.2.1 hash_size_t()	168
8.16.3 Member Function Documentation	168
8.16.3.1 operator()()	168
8.16.4 Member Data Documentation	168
8.16.4.1 n	168
8.17 glucat::index_set< LO, HI > Class Template Reference	169
8.17.1 Detailed Description	172
8.17.2 Member Typedef Documentation	172
8.17.2.1 bitset_t	172
8.17.2.2 error_t	172
8.17.2.3 index_pair_t	172
8.17.2.4 index_set_t	172
8.17.3 Constructor & Destructor Documentation	173
8.17.3.1 index_set() [1/6]	173
8.17.3.2 index_set() [2/6]	173
8.17.3.3 index_set() [3/6]	173
8.17.3.4 index_set() [4/6]	173
8.17.3.5 index_set() [5/6]	174
8.17.3.6 index_set() [6/6]	174
8.17.4 Member Function Documentation	174
8.17.4.1 BOOST_STATIC_ASSERT()	174
8.17.4.2 classname()	174
8.17.4.3 count()	174
8.17.4.4 count_neg()	175
8.17.4.5 count_pos()	175
8.17.4.6 flip() [1/2]	175
8.17.4.7 flip() [2/2]	175

8.17.4.8 fold() [1/2]	175
8.17.4.9 fold() [2/2]	176
8.17.4.10 hash_fn()	176
8.17.4.11 is_contiguous()	176
8.17.4.12 lex_less_than()	176
8.17.4.13 max()	176
8.17.4.14 min()	177
8.17.4.15 operator!=()	177
8.17.4.16 operator&=()	177
8.17.4.17 operator<()	177
8.17.4.18 operator==()	178
8.17.4.19 operator[]() [1/2]	178
8.17.4.20 operator[]() [2/2]	178
8.17.4.21 operator^=()	178
8.17.4.22 operator" =()	178
8.17.4.23 operator~()	179
8.17.4.24 reset() [1/2]	179
8.17.4.25 reset() [2/2]	179
8.17.4.26 set() [1/3]	179
8.17.4.27 set() [2/3]	179
8.17.4.28 set() [3/3]	180
8.17.4.29 sign_of_mult()	180
8.17.4.30 sign_of_square()	180
8.17.4.31 test()	180
8.17.4.32 unfold()	181
8.17.4.33 value_of_fold()	181
8.17.5 Friends And Related Symbol Documentation	181
8.17.5.1 compare	181
8.17.5.2 operator&	181
8.17.5.3 operator^	181
8.17.5.4 operator"	182
8.17.5.5 reference	182
8.17.6 Member Data Documentation	182
8.17.6.1 v_hi	182
8.17.6.2 v_lo	182
8.18 PyClical.index_set Class Reference	183
8.18.1 Detailed Description	184
8.18.2 Member Function Documentation	184
8.18.2.1 __and__()	184
8.18.2.2 __cinit__()	184
8.18.2.3 __contains__()	185
8.18.2.4 __dealloc__()	185

8.18.2.5	__getitem__()	185
8.18.2.6	__iand__()	186
8.18.2.7	__invert__()	186
8.18.2.8	__ior__()	186
8.18.2.9	__iter__()	186
8.18.2.10	__ixor__()	187
8.18.2.11	__or__()	187
8.18.2.12	__repr__()	187
8.18.2.13	__richcmp__()	188
8.18.2.14	__setitem__()	188
8.18.2.15	__str__()	188
8.18.2.16	__xor__()	189
8.18.2.17	count()	189
8.18.2.18	count_neg()	189
8.18.2.19	count_pos()	190
8.18.2.20	hash_fn()	190
8.18.2.21	max()	190
8.18.2.22	min()	191
8.18.2.23	sign_of_mult()	191
8.18.2.24	sign_of_square()	191
8.18.3	Member Data Documentation	192
8.18.3.1	instance	192
8.19	glucat::index_set_hash< LO, HI > Class Template Reference	192
8.19.1	Detailed Description	192
8.19.2	Member Typedef Documentation	192
8.19.2.1	index_set_t	192
8.19.3	Member Function Documentation	193
8.19.3.1	operator()()	193
8.20	glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > Class Template Reference	193
8.20.1	Detailed Description	198
8.20.2	Member Typedef Documentation	198
8.20.2.1	basis_matrix_t	198
8.20.2.2	error_t	198
8.20.2.3	framed_multi_t	198
8.20.2.4	index_set_t	199
8.20.2.5	matrix_index_t	199
8.20.2.6	matrix_multi_t	199
8.20.2.7	matrix_t	199
8.20.2.8	multivector_t	199
8.20.2.9	orientation_t	199
8.20.2.10	scalar_t	200
8.20.2.11	term_t	200

8.20.2.12 tune_p	200
8.20.2.13 vector_t	200
8.20.3 Constructor & Destructor Documentation	200
8.20.3.1 ~matrix_multi()	200
8.20.3.2 matrix_multi() [1/17]	200
8.20.3.3 matrix_multi() [2/17]	201
8.20.3.4 matrix_multi() [3/17]	201
8.20.3.5 matrix_multi() [4/17]	201
8.20.3.6 matrix_multi() [5/17]	201
8.20.3.7 matrix_multi() [6/17]	202
8.20.3.8 matrix_multi() [7/17]	202
8.20.3.9 matrix_multi() [8/17]	202
8.20.3.10 matrix_multi() [9/17]	202
8.20.3.11 matrix_multi() [10/17]	203
8.20.3.12 matrix_multi() [11/17]	203
8.20.3.13 matrix_multi() [12/17]	203
8.20.3.14 matrix_multi() [13/17]	203
8.20.3.15 matrix_multi() [14/17]	204
8.20.3.16 matrix_multi() [15/17]	204
8.20.3.17 matrix_multi() [16/17]	204
8.20.3.18 matrix_multi() [17/17]	204
8.20.4 Member Function Documentation	205
8.20.4.1 basis_element()	205
8.20.4.2 classname()	205
8.20.4.3 fast_framed_multi()	205
8.20.4.4 fast_matrix_multi()	205
8.20.4.5 operator+=()	206
8.20.4.6 operator=()	206
8.20.4.7 random()	206
8.20.5 Friends And Related Symbol Documentation	206
8.20.5.1 framed_multi	206
8.20.5.2 matrix_log	207
8.20.5.3 matrix_multi	207
8.20.5.4 matrix_sqrt	207
8.20.5.5 operator%	207
8.20.5.6 operator&	207
8.20.5.7 operator*	208
8.20.5.8 operator/	208
8.20.5.9 operator<<	208
8.20.5.10 operator>>	208
8.20.5.11 operator^	208
8.20.5.12 operator" 	208

8.20.5.13 reframe	209
8.20.5.14 star	209
8.20.6 Member Data Documentation	209
8.20.6.1 m_frame	209
8.20.6.2 m_matrix	209
8.21 std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > > Struct Template Reference	210
8.21.1 Detailed Description	210
8.22 std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > > Struct Template Reference	211
8.22.1 Detailed Description	211
8.23 glucat::numeric_traits< Scalar_T > Class Template Reference	212
8.23.1 Detailed Description	214
8.23.2 Member Function Documentation	214
8.23.2.1 abs()	214
8.23.2.2 acos()	214
8.23.2.3 asin()	214
8.23.2.4 atan()	215
8.23.2.5 conj()	215
8.23.2.6 cos()	215
8.23.2.7 cosh()	215
8.23.2.8 exp()	215
8.23.2.9 fmod()	216
8.23.2.10 imag()	216
8.23.2.11 isInf() [1/3]	216
8.23.2.12 isInf() [2/3]	216
8.23.2.13 isInf() [3/3]	217
8.23.2.14 isNaN() [1/3]	217
8.23.2.15 isNaN() [2/3]	217
8.23.2.16 isNaN() [3/3]	217
8.23.2.17 isNaN_or_isInf()	218
8.23.2.18 ln_2() [1/2]	218
8.23.2.19 ln_2() [2/2]	218
8.23.2.20 log()	218
8.23.2.21 log2()	219
8.23.2.22 NaN()	219
8.23.2.23 pi() [1/2]	219
8.23.2.24 pi() [2/2]	219
8.23.2.25 pow()	220
8.23.2.26 real()	220
8.23.2.27 sin()	220
8.23.2.28 sinh()	220
8.23.2.29 sqrt()	220
8.23.2.30 tan()	221

8.23.2.31 <code>tanh()</code>	221
8.23.2.32 <code>to_double()</code>	221
8.23.2.33 <code>to_int()</code>	221
8.23.2.34 <code>to_scalar_t()</code> [1/9]	221
8.23.2.35 <code>to_scalar_t()</code> [2/9]	222
8.23.2.36 <code>to_scalar_t()</code> [3/9]	222
8.23.2.37 <code>to_scalar_t()</code> [4/9]	222
8.23.2.38 <code>to_scalar_t()</code> [5/9]	222
8.23.2.39 <code>to_scalar_t()</code> [6/9]	222
8.23.2.40 <code>to_scalar_t()</code> [7/9]	223
8.23.2.41 <code>to_scalar_t()</code> [8/9]	223
8.23.2.42 <code>to_scalar_t()</code> [9/9]	223
8.24 <code>pade::pade_log_denom< Scalar_T ></code> Struct Template Reference	223
8.24.1 Detailed Description	224
8.24.2 Member Typedef Documentation	224
8.24.2.1 <code>array</code>	224
8.24.3 Member Data Documentation	224
8.24.3.1 <code>denom</code>	224
8.25 <code>pade::pade_log_denom< dd_real ></code> Struct Reference	224
8.25.1 Detailed Description	225
8.25.2 Member Typedef Documentation	225
8.25.2.1 <code>array</code>	225
8.25.3 Member Data Documentation	225
8.25.3.1 <code>denom</code>	225
8.26 <code>pade::pade_log_denom< float ></code> Struct Reference	225
8.26.1 Detailed Description	225
8.26.2 Member Typedef Documentation	226
8.26.2.1 <code>array</code>	226
8.26.3 Member Data Documentation	226
8.26.3.1 <code>denom</code>	226
8.27 <code>pade::pade_log_denom< long double ></code> Struct Reference	226
8.27.1 Detailed Description	226
8.27.2 Member Typedef Documentation	226
8.27.2.1 <code>array</code>	226
8.27.3 Member Data Documentation	227
8.27.3.1 <code>denom</code>	227
8.28 <code>pade::pade_log_denom< qd_real ></code> Struct Reference	227
8.28.1 Detailed Description	227
8.28.2 Member Typedef Documentation	227
8.28.2.1 <code>array</code>	227
8.28.3 Member Data Documentation	227
8.28.3.1 <code>denom</code>	227

8.29 pade::pade_log_numer< Scalar_T > Struct Template Reference	228
8.29.1 Detailed Description	228
8.29.2 Member Typedef Documentation	228
8.29.2.1 array	228
8.29.3 Member Data Documentation	228
8.29.3.1 numer	228
8.30 pade::pade_log_numer< dd_real > Struct Reference	229
8.30.1 Detailed Description	229
8.30.2 Member Typedef Documentation	229
8.30.2.1 array	229
8.30.3 Member Data Documentation	229
8.30.3.1 numer	229
8.31 pade::pade_log_numer< float > Struct Reference	229
8.31.1 Detailed Description	230
8.31.2 Member Typedef Documentation	230
8.31.2.1 array	230
8.31.3 Member Data Documentation	230
8.31.3.1 numer	230
8.32 pade::pade_log_numer< long double > Struct Reference	230
8.32.1 Detailed Description	230
8.32.2 Member Typedef Documentation	231
8.32.2.1 array	231
8.32.3 Member Data Documentation	231
8.32.3.1 numer	231
8.33 pade::pade_log_numer< qd_real > Struct Reference	231
8.33.1 Detailed Description	231
8.33.2 Member Typedef Documentation	231
8.33.2.1 array	231
8.33.3 Member Data Documentation	232
8.33.3.1 numer	232
8.34 pade::pade_sqrt_denom< Scalar_T > Struct Template Reference	232
8.34.1 Detailed Description	232
8.34.2 Member Typedef Documentation	232
8.34.2.1 array	232
8.34.3 Member Data Documentation	233
8.34.3.1 denom	233
8.35 pade::pade_sqrt_denom< dd_real > Struct Reference	233
8.35.1 Detailed Description	233
8.35.2 Member Typedef Documentation	233
8.35.2.1 array	233
8.35.3 Member Data Documentation	233
8.35.3.1 denom	233

8.36 pade::pade_sqrt_denom< float > Struct Reference	234
8.36.1 Detailed Description	234
8.36.2 Member Typedef Documentation	234
8.36.2.1 array	234
8.36.3 Member Data Documentation	234
8.36.3.1 denom	234
8.37 pade::pade_sqrt_denom< long double > Struct Reference	234
8.37.1 Detailed Description	235
8.37.2 Member Typedef Documentation	235
8.37.2.1 array	235
8.37.3 Member Data Documentation	235
8.37.3.1 denom	235
8.38 pade::pade_sqrt_denom< qd_real > Struct Reference	235
8.38.1 Detailed Description	235
8.38.2 Member Typedef Documentation	236
8.38.2.1 array	236
8.38.3 Member Data Documentation	236
8.38.3.1 denom	236
8.39 pade::pade_sqrt_numer< Scalar_T > Struct Template Reference	236
8.39.1 Detailed Description	236
8.39.2 Member Typedef Documentation	237
8.39.2.1 array	237
8.39.3 Member Data Documentation	237
8.39.3.1 numer	237
8.40 pade::pade_sqrt_numer< dd_real > Struct Reference	237
8.40.1 Detailed Description	237
8.40.2 Member Typedef Documentation	237
8.40.2.1 array	237
8.40.3 Member Data Documentation	238
8.40.3.1 numer	238
8.41 pade::pade_sqrt_numer< float > Struct Reference	238
8.41.1 Detailed Description	238
8.41.2 Member Typedef Documentation	238
8.41.2.1 array	238
8.41.3 Member Data Documentation	238
8.41.3.1 numer	238
8.42 pade::pade_sqrt_numer< long double > Struct Reference	239
8.42.1 Detailed Description	239
8.42.2 Member Typedef Documentation	239
8.42.2.1 array	239
8.42.3 Member Data Documentation	239
8.42.3.1 numer	239

8.43 pade::pade_sqrt_numer< qd_real > Struct Reference	239
8.43.1 Detailed Description	240
8.43.2 Member Typedef Documentation	240
8.43.2.1 array	240
8.43.3 Member Data Documentation	240
8.43.3.1 numer	240
8.44 glucat::numeric_traits< Scalar_T >::promoted Struct Reference	240
8.44.1 Detailed Description	241
8.44.2 Member Typedef Documentation	241
8.44.2.1 type [1/3]	241
8.44.2.2 type [2/3]	241
8.44.2.3 type [3/3]	241
8.45 glucat::random_generator< Scalar_T > Class Template Reference	241
8.45.1 Detailed Description	242
8.45.2 Constructor & Destructor Documentation	242
8.45.2.1 random_generator() [1/2]	242
8.45.2.2 random_generator() [2/2]	243
8.45.2.3 ~random_generator()	243
8.45.3 Member Function Documentation	243
8.45.3.1 generator()	243
8.45.3.2 normal()	243
8.45.3.3 operator=()	243
8.45.3.4 uniform()	243
8.45.4 Friends And Related Symbol Documentation	244
8.45.4.1 friend_for_private_destructor	244
8.45.5 Member Data Documentation	244
8.45.5.1 normal_dist	244
8.45.5.2 seed	244
8.45.5.3 uint_gen	244
8.45.5.4 uniform_dist	245
8.46 glucat::index_set< LO, HI >::reference Class Reference	245
8.46.1 Detailed Description	246
8.46.2 Constructor & Destructor Documentation	246
8.46.2.1 reference() [1/2]	246
8.46.2.2 reference() [2/2]	247
8.46.2.3 ~reference()	247
8.46.3 Member Function Documentation	247
8.46.3.1 flip()	247
8.46.3.2 operator bool()	247
8.46.3.3 operator=() [1/2]	248
8.46.3.4 operator=() [2/2]	248
8.46.3.5 operator==()	248

8.46.3.6 operator~()	248
8.46.4 Friends And Related Symbol Documentation	249
8.46.4.1 index_set	249
8.46.5 Member Data Documentation	249
8.46.5.1 m_idx	249
8.46.5.2 m_pst	249
8.47 glucat::sorted_range< Map_T, Sorted_Map_T > Class Template Reference	249
8.47.1 Detailed Description	250
8.47.2 Member Typedef Documentation	250
8.47.2.1 map_t	250
8.47.2.2 sorted_iterator	250
8.47.2.3 sorted_map_t	250
8.47.3 Constructor & Destructor Documentation	251
8.47.3.1 sorted_range()	251
8.47.4 Member Data Documentation	251
8.47.4.1 sorted_begin	251
8.47.4.2 sorted_end	251
8.48 glucat::sorted_range< Sorted_Map_T, Sorted_Map_T > Class Template Reference	251
8.48.1 Detailed Description	252
8.48.2 Member Typedef Documentation	252
8.48.2.1 map_t	252
8.48.2.2 sorted_iterator	252
8.48.2.3 sorted_map_t	252
8.48.3 Constructor & Destructor Documentation	252
8.48.3.1 sorted_range()	252
8.48.4 Member Data Documentation	253
8.48.4.1 sorted_begin	253
8.48.4.2 sorted_end	253
8.49 glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term Class Reference	253
8.49.1 Detailed Description	254
8.49.2 Member Typedef Documentation	255
8.49.2.1 var_pair_t	255
8.49.3 Constructor & Destructor Documentation	255
8.49.3.1 ~var_term()	255
8.49.3.2 var_term() [1/2]	255
8.49.3.3 var_term() [2/2]	255
8.49.4 Member Function Documentation	255
8.49.4.1 classname()	255
8.49.4.2 operator*=()	256
9 File Documentation	257
9.1 glucat/clifford_algebra.h File Reference	257

9.1.1 Macro Definition Documentation	265
9.1.1.1 <code>_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS</code>	265
9.2 <code>clifford_algebra.h</code>	265
9.3 <code>glucat/clifford_algebra_imp.h</code> File Reference	274
9.4 <code>clifford_algebra_imp.h</code>	282
9.5 <code>glucat/errors.h</code> File Reference	294
9.6 <code>errors.h</code>	295
9.7 <code>glucat/errors_imp.h</code> File Reference	296
9.8 <code>errors_imp.h</code>	297
9.9 <code>glucat/framed_multi.h</code> File Reference	298
9.10 <code>framed_multi.h</code>	301
9.11 <code>glucat/framed_multi_imp.h</code> File Reference	304
9.11.1 Macro Definition Documentation	307
9.11.1.1 <code>_GLUCAT_HASH_N</code>	307
9.11.1.2 <code>_GLUCAT_HASH_SIZE_T</code>	307
9.12 <code>framed_multi_imp.h</code>	308
9.13 <code>glucat/generation.h</code> File Reference	328
9.14 <code>generation.h</code>	329
9.15 <code>glucat/generation_imp.h</code> File Reference	330
9.16 <code>generation_imp.h</code>	331
9.17 <code>glucat/global.h</code> File Reference	334
9.17.1 Macro Definition Documentation	336
9.17.1.1 <code>_GLUCAT_CTAAssert</code>	336
9.18 <code>global.h</code>	336
9.19 <code>glucat/glucat.h</code> File Reference	337
9.20 <code>glucat.h</code>	338
9.21 <code>glucat/glucat_config.h</code> File Reference	340
9.21.1 Macro Definition Documentation	340
9.21.1.1 <code>GLUCAT_HAVE_CXX11</code>	340
9.21.1.2 <code>GLUCAT_HAVE_INTTYPES_H</code>	341
9.21.1.3 <code>GLUCAT_HAVE_STDINT_H</code>	341
9.21.1.4 <code>GLUCAT_HAVE_STDIO_H</code>	341
9.21.1.5 <code>GLUCAT_HAVE_STDLIB_H</code>	341
9.21.1.6 <code>GLUCAT_HAVE_STRING_H</code>	341
9.21.1.7 <code>GLUCAT_HAVE_STRINGS_H</code>	341
9.21.1.8 <code>GLUCAT_HAVE_SYS_STAT_H</code>	341
9.21.1.9 <code>GLUCAT_HAVE_SYS_TYPES_H</code>	341
9.21.1.10 <code>GLUCAT_HAVE_UNISTD_H</code>	342
9.21.1.11 <code>GLUCAT_PACKAGE</code>	342
9.21.1.12 <code>GLUCAT_PACKAGE_BUGREPORT</code>	342
9.21.1.13 <code>GLUCAT_PACKAGE_NAME</code>	342
9.21.1.14 <code>GLUCAT_PACKAGE_STRING</code>	342

9.21.1.15 GLUCAT_PACKAGE_TARNAME	342
9.21.1.16 GLUCAT_PACKAGE_URL	342
9.21.1.17 GLUCAT_PACKAGE_VERSION	343
9.21.1.18 GLUCAT_STDC_HEADERS	343
9.21.1.19 GLUCAT_VERSION	343
9.22 glucat_config.h	343
9.23 glucat/glucat_imp.h File Reference	345
9.24 glucat_imp.h	345
9.25 glucat/index_set.h File Reference	346
9.26 index_set.h	348
9.27 glucat/index_set_imp.h File Reference	350
9.28 index_set_imp.h	352
9.29 glucat/long_double.h File Reference	363
9.30 long_double.h	365
9.31 glucat/matrix.h File Reference	365
9.32 matrix.h	368
9.33 glucat/matrix_imp.h File Reference	369
9.34 matrix_imp.h	371
9.35 glucat/matrix_multi.h File Reference	378
9.36 matrix_multi.h	380
9.37 glucat/matrix_multi_imp.h File Reference	384
9.38 matrix_multi_imp.h	388
9.39 glucat/portability.h File Reference	412
9.39.1 Macro Definition Documentation	413
9.39.1.1 _GLUCAT_ISINF	413
9.39.1.2 _GLUCAT_ISNAN	414
9.39.1.3 UBLAS_ABS	414
9.39.1.4 UBLAS_SQRT	414
9.40 portability.h	414
9.41 glucat/promotion.h File Reference	415
9.42 promotion.h	416
9.43 glucat/qd.h File Reference	419
9.44 qd.h	420
9.45 glucat/random.h File Reference	423
9.46 random.h	424
9.47 glucat/scalar.h File Reference	425
9.48 scalar.h	427
9.49 glucat/scalar_imp.h File Reference	430
9.50 scalar_imp.h	431
9.51 glucat/tuning.h File Reference	433
9.51.1 Function Documentation	434
9.51.1.1 _GLUCAT_CTAssert()	434

9.52 tuning.h	435
9.53 test/tuning.h File Reference	437
9.54 tuning.h	438
9.55 pyclical/glucat.pxd File Reference	439
9.56 glucat.pxd	439
9.57 pyclical/PyClical.h File Reference	441
9.57.1 Typedef Documentation	442
9.57.1.1 Clifford	442
9.57.1.2 IndexSet	442
9.57.1.3 scalar_t	443
9.57.1.4 String	443
9.57.2 Function Documentation	443
9.57.2.1 clifford_to_repr()	443
9.57.2.2 clifford_to_str()	443
9.57.2.3 index_set_to_repr()	443
9.57.2.4 index_set_to_str()	444
9.57.2.5 PyFloat_FromDouble()	444
9.57.3 Variable Documentation	444
9.57.3.1 epsilon	444
9.57.3.2 glucat_package_version	444
9.57.3.3 hi_ndx	444
9.57.3.4 lo_ndx	445
9.58 PyClical.h	445
9.59 pyclical/PyClical.pxd File Reference	447
9.60 PyClical.pxd	447
9.61 pyclical/PyClical.pyx File Reference	447
9.62 PyClical.pyx	448
9.63 test/control.h File Reference	472
9.64 control.h	473
9.65 test/driver.h File Reference	474
9.66 driver.h	475
9.67 test/timing.h File Reference	475
9.68 timing.h	476
9.69 test/try_catch.h File Reference	477
9.70 try_catch.h	477
9.71 test/undefine.h File Reference	478
9.72 undefine.h	478

Chapter 1

Directory Hierarchy

1.1 Directories

glucat	11
clifford_algebra.h	257
clifford_algebra_imp.h	274
errors.h	294
errors_imp.h	296
framed_multi.h	298
framed_multi_imp.h	304
generation.h	328
generation_imp.h	330
global.h	334
glucat.h	337
glucat_config.h	340
glucat_imp.h	345
index_set.h	346
index_set_imp.h	350
long_double.h	363
matrix.h	365
matrix_imp.h	369
matrix_multi.h	378
matrix_multi_imp.h	384
portability.h	412
promotion.h	415
qd.h	419
random.h	423
scalar.h	425
scalar_imp.h	430
tuning.h	433
pyclical	12
glucat.pxd	439
PyClical.h	441
PyClical.pxd	447
PyClical.pyx	447
test	12
control.h	472
driver.h	474
timing.h	475
try_catch.h	477
tuning.h	437
undefine.h	478

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

cga3	Definitions for 3D Conformal Geometric Algebra [DL]	13
glucat		14
glucat::gen		69
glucat::matrix		69
glucat::timing		76
pade		78
PyClical		85
std		91

Chapter 3

Hierarchical Index

3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

std::bitset	
glucat::index_set< lo_ndx, hi_ndx >	169
glucat::index_set< DEFAULT_LO, DEFAULT_HI >	169
glucat::index_set< LO, HI >	169
glucat::bool_to_type< truth_value >	95
cdef	
PyClical.clifford	96
PyClical.index_set	183
Clifford	
PyClical.clifford	96
glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >	117
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >	142
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >	193
glucat::matrix_multi< scalar_t, lo_ndx, hi_ndx, tuning_promoted >	193
glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>> >	117
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >	142
glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>> >	117
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >	193
glucat::compare_types< LHS_T, RHS_T >	129
glucat::compare_types< T, T >	130
glucat::control_t	131
glucat::CTAssertion< bool >	135
glucat::CTAssertion< true >	136
glucat::numeric_traits< Scalar_T >demoted	136
glucat::matrix::eig_genus< Matrix_T >	137
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >hash_size_t	167
glucat::index_set_hash< LO, HI >	192
IndexSet	
PyClical.index_set	183
inline	
PyClical.clifford	96
PyClical.index_set	183

std::logic_error	
glucat::glucat_error	165
glucat::error< multivector_t >	139
glucat::error< index_set >	139
glucat::error< Class_T >	139
std::map	
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >	93
glucat::gen::generator_table< Matrix_T >	160
numeric_limits	
std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >	210
std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >	211
glucat::numeric_traits< Scalar_T >	212
obj	
PyClical.clifford	96
PyClical.index_set	183
pade::pade_log_denom< Scalar_T >	223
pade::pade_log_denom< dd_real >	224
pade::pade_log_denom< float >	225
pade::pade_log_denom< long double >	226
pade::pade_log_denom< qd_real >	227
pade::pade_log_numer< Scalar_T >	228
pade::pade_log_numer< dd_real >	229
pade::pade_log_numer< float >	229
pade::pade_log_numer< long double >	230
pade::pade_log_numer< qd_real >	231
pade::pade_sqrt_denom< Scalar_T >	232
pade::pade_sqrt_denom< dd_real >	233
pade::pade_sqrt_denom< float >	234
pade::pade_sqrt_denom< long double >	234
pade::pade_sqrt_denom< qd_real >	235
pade::pade_sqrt_numer< Scalar_T >	236
pade::pade_sqrt_numer< dd_real >	237
pade::pade_sqrt_numer< float >	238
pade::pade_sqrt_numer< long double >	239
pade::pade_sqrt_numer< qd_real >	239
std::pair	
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >var_term	253
glucat::numeric_traits< Scalar_T >promoted	240
glucat::random_generator< Scalar_T >	241
glucat::index_set< LO, HI >reference	245
glucat::sorted_range< Map_T, Sorted_Map_T >	249
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >	251
toClifford	
PyClical.clifford	96
toIndexSet	
PyClical.index_set	183
std::unordered_map	
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >	142
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >	142

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

glucat::basis_table< Scalar_T, LO, HI, Matrix_T >	93
Table of basis elements used as a cache by basis_element()	
glucat::bool_to_type< truth_value >	95
Bool to type	
PyClical.clifford	96
glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >	
Clifford_algebra<> declares the operations of a Clifford algebra	117
glucat::compare_types< LHS_T, RHS_T >	
Type comparison	129
glucat::compare_types< T, T >	130
glucat::control_t	
Parameters to control tests	131
glucat::CTAssertion< bool >	
Compile time assertion	135
glucat::CTAssertion< true >	136
glucat::numeric_traits< Scalar_T >::demoted	
Demoted type for long double	136
glucat::matrix::eig_genus< Matrix_T >	
Structure containing classification of eigenvalues	137
glucat::error< Class_T >	
Specific exception class	139
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >	
A framed_multi<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector	142
glucat::gen::generator_table< Matrix_T >	
Table of generators for specific signatures	160
glucat::glucat_error	
Abstract exception class	165
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t	167
glucat::index_set< LO, HI >	
Index set class based on std::bitset<> in Gnu standard C++ library	169
PyClical.index_set	183
glucat::index_set_hash< LO, HI >	192
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >	
A matrix_multi<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector	193
std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >	
Numeric limits for framed_multi inherit limits for the corresponding scalar type	210

std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >	
Numeric limits for matrix_multi inherit limits for the corresponding scalar type	211
glucat::numeric_traits< Scalar_T >	
Extra traits which extend numeric limits	212
pade::pade_log_denom< Scalar_T >	
Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)	
223	
pade::pade_log_denom< dd_real >	224
pade::pade_log_denom< float >	225
pade::pade_log_denom< long double >	226
pade::pade_log_denom< qd_real >	227
pade::pade_log_numer< Scalar_T >	
Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)	
228	
pade::pade_log_numer< dd_real >	229
pade::pade_log_numer< float >	229
pade::pade_log_numer< long double >	230
pade::pade_log_numer< qd_real >	231
pade::pade_sqrt_denom< Scalar_T >	
Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)	
232	
pade::pade_sqrt_denom< dd_real >	233
pade::pade_sqrt_denom< float >	234
pade::pade_sqrt_denom< long double >	234
pade::pade_sqrt_denom< qd_real >	235
pade::pade_sqrt_numer< Scalar_T >	
Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)	
236	
pade::pade_sqrt_numer< dd_real >	237
pade::pade_sqrt_numer< float >	238
pade::pade_sqrt_numer< long double >	239
pade::pade_sqrt_numer< qd_real >	239
glucat::numeric_traits< Scalar_T >::promoted	
Extra traits which extend numeric limits	240
glucat::random_generator< Scalar_T >	
Random number generator with single instance per Scalar_T	241
glucat::index_set< LO, HI >::reference	
Index set member reference	245
glucat::sorted_range< Map_T, Sorted_Map_T >	
Sorted range for use with output	249
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >	251
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term	
Variable term	253

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

glucat/clifford_algebra.h	257
glucat/clifford_algebra_imp.h	274
glucat/errors.h	294
glucat/errors_imp.h	296
glucat/framed_multi.h	298
glucat/framed_multi_imp.h	304
glucat/generation.h	328
glucat/generation_imp.h	330
glucat/global.h	334
glucat/glucat.h	337
glucat/glucat_config.h	340
glucat/glucat_imp.h	345
glucat/index_set.h	346
glucat/index_set_imp.h	350
glucat/long_double.h	363
glucat/matrix.h	365
glucat/matrix_imp.h	369
glucat/matrix_multi.h	378
glucat/matrix_multi_imp.h	384
glucat/portability.h	412
glucat/promotion.h	415
glucat/qd.h	419
glucat/random.h	423
glucat/scalar.h	425
glucat/scalar_imp.h	430
glucat/tuning.h	433
pyclical/glucat.pxd	439
pyclical/PyClical.h	441
pyclical/PyClical.pxd	447
pyclical/PyClical.pyx	447
test/control.h	472
test/driver.h	474
test/timing.h	475
test/try_catch.h	477
test/tuning.h	437
test/undefine.h	478

Chapter 6

Directory Documentation

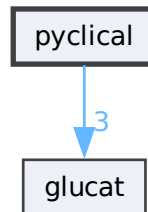
6.1 glucat Directory Reference

Files

- file [clifford_algebra.h](#)
- file [clifford_algebra_imp.h](#)
- file [errors.h](#)
- file [errors_imp.h](#)
- file [framed_multi.h](#)
- file [framed_multi_imp.h](#)
- file [generation.h](#)
- file [generation_imp.h](#)
- file [global.h](#)
- file [glucat.h](#)
- file [glucat_config.h](#)
- file [glucat_imp.h](#)
- file [index_set.h](#)
- file [index_set_imp.h](#)
- file [long_double.h](#)
- file [matrix.h](#)
- file [matrix_imp.h](#)
- file [matrix_multi.h](#)
- file [matrix_multi_imp.h](#)
- file [portability.h](#)
- file [promotion.h](#)
- file [qd.h](#)
- file [random.h](#)
- file [scalar.h](#)
- file [scalar_imp.h](#)
- file [tuning.h](#)

6.2 pyclical Directory Reference

Directory dependency graph for pyclical:

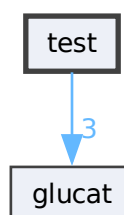


Files

- file [glucal.pxd](#)
- file [PyClical.h](#)
- file [PyClical.pxd](#)
- file [PyClical.pyx](#)

6.3 test Directory Reference

Directory dependency graph for test:



Files

- file [control.h](#)
- file [driver.h](#)
- file [timing.h](#)
- file [try_catch.h](#)
- file [tuning.h](#)
- file [undefine.h](#)

Chapter 7

Namespace Documentation

7.1 cga3 Namespace Reference

Definitions for 3D Conformal Geometric Algebra [DL].

Functions

- `template<typename Multivector_T>`
`Multivector_T cga3 (const Multivector_T &x)`
Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].
- `template<typename Multivector_T>`
`Multivector_T cga3std (const Multivector_T &X)`
Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].
- `template<typename Multivector_T>`
`Multivector_T agc3 (const Multivector_T &X)`
Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

7.1.1 Detailed Description

Definitions for 3D Conformal Geometric Algebra [DL].

7.1.2 Function Documentation

7.1.2.1 agc3()

```
template<typename Multivector_T>
Multivector_T cga3::agc3 (
    const Multivector_T & X) [inline]
```

Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Definition at line 126 of file [PyClical.h](#).

References [agc3\(\)](#), and [cga3std\(\)](#).

Referenced by [agc3\(\)](#).

7.1.2.2 cga3()

```
template<typename Multivector_T>
Multivector_T cga3::cga3 (
    const Multivector_T & x) [inline]
```

Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].

Definition at line 103 of file [PyClical.h](#).

7.1.2.3 cga3std()

```
template<typename Multivector_T>
Multivector_T cga3::cga3std (
    const Multivector_T & X) [inline]
```

Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].

Definition at line 114 of file [PyClical.h](#).

References [cga3std\(\)](#).

Referenced by [agc3\(\)](#), and [cga3std\(\)](#).

7.2 glucat Namespace Reference

Namespaces

- namespace [gen](#)
- namespace [matrix](#)
- namespace [timing](#)

Classes

- class [basis_table](#)
Table of basis elements used as a cache by basis_element().
- class [bool_to_type](#)
Bool to type.
- class [clifford_algebra](#)
clifford_algebra<> declares the operations of a Clifford algebra
- class [compare_types](#)
Type comparison.
- class [compare_types< T, T >](#)
- class [control_t](#)
Parameters to control tests.
- struct [CTAssertion](#)
Compile time assertion.
- struct [CTAssertion< true >](#)
- class [error](#)
Specific exception class.

- class [framed_multi](#)
A [framed_multi](#)<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector.
- class [glucat_error](#)
Abstract exception class.
- class [index_set](#)
Index set class based on `std::bitset<>` in Gnu standard C++ library.
- class [index_set_hash](#)
- class [matrix_multi](#)
A [matrix_multi](#)<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector.
- class [numeric_traits](#)
Extra traits which extend numeric limits.
- class [random_generator](#)
Random number generator with single instance per Scalar_T.
- class [sorted_range](#)
Sorted range for use with output.
- class [sorted_range](#)<Sorted_Map_T, Sorted_Map_T >

Typedefs

- using [index_t](#) = int
Size of [index_t](#) should be enough to represent LO, HI.
- using [set_value_t](#) = unsigned long
Size of [set_value_t](#) should be enough to contain [index_set](#)<LO,HI>.
- typedef int(*) [intfn](#)()
For exception catching: pointer to function returning int.
- typedef int(*) [intintfn](#)(int)
For exception catching: pointer to function of int returning int.
- using [tuning_slow](#)
- using [tuning_naive](#)
- using [tuning_fast](#)

Functions

- template<template< typename, const [index_t](#), const [index_t](#), typename > class Multivector, template< typename, const [index_t](#), const [index_t](#), typename > class RHS, typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
auto [operator!=](#) (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool
Test for inequality of multivectors.
- template<template< typename, const [index_t](#), const [index_t](#), typename > class Multivector, typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
auto [operator!=](#) (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool
Test for inequality of multivector and scalar.
- template<template< typename, const [index_t](#), const [index_t](#), typename > class Multivector, typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
auto [operator!=](#) (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool
Test for inequality of scalar and multivector.
- template<template< typename, const [index_t](#), const [index_t](#), typename > class Multivector, typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
auto [error_squared_tol](#) (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T
Quadratic norm error tolerance relative to a specific multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`

Relative or absolute error using the quadratic norm.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`

Test for approximate equality of multivectors.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`

Test for approximate equality of multivectors.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Product of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator\* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Product of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric product.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Outer product.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inner product.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Left contraction.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`

Hestenes scalar product.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Quotient of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Quotient of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric quotient.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Transformation via twisted adjoint action.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric multiplicative inverse.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Integer power of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Multivector power of multivector.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Outer product power of multivector.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Scalar part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Real part: synonym for scalar part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Imaginary part: deprecated (always 0).
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Pure part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Even part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Odd part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`
Vector part of multivector, as a [vector_t](#) with respect to frame().
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Main involution, each {i} is replaced by -{i} in each term, eg. {1}{2} -> (-{2})*(-{1}).*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Reversion, eg. {1}{2} -> {2}*{1}.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Conjugation, rev o invo == invo o rev.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar_T quadratic form == (rev(x)*x)(0).*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Scalar_T norm == sum of norm of coordinates.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Absolute value == sqrt(norm).

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Maximum of absolute values of components of multivector: multivector infinity norm.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of -1 which commutes with all members of the frame of the given multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Exponential of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Hyperbolic sine of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse hyperbolic sine of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse hyperbolic sine of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Tangent of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Tangent of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse tangent of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse tangent of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Hyperbolic tangent of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse hyperbolic tangent of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Inverse hyperbolic tangent of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`static void check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

Check that i is a valid complexifier for val.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator\* (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

Geometric product.

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Outer product.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Inner product.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Left contraction.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`
Hestenes scalar product.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Geometric quotient.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Transformation via twisted adjoint action.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`
Read multivector from input.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`
Write multivector to output.
- `template<typename Scalar_T, const index_t LO, const index_t HI>`
`auto operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term) -> std::ostream &`
Write term to output.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Exponential of multivector.
- `template<typename Scalar_T, const index_t LO, const index_t HI>`
`static auto crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`
Coordinate of product of terms.
- `template<typename Scalar_T, const index_t LO, const index_t HI>`
`auto operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`
Product of terms.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`
Square root of multivector with specified complexifier.
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`
`auto log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`

- Natural logarithm of multivector with specified complexifier.*

 - `template<typename Scalar_T, const index\_t LO, const index\_t HI>`
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`
- Coordinate of product of terms.*

 - `\_GLUCAT\_CTAssert (std::numeric_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const index\_t BITS_PER_CHAR`

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

 - `\_GLUCAT\_CTAssert (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const index\_t DEFAULT_LO`

Default lowest index in an index set.

 - `template<typename LHS_T, typename RHS_T>`
`auto pos\_mod (LHS_T lhs, RHS_T rhs) -> LHS_T`

Modulo function which works reliably for lhs < 0.

 - `template<const index\_t LO, const index\_t HI>`
`auto operator^ (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`

Symmetric set difference: exclusive or.

 - `template<const index\_t LO, const index\_t HI>`
`auto operator& (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`

Set intersection: and.

 - `template<const index\_t LO, const index\_t HI>`
`auto operator| (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`

Set union: or.

 - `template<const index\_t LO, const index\_t HI>`
`auto compare (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int`

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

 - `\_GLUCAT\_CTAssert (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >), Default_index_set_too_big_for_value) template< const index\_t LO`

Size of [set_value_t](#) should be enough to contain [bitset](#)<[DEFAULT_HI](#)-[DEFAULT_LO](#)>.

 - `const index\_t HI auto operator<< (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &`
 - `template<const index\_t LO, const index\_t HI>`
`auto operator>> (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &`

Read in index set.

 - `auto sign\_of\_square (index\_t j) -> int`

Square of generator [j].

 - `template<const index\_t LO, const index\_t HI>`
`auto min\_neg (const index\_set< LO, HI > &ist) -> index\_t`

Minimum negative index, or 0 if none.

 - `template<const index\_t LO, const index\_t HI>`
`auto max\_pos (const index\_set< LO, HI > &ist) -> index\_t`

Maximum positive index, or 0 if none.

 - `template<const index\_t LO, const index\_t HI>`
`auto operator<< (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &`

Write out index set.

 - `static auto inverse\_reversed\_gray (unsigned long x) -> unsigned long`

Inverse reversed Gray code.

 - `static auto inverse\_gray (unsigned long x) -> unsigned long`

Inverse Gray code.

 - `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator\* (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Geometric product.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator^ (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO,`
`HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Outer product.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator& (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO,`
`HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Inner product.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator% (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO,`
`HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Left contraction.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto star (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &rhs) -> Scalar_T`

Hestenes scalar product.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator/ (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Geometric quotient.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator| (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Transformation via twisted adjoint action.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator>> (std::istream &s, matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`

Read multivector from input.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto operator<< (std::ostream &os, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream`
`&`

Write multivector to output.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto reframe (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &rhs, matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &rhs_reframed) -> const index\_set< LO, HI >`

Find a common frame for operands of a binary operator.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI,`
`Tune_P > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Square root of multivector with specified complexifier.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto matrix\_sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO,`
`HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Square root of multivector with specified complexifier.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI, Tune_P`
`< > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector with specified complexifier.

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto matrix\_log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO,`
`HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector with specified complexifier.

- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
auto [exp](#) (const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &val) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Exponential of multivector.
- auto [offset_level](#) (const [index_t](#) p, const [index_t](#) q) -> [index_t](#)
Determine the log2 dim corresponding to signature p, q.
- template<typename Matrix_Index_T, const [index_t](#) LO, const [index_t](#) HI>
static auto [folded_dim](#) (const [index_set](#)< LO, HI > &sub) -> Matrix_Index_T
Determine the matrix dimension of the fold of a subalgebra.
- template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
static auto [fast](#) (const Matrix_T &X, [index_t](#) level) -> Multivector_T
Inverse generalized Fast Fourier Transform.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P, const size_t Size>
static auto [pade_approx](#) (const std::array< Scalar_T, Size > &numer, const std::array< Scalar_T, Size > &denom, const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &X) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Pade' approximation.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
static void [db_step](#) ([matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &M, [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &Y)
Single step of product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
static auto [db_sqrt](#) (const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_tol=std::numeric_limits< Scalar_T >::epsilon(), 4) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
static auto [cr_sqrt](#) (const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_Y_tol=std::numeric_limits< Scalar_T >::epsilon(), 1) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Cyclic reduction square root iteration.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
static auto [pade_log](#) (const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &val) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Pade' approximation of log.
- template<typename Scalar_T, const [index_t](#) LO, const [index_t](#) HI, typename Tune_P>
static auto [cascade_log](#) (const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P > &val) -> const [matrix_multi](#)< Scalar_T, LO, HI, Tune_P >
Incomplete square root cascade and Pade' approximation of log.
- template<typename Scalar_T>
auto [log2](#) (const Scalar_T &x) -> Scalar_T
Log base 2 of scalar.
- template<typename Scalar_T>
auto [to_promote](#) (const Scalar_T &val) -> typename [numeric_traits](#)< Scalar_T >::promoted::type
Cast to promote.
- template<typename Scalar_T>
auto [to_demote](#) (const Scalar_T &val) -> typename [numeric_traits](#)< Scalar_T >::demoted::type
Cast to demote.
- int [try_catch](#) (intfn f)
Exception catching for functions returning int.
- int [try_catch](#) (intintfn f, int arg)
Exception catching for functions of int returning int.

Variables

- `template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>`
`const Scalar_T clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation = std::numeric_limits<Scalar_T>::epsilon()`
Default for truncation.
- `const double MS_PER_S = 1000.0`
Timing constant: deprecated here - moved to [test/timing.h](#).
- `const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits`
Number of bits in [set_value_t](#).
- `const index_t DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)`
Default highest index in an index set.
- `static const long double l_pi = 3.1415926535897932384626433832795029L`
- `static const long double l_ln2 = 0.6931471805599453094172321214581766L`
- `const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits`
- `const unsigned int Tuning_Max_Threshold = 1 << Tuning_Int_Digits`
- `const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Slow_Basis_Max_Count = 0`
- `const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0`
- `const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold`
- `const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold`
- `const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0`
- `const unsigned int Tuning_Fast_Div_Max_Steps = 0`
- `const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256`
- `const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256`
- `const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16`
- `const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8`
- `const unsigned int Tuning_Fast_Basis_Max_Count = 1`
- `const unsigned int Tuning_Fast_Fast_Size_Threshold = 0`
- `const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0`
- `const unsigned int Tuning_Fast_Products_Size_Threshold = 0`

7.2.1 Typedef Documentation

7.2.1.1 index_t

using `glucat::index_t` = int

Size of `index_t` should be enough to represent LO, HI.

Definition at line 77 of file [global.h](#).

7.2.1.2 intfn

typedef int(*) `glucat::intfn()`

For exception catching: pointer to function returning int.

Definition at line 37 of file [try_catch.h](#).

7.2.1.3 intintfn

```
typedef int(*) glucat::intintfn(int)
```

For exception catching: pointer to function of int returning int.

Definition at line 40 of file [try_catch.h](#).

7.2.1.4 set_value_t

```
using glucat::set_value_t = unsigned long
```

Size of [set_value_t](#) should be enough to contain [index_set<LO,HI>](#).

Definition at line 79 of file [global.h](#).

7.2.1.5 tuning_fast

```
using glucat::tuning_fast
```

Initial value:

```
tuning
<
  Tuning_Fast_Mult_Matrix_Threshold,
  Tuning_Fast_Div_Max_Steps,
  Tuning_Fast_CR_Sqrt_Max_Steps,
  Tuning_Fast_DB_Sqrt_Max_Steps,
  Tuning_Fast_Log_Max_Outer_Steps,
  Tuning_Fast_Log_Max_Inner_Steps,
  Tuning_Fast_Basis_Max_Count,
  Tuning_Fast_Fast_Size_Threshold,
  Tuning_Fast_Inv_Fast_Dim_Threshold,
  Tuning_Fast_Products_Size_Threshold,
  Tuning_Default_Denom_Different_Bits,
  Tuning_Default_Extra_Different_Bits,
  Tuning_Default_Function_Precision
>
```

Definition at line 97 of file [tuning.h](#).

7.2.1.6 tuning_naive

```
using glucat::tuning_naive
```

Initial value:

```
tuning
<
  Tuning_Naive_Mult_Matrix_Threshold,
  Tuning_Default_Div_Max_Steps,
  Tuning_Default_CR_Sqrt_Max_Steps,
  Tuning_Default_DB_Sqrt_Max_Steps,
  Tuning_Default_Log_Max_Outer_Steps,
  Tuning_Default_Log_Max_Inner_Steps,
  Tuning_Naive_Basis_Max_Count,
  Tuning_Naive_Fast_Size_Threshold,
  Tuning_Naive_Inv_Fast_Dim_Threshold,
  Tuning_Default_Products_Size_Threshold,
  Tuning_Default_Denom_Different_Bits,
  Tuning_Default_Extra_Different_Bits,
  Tuning_Default_Function_Precision
>
```

Definition at line 69 of file [tuning.h](#).

7.2.1.7 tuning_slow

using [glucat::tuning_slow](#)

Initial value:

```
tuning
<
    Tuning_Slow_Mult_Matrix_Threshold,
    Tuning_Default_Div_Max_Steps,
    Tuning_Default_CR_Sqrt_Max_Steps,
    Tuning_Default_DB_Sqrt_Max_Steps,
    Tuning_Default_Log_Max_Outer_Steps,
    Tuning_Default_Log_Max_Inner_Steps,
    Tuning_Slow_Basis_Max_Count,
    Tuning_Slow_Fast_Size_Threshold,
    Tuning_Slow_Inv_Fast_Dim_Threshold,
    Tuning_Slow_Products_Size_Threshold,
    Tuning_Default_Denom_Different_Bits,
    Tuning_Default_Extra_Different_Bits,
    Tuning_Default_Function_Precision
>
```

Definition at line 47 of file [tuning.h](#).

7.2.2 Function Documentation

7.2.2.1 _GLUCAT_CTAssert() [1/3]

```
glucat::_GLUCAT_CTAssert (
    _GLUCAT_BITS_PER_ULONG == BITS\_PER\_SET\_VALUE,
    BitsPerUlongDoesNotMatchSetValueT ) const
```

Default lowest index in an index set.

References [_GLUCAT_CTAssert\(\)](#), and [BITS_PER_SET_VALUE](#).

7.2.2.2 _GLUCAT_CTAssert() [2/3]

```
glucat::_GLUCAT_CTAssert (
    sizeof(set\_value\_t) >= sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >) ,
    Default_index_set_too_big_for_value ) const
```

Size of [set_value_t](#) should be enough to contain `bitset<DEFAULT_HI-DEFAULT_LO>`.

Write out index set

References [_GLUCAT_CTAssert\(\)](#), and [DEFAULT_HI](#).

7.2.2.3 _GLUCAT_CTAssert() [3/3]

```
glucat::_GLUCAT_CTAssert (
    std::numeric_limits< unsigned char >::radix == 2,
    CannotDetermineBitsPerChar ) const
```

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

Number of bits per char is used to determine number of bits in [set_value_t](#)

References [_GLUCAT_CTAssert\(\)](#).

Referenced by [_GLUCAT_CTAssert\(\)](#), [_GLUCAT_CTAssert\(\)](#), and [_GLUCAT_CTAssert\(\)](#).

7.2.2.4 abs()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Absolute value == sqrt(norm).

Definition at line 577 of file [clifford_algebra_imp.h](#).

References [abs\(\)](#), and [glucat::numeric_traits< Scalar_T >::sqrt\(\)](#).

Referenced by [abs\(\)](#), [PyClical.clifford::abs\(\)](#), [clifford_to_str\(\)](#), [matrix_log\(\)](#), and [matrix_sqrt\(\)](#).

7.2.2.5 acos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Inverse cosine of multivector.

Definition at line 902 of file [clifford_algebra_imp.h](#).

References [acos\(\)](#), and [complexifier\(\)](#).

7.2.2.6 acos() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse cosine of multivector with specified complexifier.

Definition at line 882 of file [clifford_algebra_imp.h](#).

References [acos\(\)](#), [acosh\(\)](#), and [check_complex\(\)](#).

Referenced by [acos\(\)](#), and [acos\(\)](#).

7.2.2.7 acosh() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_
_T, LO, HI, Tune_P > [inline]
```

Inverse hyperbolic cosine of multivector.

Definition at line 843 of file [clifford_algebra_imp.h](#).

References [acosh\(\)](#), and [complexifier\(\)](#).

7.2.2.8 acosh() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse hyperbolic cosine of multivector with specified complexifier.

Definition at line 824 of file [clifford_algebra_imp.h](#).

References [acosh\(\)](#), [check_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), and [acosh\(\)](#).

7.2.2.9 approx_equal() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T,
const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 169 of file [clifford_algebra_imp.h](#).

References [approx_equal\(\)](#), and [error_squared_tol\(\)](#).

7.2.2.10 approx_equal() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold,
    const Scalar_T tolerance) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 154 of file [clifford_algebra_imp.h](#).

References [approx_equal\(\)](#), and [error_squared\(\)](#).

Referenced by [approx_equal\(\)](#), [approx_equal\(\)](#), and [matrix_sqrt\(\)](#).

7.2.2.11 asin() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_
_T, LO, HI, Tune_P > [inline]
```

Inverse sine of multivector.

Definition at line 1007 of file [clifford_algebra_imp.h](#).

References [asin\(\)](#), and [complexifier\(\)](#).

7.2.2.12 asin() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse sine of multivector with specified complexifier.

Definition at line 987 of file [clifford_algebra_imp.h](#).

References [asin\(\)](#), [asinh\(\)](#), and [check_complex\(\)](#).

Referenced by [asin\(\)](#), and [asin\(\)](#).

7.2.2.13 asinh() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Inverse hyperbolic sine of multivector.

Definition at line 948 of file [clifford_algebra_imp.h](#).

References [asinh\(\)](#), and [complexifier\(\)](#).

7.2.2.14 asinh() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Inverse hyperbolic sine of multivector with specified complexifier.

Definition at line 929 of file [clifford_algebra_imp.h](#).

References [asinh\(\)](#), [check_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [asin\(\)](#), [asinh\(\)](#), and [asinh\(\)](#).

7.2.2.15 atan() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Inverse tangent of multivector.

Definition at line 1107 of file [clifford_algebra_imp.h](#).

References [atan\(\)](#), and [complexifier\(\)](#).

7.2.2.16 atan() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse tangent of multivector with specified complexifier.

Definition at line [1087](#) of file [clifford_algebra_imp.h](#).

References [atan\(\)](#), [atanh\(\)](#), and [check_complex\(\)](#).

Referenced by [atan\(\)](#), and [atan\(\)](#).

7.2.2.17 atanh() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↔
_T, LO, HI, Tune_P > [inline]
```

Inverse hyperbolic tangent of multivector.

Definition at line [1051](#) of file [clifford_algebra_imp.h](#).

References [atanh\(\)](#), and [complexifier\(\)](#).

7.2.2.18 atanh() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse hyperbolic tangent of multivector with specified complexifier.

Definition at line [1034](#) of file [clifford_algebra_imp.h](#).

References [atanh\(\)](#), [check_complex\(\)](#), [log\(\)](#), and [norm\(\)](#).

Referenced by [atan\(\)](#), [atanh\(\)](#), and [atanh\(\)](#).

7.2.2.19 `cascade_log()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::cascade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix\_multi<
Scalar_T, LO, HI, Tune_P > [static]
```

Incomplete square root cascade and Pade' approximation of log.

Definition at line 1910 of file [matrix_multi_imp.h](#).

References [cascade_log\(\)](#), [db_step\(\)](#), [epsilon](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), [norm\(\)](#), and [pade_log\(\)](#).

Referenced by [cascade_log\(\)](#), and [matrix_log\(\)](#).

7.2.2.20 `check_complex()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
void glucat::check_complex (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) [inline], [static]
```

Check that i is a valid complexifier for val.

Definition at line 652 of file [clifford_algebra_imp.h](#).

References [check_complex\(\)](#), and [complexifier\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [check_complex\(\)](#), [cos\(\)](#), [log\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

7.2.2.21 `clifford_exp()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::clifford_exp (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↵
_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 689 of file [clifford_algebra_imp.h](#).

References [clifford_exp\(\)](#), and [log2\(\)](#).

Referenced by [clifford_exp\(\)](#), [exp\(\)](#), and [exp\(\)](#).

7.2.2.22 compare()

```
template<const index_t LO, const index_t HI>
auto glucat::compare (
    const index_set< LO, HI > & a,
    const index_set< LO, HI > & b) -> int [inline]
```

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

Lexicographic ordering of two sets: -1 if $a < b$, +1 if $a > b$, 0 if $a == b$.

Definition at line 574 of file [index_set_imp.h](#).

References [compare\(\)](#).

Referenced by [compare\(\)](#).

7.2.2.23 complexifier()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::complexifier (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P >
```

Square root of -1 which commutes with all members of the frame of the given multivector.

Definition at line 592 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [pos_mod\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [check_complex\(\)](#), [complexifier\(\)](#), [cos\(\)](#), [elliptic\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

7.2.2.24 conj()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::conj (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Conjugation, $rev \circ invo == invo \circ rev$.

Definition at line 553 of file [clifford_algebra_imp.h](#).

References [conj\(\)](#).

Referenced by [conj\(\)](#).

7.2.2.25 cos() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_←
_T, LO, HI, Tune_P > [inline]
```

Cosine of multivector.

Definition at line 873 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [cos\(\)](#).

7.2.2.26 cos() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Cosine of multivector with specified complexifier.

Definition at line 850 of file [clifford_algebra_imp.h](#).

References [check_complex\(\)](#), [cos\(\)](#), and [exp\(\)](#).

Referenced by [cos\(\)](#), [cos\(\)](#), and [tan\(\)](#).

7.2.2.27 cosh()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::cosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_←
_T, LO, HI, Tune_P > [inline]
```

Hyperbolic cosine of multivector.

Definition at line 806 of file [clifford_algebra_imp.h](#).

References [cosh\(\)](#), and [exp\(\)](#).

Referenced by [cosh\(\)](#), and [tanh\(\)](#).

7.2.2.28 cr_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cr_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_Y_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) ->
const matrix_multi< Scalar_T, LO, HI, Tune_P > [static]
```

Cyclic reduction square root iteration.

Definition at line 1344 of file [matrix_multi_imp.h](#).

References [cr_sqrt\(\)](#), [glucat::numeric_traits< Scalar_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [cr_sqrt\(\)](#), and [matrix_sqrt\(\)](#).

7.2.2.29 crd_of_mult() [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T [inline],
[static]
```

Coordinate of product of terms.

References [crd_of_mult\(\)](#), [exp\(\)](#), [log\(\)](#), and [sqrt\(\)](#).

Referenced by [crd_of_mult\(\)](#), [operator%\(\)](#), [operator&\(\)](#), [operator*\(\)](#), and [operator^\(\)](#).

7.2.2.30 crd_of_mult() [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T [inline],
[static]
```

Coordinate of product of terms.

Definition at line 1704 of file [framed_multi_imp.h](#).

References [crd_of_mult\(\)](#).

7.2.2.31 db_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::db_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) ->
const matrix_multi< Scalar_T, LO, HI, Tune_P > [static]
```

Product form of Denman-Beavers square root iteration.

Definition at line 1317 of file [matrix_multi_imp.h](#).

References [db_sqrt\(\)](#), [db_step\(\)](#), [glucat::numeric_traits< Scalar_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [db_sqrt\(\)](#), and [matrix_sqrt\(\)](#).

7.2.2.32 db_step()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
void glucat::db_step (
    matrix_multi< Scalar_T, LO, HI, Tune_P > & M,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & Y) [inline], [static]
```

Single step of product form of Denman-Beavers square root iteration.

Definition at line 1305 of file [matrix_multi_imp.h](#).

References [db_step\(\)](#), and [inv\(\)](#).

Referenced by [cascade_log\(\)](#), [db_sqrt\(\)](#), and [db_step\(\)](#).

7.2.2.33 elliptic()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::elliptic (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Square root of -1 which commutes with all members of the frame of the given multivector The name "elliptic" is now deprecated: use "complexifier" instead.

Definition at line 643 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [elliptic\(\)](#).

Referenced by [elliptic\(\)](#).

7.2.2.34 error_squared()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::error_squared (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold) -> Scalar_T [inline]
```

Relative or absolute error using the quadratic norm.

Definition at line 134 of file [clifford_algebra_imp.h](#).

References [error_squared\(\)](#), and [norm\(\)](#).

Referenced by [approx_equal\(\)](#), and [error_squared\(\)](#).

7.2.2.35 error_squared_tol()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::error_squared_tol (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T
```

Quadratic norm error tolerance relative to a specific multivector.

Definition at line 112 of file [clifford_algebra_imp.h](#).

References [error_squared_tol\(\)](#), and [glucat::numeric_traits< Scalar_T >::pow\(\)](#).

Referenced by [approx_equal\(\)](#), and [error_squared_tol\(\)](#).

7.2.2.36 even()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::even (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_←
_T, LO, HI, Tune_P > [inline]
```

Even part.

Definition at line 513 of file [clifford_algebra_imp.h](#).

References [even\(\)](#).

Referenced by [even\(\)](#).

7.2.2.37 exp() [1/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::exp (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const framed\_multi<
Scalar_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 1745 of file [framed_multi_imp.h](#).

References [clifford_exp\(\)](#), [exp\(\)](#), and [scalar\(\)](#).

Referenced by [cos\(\)](#), [cosh\(\)](#), [crd_of_mult\(\)](#), [exp\(\)](#), [exp\(\)](#), [matrix_log\(\)](#), [matrix_sqrt\(\)](#), [pow\(\)](#), [sin\(\)](#), and [sinh\(\)](#).

7.2.2.38 exp() [2/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::exp (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix\_multi<
Scalar_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 2074 of file [matrix_multi_imp.h](#).

References [clifford_exp\(\)](#), [exp\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), and [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar\(\)](#).

7.2.2.39 fast()

```
template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
auto glucat::fast (
    const Matrix_T & X,
    index_t level) -> Multivector_T [static]
```

Inverse generalized Fast Fourier Transform.

Definition at line 1024 of file [matrix_multi_imp.h](#).

References [fast\(\)](#), [glucat::matrix::signed_perm_nork\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [fast\(\)](#), and [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#).

7.2.2.40 folded_dim()

```
template<typename Matrix_Index_T, const index_t LO, const index_t HI>
auto glucat::folded_dim (
    const index_set< LO, HI > & sub) -> Matrix_Index_T [inline], [static]
```

Determine the matrix dimension of the fold of a subalgebra.

Definition at line 101 of file [matrix_multi_imp.h](#).

References [folded_dim\(\)](#), and [offset_level\(\)](#).

Referenced by [folded_dim\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), and [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#).

7.2.2.41 imag()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::imag (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Imaginary part: deprecated (always 0).

Definition at line 497 of file [clifford_algebra_imp.h](#).

References [imag\(\)](#).

Referenced by [imag\(\)](#).

7.2.2.42 inv()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::inv (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Geometric multiplicative inverse.

Definition at line 400 of file [clifford_algebra_imp.h](#).

References [inv\(\)](#).

Referenced by [db_step\(\)](#), [inv\(\)](#), and [matrix_log\(\)](#).

7.2.2.43 inverse_gray()

```
auto glucat::inverse_gray (
    unsigned long x) -> unsigned long [inline], [static]
```

Inverse Gray code.

Definition at line 863 of file [index_set_imp.h](#).

References [inverse_gray\(\)](#).

Referenced by [inverse_gray\(\)](#), and [glucat::index_set< LO, HI >::sign_of_mult\(\)](#).

7.2.2.44 inverse_reversed_gray()

```
auto glucat::inverse_reversed_gray (
    unsigned long x) -> unsigned long [inline], [static]
```

Inverse reversed Gray code.

Definition at line 846 of file [index_set_imp.h](#).

References [inverse_reversed_gray\(\)](#).

Referenced by [inverse_reversed_gray\(\)](#), and [glucat::index_set< LO, HI >::sign_of_mult\(\)](#).

7.2.2.45 involute()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::involute (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1}).

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

Definition at line 537 of file [clifford_algebra_imp.h](#).

References [involute\(\)](#).

Referenced by [involute\(\)](#).

7.2.2.46 log() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1795 of file [framed_multi_imp.h](#).

References [check_complex\(\)](#), and [log\(\)](#).

7.2.2.47 log() [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 2033 of file [matrix_multi_imp.h](#).

References [check_complex\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), [log\(\)](#), and [matrix_log\(\)](#).

7.2.2.48 log() [3/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Natural logarithm of multivector.

Definition at line 798 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [log\(\)](#).

7.2.2.49 log() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Natural logarithm of multivector with specified complexifier.

Definition at line 790 of file [clifford_algebra_imp.h](#).

References [log\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [crd_of_mult\(\)](#), [log\(\)](#), [log\(\)](#), [log\(\)](#), [log\(\)](#), and [pow\(\)](#).

7.2.2.50 log2()

```
template<typename Scalar_T>
auto glucat::log2 (
    const Scalar_T & x) -> Scalar_T [inline]
```

Log base 2 of scalar.

Definition at line 303 of file [scalar.h](#).

References [log2\(\)](#), and [glucat::numeric_traits< Scalar_T >::log2\(\)](#).

Referenced by [clifford_exp\(\)](#), and [log2\(\)](#).

7.2.2.51 matrix_log()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1957 of file [matrix_multi_imp.h](#).

References [abs\(\)](#), [cascade_log\(\)](#), [glucat::matrix::classify_eigenvalues\(\)](#), [exp\(\)](#), [inv\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, matrix_log\(\), norm\(\), and glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar\(\)](#).

Referenced by [log\(\)](#), and [matrix_log\(\)](#).

7.2.2.52 matrix_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1563 of file [matrix_multi_imp.h](#).

References [abs\(\)](#), [approx_equal\(\)](#), [glucat::matrix::classify_eigenvalues\(\)](#), [cr_sqrt\(\)](#), [db_sqrt\(\)](#), [pade::pade_sqrt_denom< Scalar_T >::exp\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), [matrix_sqrt\(\)](#), [norm\(\)](#), [pade::pade_sqrt_numer< Scalar_T >::pade_approx\(\)](#), [pow\(\)](#), and [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar\(\)](#).

Referenced by [matrix_sqrt\(\)](#), and [sqrt\(\)](#).

7.2.2.53 max_abs()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::max_abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

Definition at line 585 of file [clifford_algebra_imp.h](#).

References [max_abs\(\)](#).

Referenced by [max_abs\(\)](#).

7.2.2.54 max_pos()

```
template<const index\_t LO, const index\_t HI>
auto glucat::max_pos (
    const index\_set< LO, HI > & ist) -> index\_t [inline]
```

Maximum positive index, or 0 if none.

Definition at line 977 of file [index_set_imp.h](#).

References [max_pos\(\)](#).

Referenced by [max_pos\(\)](#), and [operator<<\(\)](#).

7.2.2.55 min_neg()

```
template<const index\_t LO, const index\_t HI>
auto glucat::min_neg (
    const index\_set< LO, HI > & ist) -> index\_t [inline]
```

Minimum negative index, or 0 if none.

Definition at line 970 of file [index_set_imp.h](#).

References [min_neg\(\)](#).

Referenced by [min_neg\(\)](#), and [operator<<\(\)](#).

7.2.2.56 norm()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::norm (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar_T norm == sum of norm of coordinates.

Definition at line 569 of file [clifford_algebra_imp.h](#).

References [norm\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [cascade_log\(\)](#), [cr_sqrt\(\)](#), [db_sqrt\(\)](#), [error_squared\(\)](#), [matrix_log\(\)](#), [matrix_sqrt\(\)](#), and [norm\(\)](#).

7.2.2.57 odd()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::odd (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_←
_T, LO, HI, Tune_P > [inline]
```

Odd part.

Definition at line 521 of file [clifford_algebra_imp.h](#).

References [odd\(\)](#).

Referenced by [odd\(\)](#).

7.2.2.58 offset_level()

```
auto glucat::offset_level (
    const index\_t p,
    const index\_t q) -> index\_t [inline]
```

Determine the log2 dim corresponding to signature p, q.

Definition at line 86 of file [matrix_multi_imp.h](#).

References [offset_level\(\)](#), and [pos_mod\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#), [folded_dim\(\)](#), and [offset_level\(\)](#).

7.2.2.59 operator!=() [1/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T,
const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator!=(
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for inequality of multivectors.

Definition at line 86 of file [clifford_algebra_imp.h](#).

7.2.2.60 operator!=() [2/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator!=(
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> bool [inline]
```

Test for inequality of multivector and scalar.

Definition at line 94 of file [clifford_algebra_imp.h](#).

7.2.2.61 operator!=(()) [3/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator!=(
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> bool [inline]
```

Test for inequality of scalar and multivector.

Definition at line 102 of file [clifford_algebra_imp.h](#).

7.2.2.62 operator%() [1/3]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator% (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed\_multi<
Scalar_T, LO, HI, Tune_P >
```

Left contraction.

Definition at line 597 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_SIZE_T](#), and [crd_of_mult\(\)](#).

7.2.2.63 operator%() [2/3]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator% (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix\_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Left contraction.

Definition at line 579 of file [matrix_multi_imp.h](#).

7.2.2.64 operator%() [3/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T,
const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator% (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Left contraction.

Definition at line 322 of file [clifford_algebra_imp.h](#).

7.2.2.65 operator&() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Inner product.

Definition at line 495 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_SIZE_T](#), and [crd_of_mult\(\)](#).

7.2.2.66 operator&() [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator& (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set< LO, HI > [inline]
```

Set intersection: and.

Definition at line 186 of file [index_set_imp.h](#).

7.2.2.67 operator&() [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Inner product.

Definition at line 560 of file [matrix_multi_imp.h](#).

7.2.2.68 operator&() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Inner product.

Definition at line 307 of file [clifford_algebra_imp.h](#).

7.2.2.69 operator*() [1/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Geometric product.

Definition at line 374 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_SIZE_T](#).

7.2.2.70 operator*() [2/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric product.

Definition at line 500 of file [matrix_multi_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::NaN\(\)](#), and [reframe\(\)](#).

7.2.2.71 operator*() [3/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Geometric product.

Definition at line 277 of file [clifford_algebra_imp.h](#).

7.2.2.72 operator*() [4/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Product of multivector and scalar.

Definition at line 251 of file [clifford_algebra_imp.h](#).

7.2.2.73 operator*() [5/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Product of scalar and multivector.

Definition at line 262 of file [clifford_algebra_imp.h](#).

7.2.2.74 operator*() [6/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::operator* (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T > [inline]
```

Product of terms.

Definition at line 1712 of file [framed_multi_imp.h](#).

References [ord_of_mult\(\)](#).

7.2.2.75 operator+() [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric sum.

Definition at line 206 of file [clifford_algebra_imp.h](#).

7.2.2.76 operator+() [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric sum of multivector and scalar.

Definition at line 181 of file [clifford_algebra_imp.h](#).

7.2.2.77 operator+() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric sum of scalar and multivector.

Definition at line 192 of file [clifford_algebra_imp.h](#).

7.2.2.78 operator-() [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric difference.

Definition at line 240 of file [clifford_algebra_imp.h](#).

7.2.2.79 operator-() [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric difference of multivector and scalar.

Definition at line 217 of file [clifford_algebra_imp.h](#).

7.2.2.80 operator-() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric difference of scalar and multivector.

Definition at line 228 of file [clifford_algebra_imp.h](#).

7.2.2.81 operator/() [1/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Geometric quotient.

Definition at line 731 of file [framed_multi_imp.h](#).

7.2.2.82 operator/() [2/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >
```

Geometric quotient.

Definition at line 612 of file [matrix_multi_imp.h](#).

References [glucat::matrix::isnan\(\)](#), and [reframe\(\)](#).

7.2.2.83 operator/() [3/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Geometric quotient.

Definition at line 374 of file [clifford_algebra_imp.h](#).

7.2.2.84 operator/() [4/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Quotient of multivector and scalar.

Definition at line 348 of file [clifford_algebra_imp.h](#).

7.2.2.85 operator/() [5/5]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator/ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_
_T, LO, HI, Tune_P > [inline]
```

Quotient of scalar and multivector.

Definition at line 359 of file [clifford_algebra_imp.h](#).

7.2.2.86 operator<<() [1/5]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator<< (
    std::ostream & os,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream &
```

Write multivector to output.

Definition at line 1144 of file [framed_multi_imp.h](#).

References [scalar\(\)](#), [glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range\(\)](#), and [glucat::clifford_algebra< Scalar_T, Index](#)

7.2.2.87 operator<<() [2/5]

```
const index\_t HI auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist) -> std::ostream &
```

References [max_pos\(\)](#), [min_neg\(\)](#), and [sign_of_square\(\)](#).

7.2.2.88 operator<<() [3/5]

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist) -> std::ostream &
```

Write out index set.

Definition at line 611 of file [index_set_imp.h](#).

7.2.2.89 operator<<() [4/5]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::operator<< (
    std::ostream & os,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream & [inline]
```

Write multivector to output.

Definition at line 952 of file [matrix_multi_imp.h](#).

7.2.2.90 operator<<() [5/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const std::pair< const index_set< LO, HI >, Scalar_T > & term) -> std::ostream &
```

Write term to output.

Definition at line 1204 of file [framed_multi_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::to_double\(\)](#).

7.2.2.91 operator>>() [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator>> (
    std::istream & s,
    framed_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream &
```

Read multivector from input.

Definition at line 1243 of file [framed_multi_imp.h](#).

References [glucat::index_set< LO, HI >::index_set\(\)](#).

7.2.2.92 operator>>() [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::operator>> (
    std::istream & s,
    index_set< LO, HI > & ist) -> std::istream &
```

Read in index set.

Definition at line 634 of file [index_set_imp.h](#).

7.2.2.93 operator>>() [3/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator>> (
    std::istream & s,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream & [inline]
```

Read multivector from input.

Definition at line 963 of file [matrix_multi_imp.h](#).

7.2.2.94 operator^() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Outer product.

Definition at line 416 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_SIZE_T](#), and [crd_of_mult\(\)](#).

7.2.2.95 operator^() [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator^ (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set< LO, HI > [inline]
```

Symmetric set difference: exclusive or.

Definition at line 161 of file [index_set_imp.h](#).

7.2.2.96 operator^() [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Outer product.

Definition at line 541 of file [matrix_multi_imp.h](#).

7.2.2.97 operator^() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Outer product.

Definition at line 292 of file [clifford_algebra_imp.h](#).

7.2.2.98 operator" | () [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Transformation via twisted adjoint action.

Definition at line 756 of file [framed_multi_imp.h](#).

7.2.2.99 operator" | () [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator| (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs) -> const index_set< LO, HI > [inline]
```

Set union: or.

Definition at line 211 of file [index_set_imp.h](#).

7.2.2.100 operator" | () [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P > [inline]
```

Transformation via twisted adjoint action.

Definition at line 714 of file [matrix_multi_imp.h](#).

7.2.2.101 operator" | () [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Transformation via twisted adjoint action.

Definition at line 389 of file [clifford_algebra_imp.h](#).

7.2.2.102 `outer_pow()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::outer_pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Outer product power of multivector.

Definition at line 470 of file [clifford_algebra_imp.h](#).

References [outer_pow\(\)](#).

Referenced by [outer_pow\(\)](#).

7.2.2.103 `pade_approx()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P, const size_t
Size>
auto glucat::pade_approx (
    const std::array< Scalar_T, Size > & numer,
    const std::array< Scalar_T, Size > & denom,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & X) -> const matrix\_multi< Scalar_
_T, LO, HI, Tune_P > [inline], [static]
```

Pade' approximation.

Definition at line 1242 of file [matrix_multi_imp.h](#).

References [pade_approx\(\)](#).

Referenced by [matrix_sqrt\(\)](#), [pade_approx\(\)](#), and [pade_log\(\)](#).

7.2.2.104 `pade_log()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::pade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix\_multi<
Scalar_T, LO, HI, Tune_P > [static]
```

Pade' approximation of log.

Definition at line 1890 of file [matrix_multi_imp.h](#).

References [pade::pade_log_denom< Scalar_T >::denom](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), [pade::pade_log_numer< Scalar_T >::numer](#), [pade_approx\(\)](#), and [pade_log\(\)](#).

Referenced by [cascade_log\(\)](#), and [pade_log\(\)](#).

7.2.2.105 pos_mod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::pos_mod (
    LHS_T lhs,
    RHS_T rhs) -> LHS_T [inline]
```

Modulo function which works reliably for lhs < 0.

Definition at line 117 of file [global.h](#).

References [pos_mod\(\)](#).

Referenced by [complexifier\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#), [glucat::gen::generator_table< Matrix_T >::gen_vector\(\)](#), [offset_level\(\)](#), [glucat::gen::generator_table< Matrix_T >::operator\(\)](#), and [pos_mod\(\)](#).

7.2.2.106 pow() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P > [inline]
```

Multivector power of multivector.

Definition at line 446 of file [clifford_algebra_imp.h](#).

References [exp\(\)](#), [log\(\)](#), and [pow\(\)](#).

7.2.2.107 pow() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Integer power of multivector.

Definition at line 407 of file [clifford_algebra_imp.h](#).

References [pow\(\)](#).

Referenced by [matrix_sqrt\(\)](#), [pow\(\)](#), and [pow\(\)](#).

7.2.2.108 pure()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::pure (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_
_T, LO, HI, Tune_P > [inline]
```

Pure part.

Definition at line 505 of file [clifford_algebra_imp.h](#).

References [pure\(\)](#).

Referenced by [pure\(\)](#).

7.2.2.109 quad()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::quad (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar_T quadratic form == (rev(x)*x)(0).

Definition at line 561 of file [clifford_algebra_imp.h](#).

References [quad\(\)](#).

Referenced by [quad\(\)](#).

7.2.2.110 real()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::real (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Real part: synonym for scalar part.

Definition at line 486 of file [clifford_algebra_imp.h](#).

References [real\(\)](#).

Referenced by [real\(\)](#).

7.2.2.111 `reframe()`

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::reframe (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs_reframed,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs_reframed) -> const index\_set< LO,
HI > [inline]
```

Find a common frame for operands of a binary operator.

Definition at line 343 of file [matrix_multi_imp.h](#).

References [reframe\(\)](#).

Referenced by [operator*\(\)](#), [operator/\(\)](#), and [reframe\(\)](#).

7.2.2.112 `reverse()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::reverse (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↔
_T, LO, HI, Tune_P > [inline]
```

Reversion, eg. $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$.

Definition at line 545 of file [clifford_algebra_imp.h](#).

References [reverse\(\)](#).

Referenced by [reverse\(\)](#).

7.2.2.113 `scalar()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::scalar (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T [inline]
```

Scalar part.

Definition at line 478 of file [clifford_algebra_imp.h](#).

References [scalar\(\)](#).

Referenced by [exp\(\)](#), [operator<<\(\)](#), and [scalar\(\)](#).

7.2.2.114 sign_of_square()

```
auto glucat::sign_of_square (
    index_t j) -> int [inline]
```

Square of generator $\{j\}$.

Square of generator index j .

Definition at line 963 of file [index_set_imp.h](#).

References [sign_of_square\(\)](#).

Referenced by [operator<<\(\)](#), and [sign_of_square\(\)](#).

7.2.2.115 sin() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Sine of multivector.

Definition at line 978 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [sin\(\)](#).

7.2.2.116 sin() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Sine of multivector with specified complexifier.

Definition at line 955 of file [clifford_algebra_imp.h](#).

References [check_complex\(\)](#), [exp\(\)](#), and [sin\(\)](#).

Referenced by [sin\(\)](#), [sin\(\)](#), and [tan\(\)](#).

7.2.2.117 sinh()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_T, LO, HI, Tune_P > [inline]
```

Hyperbolic sine of multivector.

Definition at line 910 of file [clifford_algebra_imp.h](#).

References [exp\(\)](#), and [sinh\(\)](#).

Referenced by [sinh\(\)](#), and [tanh\(\)](#).

7.2.2.118 sqrt() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1722 of file [framed_multi_imp.h](#).

References [check_complex\(\)](#), and [sqrt\(\)](#).

7.2.2.119 sqrt() [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1657 of file [matrix_multi_imp.h](#).

References [check_complex\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan\(\)](#), [matrix_sqrt\(\)](#), and [sqrt\(\)](#).

7.2.2.120 sqrt() [3/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Square root of multivector.

Definition at line 682 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [sqrt\(\)](#).

7.2.2.121 sqrt() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Square root of multivector with specified complexifier.

Definition at line 674 of file [clifford_algebra_imp.h](#).

References [sqrt\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [crd_of_mult\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [sqrt\(\)](#).

7.2.2.122 star() [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T
```

Hestenes scalar product.

Definition at line 683 of file [framed_multi_imp.h](#).

References [star\(\)](#).

7.2.2.123 star() [2/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 598 of file [matrix_multi_imp.h](#).

References [star\(\)](#).

7.2.2.124 star() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 337 of file [clifford_algebra_imp.h](#).

References [star\(\)](#).

Referenced by [star\(\)](#), [star\(\)](#), and [star\(\)](#).

7.2.2.125 tan() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↵
_T, LO, HI, Tune_P > [inline]
```

Tangent of multivector.

Definition at line 1078 of file [clifford_algebra_imp.h](#).

References [complexifier\(\)](#), and [tan\(\)](#).

7.2.2.126 tan() [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Tangent of multivector with specified complexifier.

Definition at line [1059](#) of file [clifford_algebra_imp.h](#).

References [check_complex\(\)](#), [cos\(\)](#), [sin\(\)](#), and [tan\(\)](#).

Referenced by [tan\(\)](#), and [tan\(\)](#).

7.2.2.127 tanh()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::tanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar_↔
_T, LO, HI, Tune_P > [inline]
```

Hyperbolic tangent of multivector.

Definition at line [1015](#) of file [clifford_algebra_imp.h](#).

References [cosh\(\)](#), [sinh\(\)](#), and [tanh\(\)](#).

Referenced by [tanh\(\)](#).

7.2.2.128 to_demote()

```
template<typename Scalar_T>
auto glucat::to_demote (
    const Scalar_T & val) -> typename numeric\_traits< Scalar_T >::demoted::type
[inline]
```

Cast to demote.

Definition at line [135](#) of file [scalar_imp.h](#).

References [to_demote\(\)](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

Referenced by [to_demote\(\)](#).

7.2.2.129 to_promote()

```
template<typename Scalar_T>
auto glucat::to_promote (
    const Scalar_T & val) -> typename numeric_traits< Scalar_T >::promoted::type
[inline]
```

Cast to promote.

Definition at line 125 of file [scalar_imp.h](#).

References [to_promote\(\)](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

Referenced by [to_promote\(\)](#).

7.2.2.130 try_catch() [1/2]

```
int glucat::try_catch (
    intfn f)
```

Exception catching for functions returning int.

Definition at line 49 of file [try_catch.h](#).

References [try_catch\(\)](#).

Referenced by [glucat::control_t::call\(\)](#), [glucat::control_t::call\(\)](#), [try_catch\(\)](#), and [try_catch\(\)](#).

7.2.2.131 try_catch() [2/2]

```
int glucat::try_catch (
    intintfn f,
    int arg)
```

Exception catching for functions of int returning int.

Definition at line 64 of file [try_catch.h](#).

References [try_catch\(\)](#).

7.2.2.132 vector_part()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::vector_part (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const std::vector< Scalar_
_T > [inline]
```

Vector part of multivector, as a [vector_t](#) with respect to [frame\(\)](#).

Definition at line 529 of file [clifford_algebra_imp.h](#).

References [vector_part\(\)](#).

Referenced by [vector_part\(\)](#).

7.2.3 Variable Documentation

7.2.3.1 BITS_PER_SET_VALUE

```
const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits
```

Number of bits in [set_value_t](#).

Definition at line 103 of file [global.h](#).

Referenced by [_GLUCAT_CTAssert\(\)](#).

7.2.3.2 clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↵
truncation = std::numeric_limits<Scalar_T>::epsilon()
```

Default for truncation.

Definition at line 74 of file [clifford_algebra_imp.h](#).

7.2.3.3 DEFAULT_HI

```
const index_t glucat::DEFAULT_HI = index_t (BITS_PER_SET_VALUE / 2)
```

Default highest index in an index set.

Definition at line 111 of file [global.h](#).

Referenced by [_GLUCAT_CTAssert\(\)](#).

7.2.3.4 l_ln2

```
const long double glucat::l_ln2 = 0.6931471805599453094172321214581766L [static]
```

Definition at line 44 of file [long_double.h](#).

Referenced by [glucat::numeric_traits< Scalar_T >::ln_2\(\)](#).

7.2.3.5 l_pi

```
const long double glucat::l_pi = 3.1415926535897932384626433832795029L [static]
```

Definition at line 43 of file [long_double.h](#).

Referenced by [glucat::numeric_traits< Scalar_T >::pi\(\)](#).

7.2.3.6 MS_PER_S

```
const double glucat::MS_PER_S = 1000.0
```

Timing constant: deprecated here - moved to [test/timing.h](#).

Definition at line 83 of file [global.h](#).

7.2.3.7 Tuning_Fast_Basis_Max_Count

```
const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
```

Definition at line 92 of file [tuning.h](#).

7.2.3.8 Tuning_Fast_CR_Sqrt_Max_Steps

```
const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
```

Definition at line 88 of file [tuning.h](#).

7.2.3.9 Tuning_Fast_DB_Sqrt_Max_Steps

```
const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
```

Definition at line 89 of file [tuning.h](#).

7.2.3.10 Tuning_Fast_Div_Max_Steps

```
const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
```

Definition at line 87 of file [tuning.h](#).

7.2.3.11 Tuning_Fast_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
```

Definition at line 93 of file [tuning.h](#).

7.2.3.12 Tuning_Fast_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
```

Definition at line 94 of file [tuning.h](#).

7.2.3.13 Tuning_Fast_Log_Max_Inner_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
```

Definition at line 91 of file [tuning.h](#).

7.2.3.14 Tuning_Fast_Log_Max_Outer_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
```

Definition at line 90 of file [tuning.h](#).

7.2.3.15 Tuning_Fast_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
```

Definition at line 86 of file [tuning.h](#).

7.2.3.16 Tuning_Fast_Products_Size_Threshold

```
const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0
```

Definition at line 95 of file [tuning.h](#).

7.2.3.17 Tuning_Int_Digits

```
const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
```

Definition at line 36 of file [tuning.h](#).

7.2.3.18 Tuning_Max_Threshold

```
const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning_Int_Digits
```

Definition at line 37 of file [tuning.h](#).

7.2.3.19 Tuning_Naive_Basis_Max_Count

```
const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
```

Definition at line 65 of file [tuning.h](#).

7.2.3.20 Tuning_Naive_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 66 of file [tuning.h](#).

7.2.3.21 Tuning_Naive_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 67 of file [tuning.h](#).

7.2.3.22 Tuning_Naive_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
```

Definition at line 64 of file [tuning.h](#).

7.2.3.23 Tuning_Slow_Basis_Max_Count

```
const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
```

Definition at line 42 of file [tuning.h](#).

7.2.3.24 Tuning_Slow_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 43 of file [tuning.h](#).

7.2.3.25 Tuning_Slow_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 44 of file [tuning.h](#).

7.2.3.26 Tuning_Slow_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
```

Definition at line 41 of file [tuning.h](#).

7.2.3.27 Tuning_Slow_Products_Size_Threshold

```
const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 45 of file [tuning.h](#).

7.3 glucat::gen Namespace Reference

Classes

- class [generator_table](#)
Table of generators for specific signatures.

Typedefs

- using [signature_t](#) = std::pair<[index_t](#), [index_t](#)>
A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min().

Variables

- static const std::array< [index_t](#), 8 > [offset_to_super](#) = {0,-1, 0,-1,-2, 3, 2, 1}
Offsets between the current signature and that of the real superalgebra.

7.3.1 Typedef Documentation

7.3.1.1 signature_t

```
using glucat::gen::signature_t = std::pair<index_t, index_t>
```

A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min().

Definition at line 48 of file [generation.h](#).

7.3.2 Variable Documentation

7.3.2.1 offset_to_super

```
const std::array<index_t, 8> glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1} [static]
```

Offsets between the current signature and that of the real superalgebra.

Definition at line 86 of file [generation.h](#).

7.4 glucat::matrix Namespace Reference

Classes

- struct [eig_genus](#)
Structure containing classification of eigenvalues.

Typedefs

- using [eig_case_t](#)
Classification of eigenvalues of a matrix.

Functions

- template<typename LHS_T, typename RHS_T>
auto [kron](#) (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T
Kronecker tensor product of matrices - as per Matlab kron.
- template<typename LHS_T, typename RHS_T>
auto [mono_kron](#) (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T
Sparse Kronecker tensor product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
auto [nork](#) (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T
Left inverse of Kronecker product.
- template<typename LHS_T, typename RHS_T>
auto [signed_perm_nork](#) (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T
Left inverse of Kronecker product where lhs is a signed permutation matrix.
- template<typename Matrix_T>
auto [nnz](#) (const Matrix_T &m) -> typename Matrix_T::size_type
Number of non-zeros.
- template<typename Matrix_T>
auto [isinf](#) (const Matrix_T &m) -> bool
Infinite.
- template<typename Matrix_T>
auto [isnan](#) (const Matrix_T &m) -> bool
Not a Number.
- template<typename Matrix_T>
auto [unit](#) (const typename Matrix_T::size_type n) -> const Matrix_T
Unit matrix - as per Matlab eye.
- template<typename LHS_T, typename RHS_T>
auto [mono_prod](#) (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type
Product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
auto [sparse_prod](#) (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type
Product of sparse matrices.
- template<typename LHS_T, typename RHS_T>
auto [prod](#) (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type
Product of matrices.
- template<typename Scalar_T, typename LHS_T, typename RHS_T>
auto [inner](#) (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T
*Inner product: $\sum(x(i,j)*y(i,j))/x.nrows()$.*
- template<typename Matrix_T>
auto [norm_frob2](#) (const Matrix_T &val) -> typename Matrix_T::value_type
Square of Frobenius norm.
- template<typename Matrix_T>
auto [trace](#) (const Matrix_T &val) -> typename Matrix_T::value_type
Matrix trace.

- `template<typename Matrix_T>`
`auto eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >`
Eigenvalues of a matrix.
- `template<typename Matrix_T>`
`auto classify\_eigenvalues (const Matrix_T &val) -> eig\_genus< Matrix_T >`
Classify the eigenvalues of a matrix.
- `template<typename LHS_T, typename RHS_T>`
`void nork\_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const`
`typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`
Utility routine for nork: calculate result for a range of indices.
- `template<typename Matrix_T>`
`static auto to\_blaze (const Matrix_T &val) -> blaze::DynamicMatrix< double, blaze::rowMajor >`
Convert matrix to Blaze format.

7.4.1 Typedef Documentation

7.4.1.1 eig_case_t

using [glucat::matrix::eig_case_t](#)

Initial value:

```
enum {
    safe_eigs,
    neg_real_eigs,
    both_eigs}
```

Classification of eigenvalues of a matrix.

Definition at line 133 of file [matrix.h](#).

7.4.2 Function Documentation

7.4.2.1 classify_eigenvalues()

```
template<typename Matrix_T>
auto glucat::matrix::classify_eigenvalues (
    const Matrix_T & val) -> eig\_genus< Matrix_T >
```

Classify the eigenvalues of a matrix.

Definition at line 492 of file [matrix_imp.h](#).

References [classify_eigenvalues\(\)](#), [eigenvalues\(\)](#), [epsilon](#), [glucat::matrix::eig_genus< Matrix_T >::m_eig_case](#), [glucat::matrix::eig_genus< Matrix_T >::m_is_singular](#), [glucat::matrix::eig_genus< Matrix_T >::m_safe_arg](#), [glucat::numeric_traits< Scalar_T >::pi\(\)](#), and [glucat::numeric_traits< Scalar_T >::to_double\(\)](#).

Referenced by [classify_eigenvalues\(\)](#), [glucat::matrix_log\(\)](#), and [glucat::matrix_sqrt\(\)](#).

7.4.2.2 `eigenvalues()`

```
template<typename Matrix_T>
auto glucat::matrix::eigenvalues (
    const Matrix_T & val) -> std::vector< std::complex< double > >
```

Eigenvalues of a matrix.

Definition at line 464 of file [matrix_imp.h](#).

References [eigenvalues\(\)](#), and [to_blaze\(\)](#).

Referenced by [classify_eigenvalues\(\)](#), and [eigenvalues\(\)](#).

7.4.2.3 `inner()`

```
template<typename Scalar_T, typename LHS_T, typename RHS_T>
auto glucat::matrix::inner (
    const LHS_T & lhs,
    const RHS_T & rhs) -> Scalar_T
```

Inner product: $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$.

Inner product: $\text{sum}(lhs(i,j)*rhs(i,j))/lhs.\text{nrows}()$.

Definition at line 368 of file [matrix_imp.h](#).

References [inner\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), and [inner\(\)](#).

7.4.2.4 `isinf()`

```
template<typename Matrix_T>
auto glucat::matrix::isinf (
    const Matrix_T & m) -> bool
```

Infinite.

Definition at line 270 of file [matrix_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::isInf\(\)](#), and [isinf\(\)](#).

Referenced by [isinf\(\)](#).

7.4.2.5 `isnan()`

```
template<typename Matrix_T>
auto glucat::matrix::isnan (
    const Matrix_T & m) -> bool
```

Not a Number.

Definition at line 287 of file [matrix_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::isNaN\(\)](#), and [isnan\(\)](#).

Referenced by [isnan\(\)](#), and [glucat::operator/\(\)](#).

7.4.2.6 kron()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::kron (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Kronecker tensor product of matrices - as per Matlab kron.

Definition at line 78 of file [matrix_imp.h](#).

References [kron\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast\(\)](#), and [kron\(\)](#).

7.4.2.7 mono_kron()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_kron (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Sparse Kronecker tensor product of monomial matrices.

Definition at line 114 of file [matrix_imp.h](#).

References [mono_kron\(\)](#).

Referenced by [glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1\(\)](#), and [mono_kron\(\)](#).

7.4.2.8 mono_prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↔
_type
```

Product of monomial matrices.

Definition at line 315 of file [matrix_imp.h](#).

References [mono_prod\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#), [glucat::gen::generator_table< Matrix_T >::gen_f](#), [glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4\(\)](#), [glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1\(\)](#), and [mono_prod\(\)](#).

7.4.2.9 nnz()

```
template<typename Matrix_T>
auto glucat::matrix::nnz (
    const Matrix_T & m) -> typename Matrix_T::size_type
```

Number of non-zeros.

Definition at line 253 of file [matrix_imp.h](#).

References [nnz\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), and [nnz\(\)](#).

7.4.2.10 nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::nork (
    const LHS_T & lhs,
    const RHS_T & rhs,
    const bool mono = true) -> const RHS_T
```

Left inverse of Kronecker product.

Definition at line 177 of file [matrix_imp.h](#).

References [nork\(\)](#), [nork_range\(\)](#), and [norm_frob2\(\)](#).

Referenced by [nork\(\)](#).

7.4.2.11 nork_range()

```
template<typename LHS_T, typename RHS_T>
void glucat::matrix::nork_range (
    RHS_T & result,
    const typename LHS_T::const_iterator2 lhs_it2,
    const RHS_T & rhs,
    const typename RHS_T::size_type res_s1,
    const typename RHS_T::size_type res_s2)
```

Utility routine for nork: calculate result for a range of indices.

Definition at line 147 of file [matrix_imp.h](#).

References [nork_range\(\)](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

Referenced by [nork\(\)](#), [nork_range\(\)](#), and [signed_perm_nork\(\)](#).

7.4.2.12 norm_frob2()

```
template<typename Matrix_T>
auto glucat::matrix::norm_frob2 (
    const Matrix_T & val) -> typename Matrix_T::value_type
```

Square of Frobenius norm.

Definition at line 390 of file [matrix_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::isNaN\(\)](#), [glucat::numeric_traits< Scalar_T >::NaN\(\)](#), and [norm_frob2\(\)](#).

Referenced by [nork\(\)](#), and [norm_frob2\(\)](#).

7.4.2.13 prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression←
_type [inline]
```

Product of matrices.

Definition at line 356 of file [matrix_imp.h](#).

References [prod\(\)](#).

Referenced by [prod\(\)](#).

7.4.2.14 signed_perm_nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::signed_perm_nork (
    const LHS_T & lhs,
    const RHS_T & rhs) -> const RHS_T
```

Left inverse of Kronecker product where lhs is a signed permutation matrix.

Definition at line 223 of file [matrix_imp.h](#).

References [nork_range\(\)](#), and [signed_perm_nork\(\)](#).

Referenced by [glucat::fast\(\)](#), and [signed_perm_nork\(\)](#).

7.4.2.15 sparse_prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::sparse_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression←
_type [inline]
```

Product of sparse matrices.

Definition at line 345 of file [matrix_imp.h](#).

References [sparse_prod\(\)](#).

Referenced by [sparse_prod\(\)](#).

7.4.2.16 to_blaze()

```
template<typename Matrix_T>
auto glucat::matrix::to_blaze (
    const Matrix_T & val) -> blaze::DynamicMatrix< double, blaze::rowMajor > [static]
```

Convert matrix to Blaze format.

Definition at line 435 of file [matrix_imp.h](#).

References [to_blaze\(\)](#).

Referenced by [eigenvalues\(\)](#), and [to_blaze\(\)](#).

7.4.2.17 trace()

```
template<typename Matrix_T>
auto glucat::matrix::trace (
    const Matrix_T & val) -> typename Matrix_T::value_type
```

Matrix trace.

Definition at line 411 of file [matrix_imp.h](#).

References [glucat::numeric_traits< Scalar_T >::isNaN\(\)](#), [glucat::numeric_traits< Scalar_T >::NaN\(\)](#), and [trace\(\)](#).

Referenced by [trace\(\)](#).

7.4.2.18 unit()

```
template<typename Matrix_T>
auto glucat::matrix::unit (
    const typename Matrix_T::size_type n) -> const Matrix_T [inline]
```

Unit matrix - as per Matlab eye.

Definition at line 305 of file [matrix_imp.h](#).

References [unit\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#), [glucat::fast\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#), [glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1\(\)](#), [glucat::gen::generator_table< Matrix_T >::gen_vector\(\)](#), and [unit\(\)](#).

7.5 glucat::timing Namespace Reference

Functions

- static double [elapsed](#) (clock_t cpu_time)
Elapsed time in milliseconds.

Variables

- const double [MS_PER_SEC](#) = 1000.0
Timing constant: milliseconds per second.
- const double [MS_PER_CLOCK](#) = [MS_PER_SEC](#) / double(CLOCKS_PER_SEC)
Timing constant: milliseconds per clock.
- const int [EXTRA_TRIALS](#) = 2
Timing constant: trial expansion factor.

7.5.1 Function Documentation

7.5.1.1 elapsed()

```
double glucat::timing::elapsed (  
    clock_t cpu_time) [inline], [static]
```

Elapsed time in milliseconds.

Definition at line 51 of file [timing.h](#).

References [elapsed\(\)](#), and [MS_PER_CLOCK](#).

Referenced by [elapsed\(\)](#).

7.5.2 Variable Documentation

7.5.2.1 EXTRA_TRIALS

```
const int glucat::timing::EXTRA_TRIALS = 2
```

Timing constant: trial expansion factor.

Definition at line 45 of file [timing.h](#).

7.5.2.2 MS_PER_CLOCK

```
const double glucat::timing::MS_PER_CLOCK = MS\_PER\_SEC / double(CLOCKS_PER_SEC)
```

Timing constant: milliseconds per clock.

Definition at line 42 of file [timing.h](#).

Referenced by [elapsed\(\)](#).

7.5.2.3 MS_PER_SEC

```
const double glucat::timing::MS_PER_SEC = 1000.0
```

Timing constant: milliseconds per second.

Definition at line 39 of file [timing.h](#).

7.6 pade Namespace Reference

Classes

- struct [pade_sqrt_numer](#)
Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct [pade_sqrt_denom](#)
Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct [pade_sqrt_numer< float >](#)
- struct [pade_sqrt_denom< float >](#)
- struct [pade_sqrt_numer< long double >](#)
- struct [pade_sqrt_denom< long double >](#)
- struct [pade_sqrt_numer< dd_real >](#)
- struct [pade_sqrt_denom< dd_real >](#)
- struct [pade_sqrt_numer< qd_real >](#)
- struct [pade_sqrt_denom< qd_real >](#)
- struct [pade_log_numer](#)
Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct [pade_log_denom](#)
Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct [pade_log_numer< float >](#)
- struct [pade_log_denom< float >](#)
- struct [pade_log_numer< long double >](#)
- struct [pade_log_denom< long double >](#)
- struct [pade_log_numer< dd_real >](#)
- struct [pade_log_denom< dd_real >](#)
- struct [pade_log_numer< qd_real >](#)
- struct [pade_log_denom< qd_real >](#)

Variables

- template<typename Scalar_T>
const [pade_sqrt_numer](#)< Scalar_T >::array [pade_sqrt_numer](#)< Scalar_T >::numer
- template<typename Scalar_T>
const [pade_sqrt_denom](#)< Scalar_T >::array [pade_sqrt_denom](#)< Scalar_T >::denom
- const [pade_sqrt_numer](#)< float >::array [pade_sqrt_numer](#)< float >::numer
- const [pade_sqrt_denom](#)< float >::array [pade_sqrt_denom](#)< float >::denom
- const [pade_sqrt_numer](#)< longdouble >::array [pade_sqrt_numer](#)< longdouble >::numer
- const [pade_sqrt_denom](#)< longdouble >::array [pade_sqrt_denom](#)< longdouble >::denom
- const [pade_sqrt_numer](#)< dd_real >::array [pade_sqrt_numer](#)< dd_real >::numer
- const [pade_sqrt_denom](#)< dd_real >::array [pade_sqrt_denom](#)< dd_real >::denom
- const [pade_sqrt_numer](#)< qd_real >::array [pade_sqrt_numer](#)< qd_real >::numer
- const [pade_sqrt_denom](#)< qd_real >::array [pade_sqrt_denom](#)< qd_real >::denom
- template<typename Scalar_T>
const [pade_log_numer](#)< Scalar_T >::array [pade_log_numer](#)< Scalar_T >::numer
- template<typename Scalar_T>
const [pade_log_denom](#)< Scalar_T >::array [pade_log_denom](#)< Scalar_T >::denom
- const [pade_log_numer](#)< float >::array [pade_log_numer](#)< float >::numer
- const [pade_log_denom](#)< float >::array [pade_log_denom](#)< float >::denom
- const [pade_log_numer](#)< longdouble >::array [pade_log_numer](#)< longdouble >::numer
- const [pade_log_denom](#)< longdouble >::array [pade_log_denom](#)< longdouble >::denom
- const [pade_log_numer](#)< dd_real >::array [pade_log_numer](#)< dd_real >::numer
- const [pade_log_denom](#)< dd_real >::array [pade_log_denom](#)< dd_real >::denom
- const [pade_log_numer](#)< qd_real >::array [pade_log_numer](#)< qd_real >::numer
- const [pade_log_denom](#)< qd_real >::array [pade_log_denom](#)< qd_real >::denom

7.6.1 Variable Documentation

7.6.1.1 pade_log_denom< dd_real >::denom

const [pade_log_denom](#)<dd_real>::array [pade::pade_log_denom](#)< dd_real >::denom

Initial value:

```

=
{
    dd_real("1"),
    dd_real("2100")/dd_real("41"),
    dd_real("341145")/dd_real("1066"),
    dd_real("11069856")/dd_real("19721"),
    dd_real("6918660")/dd_real("19721"),
    dd_real("1410864")/dd_real("16687"),
    dd_real("734825")/dd_real("94054"),
    dd_real("348840")/dd_real("1363783"),
    dd_real("6783")/dd_real("2727566"),
    dd_real("266")/dd_real("53187537"),
    dd_real("7")/dd_real("8155422340"),
    dd_real("21")/dd_real("2"),
    dd_real("12635")/dd_real("82"),
    dd_real("1037799")/dd_real("2132"),
    dd_real("9883800")/dd_real("19721"),
    dd_real("293930")/dd_real("1517"),
    dd_real("88179")/dd_real("3034"),
    dd_real("305235")/dd_real("188108"),
    dd_real("40698")/dd_real("1363783"),
    dd_real("9975")/dd_real("70916716"),
    dd_real("7")/dd_real("70916716"),
    dd_real("1")/dd_real("538257874440")
}
```

Definition at line 1815 of file [matrix_multi_imp.h](#).

7.6.1.2 pade_log_denom< float >::denom

const [pade_log_denom](#)<float>::array [pade::pade_log_denom](#)< float >::denom

Initial value:

```

=
{
    1.0,
    9.0/2.0,
    144.0/17.0,
    147.0/17.0,
    441.0/85.0,
    63.0/34.0,
    84.0/221.0,
    9.0/221.0,
    9.0/4862.0,
    1.0/48620.0
}
```

Definition at line 1753 of file [matrix_multi_imp.h](#).

7.6.1.3 pade_log_denom< longdouble >::denom

const [pade_log_denom](#)<longdouble>::array [pade::pade_log_denom](#)< longdouble >::denom

Initial value:

```

=
{
    1.0L,
    17.0L/2.0L,
    1088.0L/33.0L,
    850.0L/11.0L,
    41650.0L/341.0L,
    140777.0L/1023.0L,
    1126216.0L/9889.0L,
    63206.0L/899.0L,
    790075.0L/24273.0L,
    60775.0L/5394.0L,
    38896.0L/13485.0L,
    21658.0L/40455.0L,
    4165.0L/682341.0L,
    680.0L/2047023.0L,
    21658.0L/310155.0L,
    34.0L/3411705.0L,
    17.0L/129644790.0L,
    1.0L/2333606220
}
```

Definition at line 1780 of file [matrix_multi_imp.h](#).

7.6.1.4 pade_log_denom< qd_real >::denom

```
const pade_log_denom<qd_real>::array pade::pade_log_denom< qd_real >::denom
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("33")/qd_real("2"),
    qd_real("8448")/qd_real("65"),
    qd_real("42284")/qd_real("65"),
    qd_real("211420")/qd_real("91"),
    qd_real("573562")/qd_real("91"),
    qd_real("32119472")/qd_real("2379"),
    qd_real("92917044")/qd_real("3965"),
    qd_real("603960786")/qd_real("17995"),
    qd_real("144626625")/qd_real("3599"),
    qd_real("2776831200")/qd_real("68381"),
    qd_real("16692542100")/qd_real("478667"),
    qd_real("12241197540")/qd_real("478667"),
    qd_real("1098569010")/qd_real("68381"),
    qd_real("31387686000")/qd_real("3624193"),
    qd_real("9939433900")/qd_real("2479711"),
    qd_real("67091178825")/qd_real("42155087"),
    qd_real("2683647153")/qd_real("4959422"),
    qd_real("19083713088")/qd_real("121505839"),
    qd_real("4708152900")/qd_real("121505839"),
    qd_real("941630580")/qd_real("116546417"),
    qd_real("88704330")/qd_real("62755763"),
    qd_real("12902448")/qd_real("62755763"),
    qd_real("1542684")/qd_real("62755763"),
    qd_real("6427850")/qd_real("2698497809"),
    qd_real("3471039")/qd_real("18889484663"),
    qd_real("8544096")/qd_real("774468871183"),
    qd_real("39556")/qd_real("79027435835"),
    qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
    qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
    qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
}
```

Definition at line 1862 of file [matrix_multi_imp.h](#).

7.6.1.5 pade_log_denom< Scalar_T >::denom

```
template<typename Scalar_T>
const pade_log_denom<Scalar_T>::array pade::pade_log_denom< Scalar_T >::denom
```

Initial value:

```
=
{
    1.0,                13.0/2.0,                468.0/25.0,                1573.0/50.0,
    1573.0/46.0,        11583.0/460.0,            10296.0/805.0,            2574.0/575.0,
    11583.0/10925.0,    143.0/874.0,                572.0/37145.0,            117.0/148580.0,
    13.0/742900.0,     1.0/10400600.0
}
```

Definition at line 1727 of file [matrix_multi_imp.h](#).

7.6.1.6 pade_log_numer< dd_real >::numer

```
const pade_log_numer<dd_real>::array pade::pade_log_numer< dd_real >::numer
```

Initial value:

```
=
{
    dd_real("0"),
    dd_real("1"),
}
```

```

        dd_real("10"),
        dd_real("21603")/dd_real("164"),
        dd_real("978724")/dd_real("2665"),
        dd_real("12874933")/dd_real("39442"),
        dd_real("2406734")/dd_real("22755"),
        dd_real("30653165")/dd_real("2402928"),
        dd_real("25346331")/dd_real("47074027"),
        dd_real("105689791")/dd_real("15601677520"),
        dd_real("969715")/dd_real("53502994116"),
        dd_real("118999")/dd_real("26204577562592"),
        dd_real("22781")/dd_real("492"),
        dd_real("5492649")/dd_real("21320"),
        dd_real("4191605")/dd_real("10619"),
        dd_real("11473457")/dd_real("54612"),
        dd_real("166770367")/dd_real("4004880"),
        dd_real("647746389")/dd_real("215195552"),
        dd_real("278270613")/dd_real("3900419380"),
        dd_real("606046475")/dd_real("1379188292768"),
        dd_real("11098301")/dd_real("26204577562592"),
        dd_real("18858053")/dd_real("1392249205900512960")
    }

```

Definition at line 1795 of file [matrix_multi_imp.h](#).

7.6.1.7 pade_log_numer< float >::numer

```
const pade_log_numer<float>::array pade::pade_log_numer< float >::numer
```

Initial value:

```

=
{
    0.0,          1.0,          4.0,          1337.0/204.0,
    385.0/68.0,    1879.0/680.0,    193.0/255.0,    197.0/1820.0,
    419.0/61880.0, 7129.0/61261200.0
}

```

Definition at line 1741 of file [matrix_multi_imp.h](#).

7.6.1.8 pade_log_numer< longdouble >::numer

```
const pade_log_numer<longdouble>::array pade::pade_log_numer< longdouble >::numer
```

Initial value:

```

=
{
    0.0L,          1.0L,          8.0L,          3835.0L/132.0L,
    8365.0L/132.0L, 11363807.0L/122760.0L,    162981.0L/1705.0L,
    9036157.0L/125860.0L,
    18009875.0L/453096.0L,    44211925.0L/2718576.0L,    4149566.0L/849555.0L,
    16973929.0L/16020180.0L,
    172459.0L/1068012.0L,    116317061.0L/7025382936.0L,    19679783.0L/18441630207.0L,
    23763863.0L/614721006900.0L,
    50747.0L/79318839600.0L,    42142223.0L/14295951736466400.0L
}

```

Definition at line 1766 of file [matrix_multi_imp.h](#).

7.6.1.9 pade_log_numer< qd_real >::numer

```
const pade_log_numer<qd_real>::array pade::pade_log_numer< qd_real >::numer
```

Initial value:

```

=
{
    qd_real("0"),
    qd_real("16"),
    qd_real("95201")/qd_real("780"),
    qd_real("30721")/qd_real("52"),
    qd_real("7416257")/qd_real("3640"),
    qd_real("1039099")/qd_real("195"),
    qd_real("6097772319")/qd_real("555100"),
    qd_real("1564058073")/qd_real("85400"),
    qd_real("30404640205")/qd_real("1209264"),
    qd_real("725351278")/qd_real("25193"),
    qd_real("4092322670789")/qd_real("147429436"),
    qd_real("1")
}

```

```

    qd_real("4559713849589")/qd_real("201040140"),
    qd_real("5049361751189")/qd_real("320023080"),
    qd_real("74979677195")/qd_real("8000577"),
    qd_real("16569850691873")/qd_real("3481514244"),
    qd_real("1065906022369")/qd_real("515779888"),
    qd_real("335956770855841")/qd_real("438412904800"),
qd_real("1462444287585964")/qd_real("6041877844275"),
    qd_real("397242326339851")/qd_real("6122436215532"),
    qd_real("64211291334131")/qd_real("4373168725380"),
    qd_real("142322343550859")/qd_real("51080680851480"),
qd_real("154355972958659")/qd_real("351179680853925"),
    qd_real("167483568676259")/qd_real("2937139148960100"),
    qd_real("4230788929433")/qd_real("704913395750424"),
    qd_real("197968763176019")/qd_real("392923948371995600"),
qd_real("10537522306718")/qd_real("319250708052246425"),
    qd_real("236648286272519")/qd_real("144249197475035425500"),
qd_real("260715545088119")/qd_real("4375558990076074573500"),
    qd_real("289596255666839")/qd_real("19287464028255367199880"),
    qd_real("8802625510547")/qd_real("361639950529787563499775"),
    qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
    qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
}

```

Definition at line 1836 of file [matrix_multi_imp.h](#).

7.6.1.10 pade_log_numer< Scalar_T >::numer

```

template<typename Scalar_T>
const pade_log_numer<Scalar_T>::array pade::pade_log_numer< Scalar_T >::numer

```

Initial value:

```

=
{
    0.0,          1.0,          6.0,          4741.0/300.0,
    1441.0/60.0,   107091.0/4600.0,   8638.0/575.0,   263111.0/40250.0,
    153081.0/80500.0, 395243.0/1101240.0, 28549.0/688275.0, 605453.0/228813200.0,
    785633.0/10296594000.0, 1145993.0/1873980108000.0
}

```

Definition at line 1710 of file [matrix_multi_imp.h](#).

7.6.1.11 pade_sqrt_denom< dd_real >::denom

```

const pade_sqrt_denom<dd_real>::array pade::pade_sqrt_denom< dd_real >::denom

```

Initial value:

```

=
{
    dd_real("1"),          dd_real("41")/dd_real("4"),
    dd_real("195")/dd_real("4"), dd_real("9139")/dd_real("64"),
    dd_real("73815")/dd_real("256"), dd_real("435897")/dd_real("1024"),
    dd_real("121737")/dd_real("256"), dd_real("840565")/dd_real("2048"),
    dd_real("4539051")/dd_real("16384"), dd_real("9641775")/dd_real("65536"),
    dd_real("4032015")/dd_real("65536"), dd_real("84672315")/dd_real("4194304"),
    dd_real("86493225")/dd_real("16777216"), dd_real("67863915")/dd_real("67108864"),
    dd_real("5014575")/dd_real("33554432"), dd_real("4345965")/dd_real("268435456"),
    dd_real("5311735")/dd_real("4294967296"), dd_real("1081575")/dd_real("17179869184"),
    dd_real("33649")/dd_real("17179869184"), dd_real("8855")/dd_real("274877906944"),
    dd_real("231")/dd_real("1099511627776"), dd_real("1")/dd_real("4398046511104")
}

```

Definition at line 1488 of file [matrix_multi_imp.h](#).

7.6.1.12 pade_sqrt_denom< float >::denom

```
const pade_sqrt_denom<float>::array pade::pade_sqrt_denom< float >::denom
```

Initial value:

```
=
{
    1.0,          17.0/4.0,          15.0/2.0,          455.0/64.0,
    1001.0/256.0,  1287.0/1024.0,    231.0/1024.0,    165.0/8192.0,
    45.0/65536,   1.0/262144.0
}
```

Definition at line 1425 of file [matrix_multi_imp.h](#).

7.6.1.13 pade_sqrt_denom< longdouble >::denom

```
const pade_sqrt_denom<longdouble>::array pade::pade_sqrt_denom< longdouble >::denom
```

Initial value:

```
=
{
    1.0L,          33.0L/4.0L,          31.0L,          4495.0L/64.0L,
    27405.0L/256.0L,  118755.0L/1024.0L,    94185.0L/1024.0L,    444015.0L/8192.0L,
    1562275.0L/65536.0L,  2042975.0L/262144.0L,    245157.0L/131072.0L,    676039.0L/2097152.0L,
    323323.0L/8388608.0L,  101745.0L/33554432.0L,    4845.0L/33554432.0L,    969.0L/268435456.0L,
    153.0L/4294967296.0L,  1.0L/17179869184.0L
}
```

Definition at line 1452 of file [matrix_multi_imp.h](#).

7.6.1.14 pade_sqrt_denom< qd_real >::denom

```
const pade_sqrt_denom<qd_real>::array pade::pade_sqrt_denom< qd_real >::denom
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("126"),
    qd_real("557845")/qd_real("256"),
    qd_real("12515965")/qd_real("1024"),
    qd_real("1916797311")/qd_real("65536"),
    qd_real("4450881435")/qd_real("131072"),
    qd_real("171503444385")/qd_real("8388608"),
    qd_real("221120793075")/qd_real("33554432"),
    qd_real("4923689695575")/qd_real("4294967296"),
    qd_real("456864812569")/qd_real("4294967296"),
    qd_real("3486599885395")/qd_real("137438953472"),
    qd_real("2804116503573")/qd_real("549755813888"),
    qd_real("1886827875075")/qd_real("2199023255552"),
    qd_real("263012370465")/qd_real("2199023255552"),
    qd_real("240141729555")/qd_real("17592186044416"),
    qd_real("176848560525")/qd_real("140737488355328"),
    qd_real("51538723353")/qd_real("562949953421312"),
    qd_real("1450433115")/qd_real("281474976710656"),
    qd_real("977699359")/qd_real("4503599627370496"),
    qd_real("118183439")/qd_real("18014398509481984"),
    qd_real("9652005")/qd_real("72057594037927936"),
    qd_real("121737")/qd_real("72057594037927936"),
    qd_real("6545")/qd_real("576460752303423488"),
    qd_real("561")/qd_real("18446744073709551616"),
    qd_real("1")/qd_real("73786976294838206464")
}
```

Definition at line 1535 of file [matrix_multi_imp.h](#).

7.6.1.15 pade_sqrt_denom< Scalar_T >::denom

```
template<typename Scalar_T>
const pade_sqrt_denom<Scalar_T>::array pade::pade_sqrt_denom< Scalar_T >::denom
```

Initial value:

```
=
{
    1.0,          25.0/4.0,          69.0/4.0,          1771.0/64.0,
    7315.0/256.0,  20349.0/1024.0,   4845.0/512.0,   12597.0/4096.0,
    21879.0/32768.0, 12155.0/131072.0, 1001.0/131072.0, 1365.0/4194304.0,
    91.0/16777216.0, 1.0/67108864.0
}
```

Definition at line 1399 of file [matrix_multi_imp.h](#).

7.6.1.16 pade_sqrt_numer< dd_real >::numer

```
const pade_sqrt_numer<dd_real>::array pade::pade_sqrt_numer< dd_real >::numer
```

Initial value:

```
=
{
    dd_real("1"),          dd_real("43")/dd_real("4"),
    dd_real("215")/dd_real("4"), dd_real("10621")/dd_real("64"),
    dd_real("90687")/dd_real("256"), dd_real("567987")/dd_real("1024"),
    dd_real("168861")/dd_real("256"), dd_real("1246355")/dd_real("2048"),
    dd_real("7228859")/dd_real("16384"), dd_real("16583853")/dd_real("65536"),
    dd_real("7538115")/dd_real("65536"), dd_real("173376645")/dd_real("4194304"),
    dd_real("195747825")/dd_real("16777216"), dd_real("171655785")/dd_real("67108864"),
    dd_real("14375115")/dd_real("33554432"), dd_real("14375115")/dd_real("268435456"),
    dd_real("20764055")/dd_real("4294967296"), dd_real("5167525")/dd_real("17179869184"),
    dd_real("206701")/dd_real("17179869184"), dd_real("76153")/dd_real("274877906944"),
    dd_real("3311")/dd_real("1099511627776"), dd_real("43")/dd_real("4398046511104")
}
```

Definition at line 1468 of file [matrix_multi_imp.h](#).

7.6.1.17 pade_sqrt_numer< float >::numer

```
const pade_sqrt_numer<float>::array pade::pade_sqrt_numer< float >::numer
```

Initial value:

```
=
{
    1.0,          19.0/4.0,          19.0/2.0,          665.0/64.0,
    1729.0/256.0,  2717.0/1024.0,   627.0/1024.0,   627.0/8192.0,
    285.0/65536.0, 19.0/262144.0
}
```

Definition at line 1413 of file [matrix_multi_imp.h](#).

7.6.1.18 pade_sqrt_numer< longdouble >::numer

```
const pade_sqrt_numer<longdouble>::array pade::pade_sqrt_numer< longdouble >::numer
```

Initial value:

```
=
{
    1.0L,          35.0L/4.0L,          35.0L,          5425.0L/64.0L,
    35525.0L/256.0L, 166257.0L/1024.0L, 143325.0L/1024.0L, 740025.0L/8192.0L,
    2877875.0L/65536.0L, 4206125.0L/262144.0L, 572033.0L/131072.0L, 1820105.0L/2097152.0L,
    1028755.0L/8388608.0L, 395675.0L/33554432.0L, 24225.0L/33554432.0L, 6783.0L/268435456.0L,
    1785.0L/4294967296.0L, 35.0L/17179869184.0L
}
```

Definition at line 1438 of file [matrix_multi_imp.h](#).

7.6.1.19 `pade_sqrt_numer< qd_real >::numer`

```
const pade_sqrt_numer<qd_real>::array pade::pade_sqrt_numer< qd_real >::numer
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("134"),
    qd_real("633485")/qd_real("256"),
    qd_real("15246721")/qd_real("1024"),
    qd_real("2518145487")/qd_real("65536"),
    qd_real("6344873535")/qd_real("131072"),
    qd_real("267226297065")/qd_real("8388608"),
    qd_real("379874182975")/qd_real("33554432"),
    qd_real("9425348845815")/qd_real("4294967296"),
    qd_real("987417498133")/qd_real("4294967296"),
    qd_real("6958363175533")/qd_real("549755813888"),
    qd_real("766166470485")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("17592186044416"),
    qd_real("623623871325")/qd_real("140737488355328"),
    qd_real("203123203803")/qd_real("562949953421312"),
    qd_real("6478601247")/qd_real("281474976710656"),
    qd_real("5038912081")/qd_real("4503599627370496"),
    qd_real("719844583")/qd_real("18014398509481984"),
    qd_real("71853815")/qd_real("72057594037927936"),
    qd_real("1165197")/qd_real("72057594037927936"),
    qd_real("87703")/qd_real("576460752303423488"),
    qd_real("12529")/qd_real("18446744073709551616"),
    qd_real("67")/qd_real("73786976294838206464")
}
```

Definition at line 1509 of file [matrix_multi_imp.h](#).

7.6.1.20 `pade_sqrt_numer< Scalar_T >::numer`

```
template<typename Scalar_T>
const pade_sqrt_numer<Scalar_T>::array pade::pade_sqrt_numer< Scalar_T >::numer
```

Initial value:

```
=
{
    1.0,
    10395.0/256.0,
    53703.0/32768.0,
    819.0/16777216.0,
    27.0/4.0,
    32319.0/1024.0,
    36465.0/131072.0,
    27.0/67108864.0,
    81.0/4.0,
    8721.0/512.0,
    3861.0/131072.0,
    2277.0/64.0,
    26163.0/4096.0,
    7371.0/4194304.0,
}
```

Definition at line 1382 of file [matrix_multi_imp.h](#).

7.7 PyClical Namespace Reference

Classes

- class [index_set](#)
- class [clifford](#)

Functions

- [index_set_hidden_doctests](#) ()
- [clifford_hidden_doctests](#) ()
- [e](#) (obj)
- [istpq](#) (p, q)
- [_test](#) ()

Variables

- `__version__` = str(`glucat_package_version`, 'utf-8')
- `lhs`
- `rhs`
- `threshold` = error_squared_tol(`rhs`) if threshold is `None` else threshold
- `None`
- `tol` = error_squared_tol(`rhs`) if tol is `None` else tol
- `obj`
- `i`
- `ixt`
- `fill`
- `scalar_epsilon` = `epsilon`
- float `pi` = atan(`clifford`(1.0)) * 4.0
- float `tau` = atan(`clifford`(1.0)) * 8.0
- `cl` = `clifford`
- `ist` = `index_set`
- `ninf3` = `e`(4) + `e`(-1)
- `nbar3` = `e`(4) - `e`(-1)

7.7.1 Function Documentation

7.7.1.1 `_test()`

`_test ()` [protected]

Definition at line 2025 of file `PyClical.pyx`.

References `_test()`.

Referenced by `_test()`.

7.7.1.2 `clifford_hidden_doctests()`

`clifford_hidden_doctests ()`

Tests for functions that Doctest cannot see.

For `clifford.__cinit__`: Construct an object of type `clifford`.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2, index_set({1,2})))
2{1,2}
```

```

>>> print(clifford([2,3],index_set({1,2})))
2{1}+3{2}
>>> print(clifford([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'list'>.
>>> print(clifford(None))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'NoneType'>.
>>> print(clifford(None,[1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
>>> print(clifford([1,2],[1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
>>> print(clifford(""))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string ''.
>>> print(clifford("{")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{'.
>>> print(clifford("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1'.
>>> print(clifford("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{+'.
>>> print(clifford("{1-}")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1-'.
>>> print(clifford("{1}+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.

For clifford.__richcmp__: Compare objects of type clifford.

>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True

```

Definition at line 1316 of file [PyClical.pyx](#).

7.7.1.3 e()

```

e (
    obj)

```

Abbreviation for `clifford(index_set(obj))`.

```
>>> print(e(1))
{1}
>>> print(e(-1))
{-1}
>>> print(e(0))
1
```

Definition at line 1999 of file [PyClical.pyx](#).

7.7.1.4 index_set_hidden_doctests()

`index_set_hidden_doctests ()`

Tests for functions that Doctest cannot see.

For `index_set.__cinit__`: Construct `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set(""))
{}
>>> print(index_set("{")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{'.
>>> print(index_set("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
>>> print(index_set("{1,2,100}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
>>> print(index_set({1,2,100}))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid {1, 2, 100}.
>>> print(index_set([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize index_set object from <class 'list'>.
```

For `index_set.__richcmp__`: Compare two objects of class `index_set`.

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
```

```

>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
>>> None == index_set({1,2})
False
>>> None != index_set({1,2})
True
>>> None < index_set({1,2})
False
>>> None <= index_set({1,2})
False
>>> None > index_set({1,2})
False
>>> None >= index_set({1,2})
False
>>> index_set({1,2}) == None
False
>>> index_set({1,2}) != None
True
>>> index_set({1,2}) < None
False
>>> index_set({1,2}) <= None
False
>>> index_set({1,2}) > None
False
>>> index_set({1,2}) >= None
False

```

Definition at line 406 of file [PyClical.pyx](#).

7.7.1.5 istpq()

```

istpq (
    p,
    q)

```

Abbreviation for `index_set({-q,...p})`.

```

>>> print(istpq(2,3))
{-3,-2,-1,1,2}

```

Definition at line 2012 of file [PyClical.pyx](#).

7.7.2 Variable Documentation

7.7.2.1 __version__

```

PyClical.__version__ = str(glucat_package_version,'utf-8') [private]

```

Definition at line 35 of file [PyClical.pyx](#).

7.7.2.2 cl

```

PyClical.cl = clifford

```

Definition at line 1973 of file [PyClical.pyx](#).

7.7.2.3 fill

`PyClical.fill`

Definition at line 1927 of file [PyClical.pyx](#).

7.7.2.4 i

`PyClical.i`

Definition at line 1654 of file [PyClical.pyx](#).

7.7.2.5 ist

`PyClical.ist = index_set`

Definition at line 1991 of file [PyClical.pyx](#).

7.7.2.6 ixt

`PyClical.ixt`

Definition at line 1927 of file [PyClical.pyx](#).

7.7.2.7 lhs

`PyClical.lhs`

Definition at line 1422 of file [PyClical.pyx](#).

7.7.2.8 nbar3

`PyClical.nbar3 =  $e(4) - e(-1)$`

Definition at line 2022 of file [PyClical.pyx](#).

7.7.2.9 ninf3

`PyClical.ninf3 =  $e(4) + e(-1)$`

Definition at line 2021 of file [PyClical.pyx](#).

7.7.2.10 None

`PyClical.None`

Definition at line 1422 of file [PyClical.pyx](#).

7.7.2.11 obj

`PyClical.obj`

Definition at line 1654 of file [PyClical.pyx](#).

7.7.2.12 pi

`float PyClical.pi = atan(clifford(1.0)) * 4.0`

Definition at line 1970 of file [PyClical.pyx](#).

7.7.2.13 rhs

`PyClical.rhs`

Definition at line 1422 of file [PyClical.pyx](#).

7.7.2.14 scalar_epsilon

`PyClical.scalar_epsilon = epsilon`

Definition at line 1968 of file [PyClical.pyx](#).

7.7.2.15 tau

`float PyClical.tau = atan(clifford(1.0)) * 8.0`

Definition at line 1971 of file [PyClical.pyx](#).

7.7.2.16 threshold

`PyClical.threshold = error\_squared\_tol(rhs) if threshold is None else threshold`

Definition at line 1422 of file [PyClical.pyx](#).

7.7.2.17 tol

`PyClical.tol = error\_squared\_tol(rhs) if tol is None else tol`

Definition at line 1422 of file [PyClical.pyx](#).

7.8 std Namespace Reference**Classes**

- struct [numeric_limits](#)< [glucat::framed_multi](#)< [Scalar_T](#), [LO](#), [HI](#), [Tune_P](#) > >
Numeric limits for framed_multi inherit limits for the corresponding scalar type.
- struct [numeric_limits](#)< [glucat::matrix_multi](#)< [Scalar_T](#), [LO](#), [HI](#), [Tune_P](#) > >
Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

Chapter 8

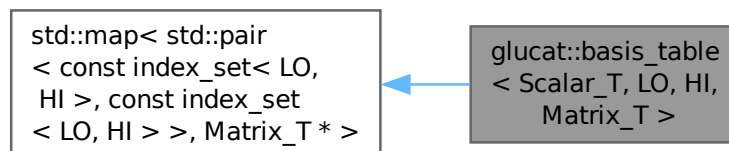
Class Documentation

8.1 glucat::basis_table< Scalar_T, LO, HI, Matrix_T > Class Template Reference

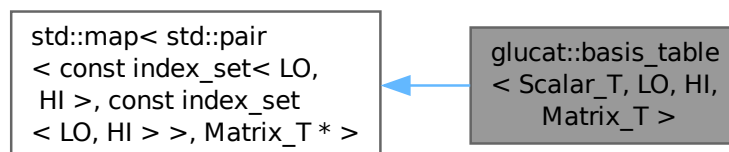
Table of basis elements used as a cache by basis_element().

```
#include <matrix_multi_imp.h>
```

Inheritance diagram for glucat::basis_table< Scalar_T, LO, HI, Matrix_T >:



Collaboration diagram for glucat::basis_table< Scalar_T, LO, HI, Matrix_T >:



Public Member Functions

- [basis_table](#) (const [basis_table](#) &)=delete
- auto [operator=](#) (const [basis_table](#) &) -> [basis_table](#) &=delete

Static Public Member Functions

- static auto [basis](#) () -> [basis_table](#) &
Single instance of basis table.

Private Member Functions

- [basis_table](#) ()=default
- [~basis_table](#) ()=default

Friends

- class [friend_for_private_destructor](#)

8.1.1 Detailed Description

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >
```

Table of basis elements used as a cache by [basis_element](#)().

Definition at line 1159 of file [matrix_multi_imp.h](#).

8.1.2 Constructor & Destructor Documentation

8.1.2.1 [basis_table](#)() [1/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis_table () [private], [default]
```

Referenced by [basis\(\)](#), [basis_table\(\)](#), and [operator=\(\)](#).

8.1.2.2 [~basis_table](#)()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::~~basis_table () [private], [default]
```

8.1.2.3 [basis_table](#)() [2/2]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis_table (
    const basis_table< Scalar_T, LO, HI, Matrix_T > & ) [delete]
```

References [basis_table\(\)](#).

8.1.3 Member Function Documentation

8.1.3.1 basis()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis () -> basis_table & [inline],
[static]
```

Single instance of basis table.

Definition at line 1165 of file [matrix_multi_imp.h](#).

References [basis_table\(\)](#).

8.1.3.2 operator=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::operator= (
    const basis_table< Scalar_T, LO, HI, Matrix_T > & ) -> basis_table &=delete
[delete]
```

References [basis_table\(\)](#).

8.1.4 Friends And Related Symbol Documentation

8.1.4.1 friend_for_private_destructor

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 1170 of file [matrix_multi_imp.h](#).

References [friend_for_private_destructor](#).

Referenced by [friend_for_private_destructor](#).

The documentation for this class was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.2 glucat::bool_to_type< truth_value > Class Template Reference

Bool to type.

```
#include <global.h>
```

Private Types

- `enum { value = truth_value }`

8.2.1 Detailed Description

```
template<bool truth_value>
class glucat::bool_to_type< truth_value >
```

Bool to type.

Definition at line 69 of file [global.h](#).

8.2.2 Member Enumeration Documentation

8.2.2.1 anonymous enum

```
template<bool truth_value>
anonymous enum [private]
```

Enumerator

value	
-------	--

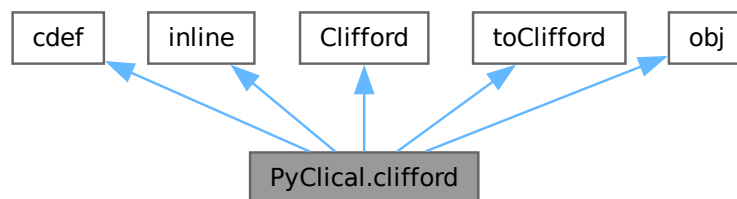
Definition at line 72 of file [global.h](#).

The documentation for this class was generated from the following file:

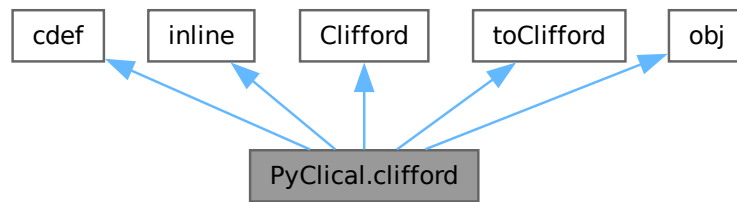
- [glucat/global.h](#)

8.3 PyClical.clifford Class Reference

Inheritance diagram for PyClical.clifford:



Collaboration diagram for PyClical.clifford:



Public Member Functions

- [__cinit__](#) (self, other=0, ixt=None)
- [__dealloc__](#) (self)
- [__contains__](#) (self, x)
- [__iter__](#) (self)
- [reframe](#) (self, ixt)
- [__richcmp__](#) (lhs, rhs, int, op)
- [__getitem__](#) (self, ixt)
- [__neg__](#) (self)
- [__pos__](#) (self)
- [__add__](#) (lhs, rhs)
- [__radd__](#) (rhs, lhs)
- [__iadd__](#) (self, rhs)
- [__sub__](#) (lhs, rhs)
- [__rsub__](#) (rhs, lhs)
- [__isub__](#) (self, rhs)
- [__mul__](#) (lhs, rhs)
- [__rmul__](#) (rhs, lhs)
- [__imul__](#) (self, rhs)
- [__mod__](#) (lhs, rhs)
- [__rmod__](#) (rhs, lhs)
- [__imod__](#) (self, rhs)
- [__and__](#) (lhs, rhs)
- [__rand__](#) (rhs, lhs)
- [__iand__](#) (self, rhs)
- [__xor__](#) (lhs, rhs)
- [__rxor__](#) (rhs, lhs)
- [__ixor__](#) (self, rhs)
- [__truediv__](#) (lhs, rhs)
- [__rtruediv__](#) (rhs, lhs)
- [__idiv__](#) (self, rhs)
- [inv](#) (self)
- [__or__](#) (lhs, rhs)
- [__ior__](#) (self, rhs)
- [__pow__](#) (self, m, dummy)
- [pow](#) (self, m)
- [outer_pow](#) (self, m)

- `__call__` (self, grade)
- `scalar` (self)
- `pure` (self)
- `even` (self)
- `odd` (self)
- `vector_part` (self, frm=None)
- `involute` (self)
- `reverse` (self)
- `conj` (self)
- `quad` (self)
- `norm` (self)
- `abs` (self)
- `max_abs` (self)
- `truncated` (self, limit)
- `isinf` (self)
- `isnan` (self)
- `frame` (self)
- `__repr__` (self)
- `__str__` (self)

Public Attributes

- `instance` = new `Clifford`((<clifford>other).unwrap())

8.3.1 Detailed Description

Python class `clifford` wraps C++ class `Clifford`.

Definition at line 532 of file `PyClical.pyx`.

8.3.2 Member Function Documentation

8.3.2.1 `__add__`()

```
PyClical.clifford.__add__ (
    lhs,
    rhs)
```

Geometric sum.

```
>>> print(clifford(1) + clifford("{2}"))
1+{2}
>>> print(clifford("{1}") + clifford("{2}"))
{1}+{2}
```

Definition at line 740 of file `PyClical.pyx`.

8.3.2.2 `__and__()`

```
PyClical.clifford.__and__ (
    lhs,
    rhs)
```

Inner product.

```
>>> print(clifford("{1}") & clifford("{2}"))
0
>>> print(clifford(2) & clifford("{2}"))
0
>>> print(clifford("{1}") & clifford("{1}"))
1
>>> print(clifford("{1}") & clifford("{1,2}"))
{2}
```

Definition at line 872 of file [PyClical.pyx](#).

8.3.2.3 `__call__()`

```
PyClical.clifford.__call__ (
    self,
    grade)
```

Pure grade-vector part.

```
>>> print(clifford("{1}") (1))
{1}
>>> print(clifford("{1}") (0))
0
>>> print(clifford("1+{1}+{1,2}") (0))
1
>>> print(clifford("1+{1}+{1,2}") (1))
{1}
>>> print(clifford("1+{1}+{1,2}") (2))
{1,2}
>>> print(clifford("1+{1}+{1,2}") (3))
0
```

Definition at line 1083 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index_set.instance](#).

8.3.2.4 `__cinit__()`

```
PyClical.clifford.__cinit__ (
    self,
    other = 0,
    ixt = None)
```

Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2,index_set({1,2})))
2{1,2}
>>> print(clifford([2,3],index_set({1,2})))
2{1}+3{2}
```

Definition at line 565 of file [PyClical.pyx](#).

8.3.2.5 `__contains__()`

```
PyClical.clifford.__contains__ (
    self,
    x)
```

Not applicable.

```
>>> x=clifford(index_set({-3,4,7})); -3 in x
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 627 of file [PyClical.pyx](#).

8.3.2.6 `__dealloc__()`

```
PyClical.clifford.__dealloc__ (
    self)
```

Clean up by deallocating the instance of C++ class Clifford.

Definition at line 621 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index_set.instance](#).

8.3.2.7 `__getitem__()`

```
PyClical.clifford.__getitem__ (
    self,
    ixt)
```

Subscripting: map from index set to scalar coordinate.

```
>>> clifford("{1}") [index_set(1)]
1.0
>>> clifford("{1}") [index_set({1})]
1.0
>>> clifford("{1}") [index_set({1,2})]
0.0
>>> clifford("2{1,2}") [index_set({1,2})]
2.0
```

Definition at line 707 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index_set.instance](#).

8.3.2.8 `__iadd__()`

```
PyClical.clifford.__iadd__ (
    self,
    rhs)
```

Geometric sum.

```
>>> x = clifford(1); x += clifford("{2}"); print(x)
1+{2}
```

Definition at line 760 of file [PyClical.pyx](#).

8.3.2.9 `__iand__()`

```
PyClical.clifford.__iand__ (
    self,
    rhs)
```

Inner product.

```
>>> x = clifford("{1}"); x &= clifford("{2}"); print(x)
0
>>> x = clifford(2); x &= clifford("{2}"); print(x)
0
>>> x = clifford("{1}"); x &= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
{2}
```

Definition at line 896 of file [PyClical.pyx](#).

8.3.2.10 `__idiv__()`

```
PyClical.clifford.__idiv__ (
    self,
    rhs)
```

Geometric quotient.

```
>>> x = clifford("{1}"); x /= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x /= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x /= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
-{2}
```

Definition at line 974 of file [PyClical.pyx](#).

8.3.2.11 `__imod__()`

```
PyClical.clifford.__imod__ (
    self,
    rhs)
```

Contraction.

```
>>> x = clifford("{1}"); x %= clifford("{2}"); print(x)
0
>>> x = clifford(2); x %= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x %= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
{2}
```

Definition at line 857 of file [PyClical.pyx](#).

8.3.2.12 `__imul__()`

```
PyClical.clifford.__imul__ (
    self,
    rhs)
```

Geometric product.

```
>>> x = clifford(2); x *= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x *= clifford("{2}"); print(x)
{1,2}
>>> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
{2}
```

Definition at line 820 of file [PyClical.pyx](#).

8.3.2.13 `__ior__()`

```
PyClical.clifford.__ior__ (
    self,
    rhs)
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
-{1}
```

Definition at line 1013 of file [PyClical.pyx](#).

8.3.2.14 `__isub__()`

```
PyClical.clifford.__isub__ (
    self,
    rhs)
```

Geometric difference.

```
>>> x = clifford(1); x -= clifford("{2}"); print(x)
1-{2}
```

Definition at line 789 of file [PyClical.pyx](#).

8.3.2.15 `__iter__()`

```
PyClical.clifford.__iter__ (
    self)
```

Not applicable.

```
>>> for a in clifford(index_set({-3,4,7})):print(a, end=",")
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 638 of file [PyClical.pyx](#).

8.3.2.16 `__ixor__()`

```
PyClical.clifford.__ixor__ (
    self,
    rhs)
```

Outer product.

```
>>> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x ^= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
0
>>> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
0
```

Definition at line 935 of file [PyClical.pyx](#).

8.3.2.17 `__mod__()`

```
PyClical.clifford.__mod__ (
    lhs,
    rhs)
```

Contraction.

```
>>> print(clifford("{1}") % clifford("{2}"))
0
>>> print(clifford(2) % clifford("{2}"))
2{2}
>>> print(clifford("{1}") % clifford("{1}"))
1
>>> print(clifford("{1}") % clifford("{1,2}"))
{2}
```

Definition at line 833 of file [PyClical.pyx](#).

8.3.2.18 `__mul__()`

```
PyClical.clifford.__mul__ (
    lhs,
    rhs)
```

Geometric product.

```
>>> print(clifford("{1}") * clifford("{2}"))
{1,2}
>>> print(clifford(2) * clifford("{2}"))
2{2}
>>> print(clifford("{1}") * clifford("{1,2}"))
{2}
```

Definition at line 798 of file [PyClical.pyx](#).

8.3.2.19 `__neg__()`

```
PyClical.clifford.__neg__ (
    self)
```

Unary `-`.

```
>>> print(-clifford("{1}"))
-{1}
```

Definition at line 722 of file [PyClical.pyx](#).

References [instance](#), and [PyClical.index_set.instance](#).

8.3.2.20 `__or__()`

```
PyClical.clifford.__or__ (
    lhs,
    rhs)
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
-{1}
```

Definition at line 1002 of file [PyClical.pyx](#).

8.3.2.21 `__pos__()`

```
PyClical.clifford.__pos__ (
    self)
```

Unary +.

```
>>> print(+clifford("{1}"))
{1}
```

Definition at line 731 of file [PyClical.pyx](#).

8.3.2.22 `__pow__()`

```
PyClical.clifford.__pow__ (
    self,
    m,
    dummy)
```

Power: self to the m.

```
>>> x=clifford("{1}"); print(x ** 2)
1
>>> x=clifford("2"); print(x ** 2)
4
>>> x=clifford("2+{1}"); print(x ** 0)
1
>>> x=clifford("2+{1}"); print(x ** 1)
2+{1}
>>> x=clifford("2+{1}"); print(x ** 2)
5+4{1}
>>> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
1
```

Definition at line 1024 of file [PyClical.pyx](#).

References [pow\(\)](#).

8.3.2.23 `__radd__()`

```
PyClical.clifford.__radd__ (
    rhs,
    lhs)
```

Geometric sum.

```
>>> print(1 + clifford("{2}"))
1+{2}
```

Definition at line 751 of file [PyClical.pyx](#).

8.3.2.24 `__rand__()`

```
PyClical.clifford.__rand__ (
    rhs,
    lhs)
```

Inner product.

```
>>> print(2 & clifford("{2}"))
0
```

Definition at line 887 of file [PyClical.pyx](#).

8.3.2.25 `__repr__()`

```
PyClical.clifford.__repr__ (
    self)
```

The “official” string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
```

Definition at line 1298 of file [PyClical.pyx](#).

References [clifford_to_repr\(\)](#).

8.3.2.26 `__richcmp__()`

```
PyClical.clifford.__richcmp__ (
    lhs,
    rhs,
    int,
    op)
```

Compare objects of type clifford.

```
>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 672 of file [PyClical.pyx](#).

8.3.2.27 `__rmod__()`

```
PyClical.clifford.__rmod__ (
    rhs,
    lhs)
```

Contraction.

```
>>> print(2 % clifford("{2}"))
2{2}
```

Definition at line 848 of file [PyClical.pyx](#).

8.3.2.28 `__rmul__()`

```
PyClical.clifford.__rmul__ (
    rhs,
    lhs)
```

Geometric product.

```
>>> print(2 * clifford("{2}"))
2{2}
```

Definition at line 811 of file [PyClical.pyx](#).

8.3.2.29 `__rsub__()`

```
PyClical.clifford.__rsub__ (
    rhs,
    lhs)
```

Geometric difference.

```
>>> print(1 - clifford("{2}"))
1-{2}
```

Definition at line 780 of file [PyClical.pyx](#).

8.3.2.30 `__rtruediv__()`

```
PyClical.clifford.__rtruediv__ (
    rhs,
    lhs)
```

Geometric quotient.

```
>>> print(2 / clifford("{2}"))
2{2}
```

Definition at line 965 of file [PyClical.pyx](#).

8.3.2.31 `__rxor__()`

```
PyClical.clifford.__rxor__ (
    rhs,
    lhs)
```

Outer product.

```
>>> print(2 ^ clifford("{2}"))
2{2}
```

Definition at line 926 of file [PyClical.pyx](#).

8.3.2.32 `__str__()`

```
PyClical.clifford.__str__ (
    self)
```

The “informal” string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
'1+3{-1}+2{1,2}+4{-2,7}'
```

Definition at line 1307 of file [PyClical.pyx](#).

References [clifford_to_str\(\)](#).

8.3.2.33 `__sub__()`

```
PyClical.clifford.__sub__ (
    lhs,
    rhs)
```

Geometric difference.

```
>>> print(clifford(1) - clifford("{2}"))
1-{2}
>>> print(clifford("{1}") - clifford("{2}"))
{1}-{2}
```

Definition at line 769 of file [PyClical.pyx](#).

8.3.2.34 `__truediv__()`

```
PyClical.clifford.__truediv__ (
    lhs,
    rhs)
```

Geometric quotient.

```
>>> print(clifford("{1}") / clifford("{2}"))
{1,2}
>>> print(clifford(2) / clifford("{2}"))
2{2}
>>> print(clifford("{1}") / clifford("{1}"))
1
>>> print(clifford("{1}") / clifford("{1,2}"))
-{2}
```

Definition at line 950 of file [PyClical.pyx](#).

8.3.2.35 `__xor__()`

```
PyClical.clifford.__xor__ (
    lhs,
    rhs)
```

Outer product.

```
>>> print(clifford("{1}") ^ clifford("{2}"))
{1,2}
>>> print(clifford(2) ^ clifford("{2}"))
2{2}
>>> print(clifford("{1}") ^ clifford("{1}"))
0
>>> print(clifford("{1}") ^ clifford("{1,2}"))
0
```

Definition at line 911 of file [PyClical.pyx](#).

8.3.2.36 abs()

```
PyClical.clifford.abs (
    self)
```

Absolute value: square root of norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
2.0
```

Definition at line 1238 of file [PyClical.pyx](#).

References [glucat.abs\(\)](#).

8.3.2.37 conj()

```
PyClical.clifford.conj (
    self)
```

Conjugation, reverse o involute == involute o reverse.

```
>>> print((clifford("{1}")).conj())
-1
>>> print((clifford("{2}") * clifford("{1}")).conj())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).conj())
-1,2
>>> print(clifford("1+{1}+{1,2}").conj())
1-{1}-{1,2}
```

Definition at line 1201 of file [PyClical.pyx](#).

References [conj\(\)](#), [instance](#), and [PyClical.index_set.instance](#).

Referenced by [conj\(\)](#).

8.3.2.38 even()

```
PyClical.clifford.even (
    self)
```

Even part of multivector, sum of even grade terms.

```
>>> print(clifford("1+{1}+{1,2}").even())
1+{1,2}
```

Definition at line 1124 of file [PyClical.pyx](#).

References [even\(\)](#), [instance](#), and [PyClical.index_set.instance](#).

Referenced by [even\(\)](#).

8.3.2.39 frame()

```
PyClical.clifford.frame (
    self)
```

Subalgebra generated by all generators of terms of given multivector.

```
>>> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
{-2,-1,1,2,7}
>>> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
<class 'PyClical.index_set'>
```

Definition at line 1287 of file [PyClical.pyx](#).

References [frame\(\)](#), [instance](#), and [PyClical.index_set.instance](#).

Referenced by [frame\(\)](#).

8.3.2.40 inv()

```
PyClical.clifford.inv (
    self)
```

Geometric multiplicative inverse.

```
>>> x = clifford("{1}"); print(x.inv())
{1}
>>> x = clifford(2); print(x.inv())
0.5
>>> x = clifford("{1,2}"); print(x.inv())
-1,2}
```

Definition at line 989 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [inv\(\)](#).

Referenced by [inv\(\)](#).

8.3.2.41 involute()

```
PyClical.clifford.involute (
    self)
```

Main involution, each {i} is replaced by -{i} in each term,
eg. `clifford("{1}") -> -clifford("{1}")`.

```
>>> print(clifford("{1}").involute())
-1}
>>> print((clifford("{2}") * clifford("{1}")).involute())
-1,2}
>>> print((clifford("{1}") * clifford("{2}")).involute())
1,2}
>>> print(clifford("1+{1}+{1,2}").involute())
1-1}+1,2}
```

Definition at line 1170 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [involute\(\)](#).

Referenced by [involute\(\)](#).

8.3.2.42 isinf()

```
PyClical.clifford.isinf (  
    self)
```

Check if a multivector contains any infinite values.

```
>>> clifford().isinf()  
False
```

Definition at line 1269 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [isnan\(\)](#).

8.3.2.43 isnan()

```
PyClical.clifford.isnan (  
    self)
```

Check if a multivector contains any IEEE NaN values.

```
>>> clifford().isnan()  
False
```

Definition at line 1278 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [isnan\(\)](#).

Referenced by [isinf\(\)](#), and [isnan\(\)](#).

8.3.2.44 max_abs()

```
PyClical.clifford.max_abs (  
    self)
```

Maximum of absolute values of components of multivector: multivector infinity norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()  
1.0  
>>> clifford("3+2{1}+{1,2}").max_abs()  
3.0
```

Definition at line 1247 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [max_abs\(\)](#).

Referenced by [max_abs\(\)](#).

8.3.2.45 norm()

```
PyClical.clifford.norm (  
    self)
```

Norm == sum of squares of coordinates.

```
>>> clifford("1+{1}+{1,2}").norm()  
3.0  
>>> clifford("1+{-1}+{1,2}+{1,2,3}").norm()  
4.0
```

Definition at line 1227 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [norm\(\)](#).

Referenced by [norm\(\)](#).

8.3.2.46 odd()

```
PyClical.clifford.odd (  
    self)
```

Odd part of multivector, sum of odd grade terms.

```
>>> print(clifford("1+{1}+{1,2}").odd())  
{1}
```

Definition at line 1133 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [odd\(\)](#).

Referenced by [odd\(\)](#).

8.3.2.47 outer_pow()

```
PyClical.clifford.outer_pow (  
    self,  
    m)
```

Outer product power.

```
>>> x=clifford("2+{1}"); print(x.outer_pow(0))  
1  
>>> x=clifford("2+{1}"); print(x.outer_pow(1))  
2+{1}  
>>> x=clifford("2+{1}"); print(x.outer_pow(2))  
4+4{1}  
>>> print(clifford("1+{1}+{1,2}").outer_pow(3))  
1+3{1}+3{1,2}
```

Definition at line 1067 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [outer_pow\(\)](#).

Referenced by [outer_pow\(\)](#).

8.3.2.48 pow()

```

PyClical.clifford.pow (
    self,
    m)

Power: self to the m.

>>> x=clifford("{1}"); print(x.pow(2))
1
>>> x=clifford("2"); print(x.pow(2))
4
>>> x=clifford("2+{1}"); print(x.pow(0))
1
>>> x=clifford("2+{1}"); print(x.pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.pow(2))
5+4{1}
>>> print(clifford("1+{1}+{1,2}").pow(3))
1+3{1}+3{1,2}
>>> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
1

```

Definition at line 1043 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [pow\(\)](#).

Referenced by [__pow__\(\)](#), and [pow\(\)](#).

8.3.2.49 pure()

```

PyClical.clifford.pure (
    self)

Pure part.

>>> print(clifford("1+{1}+{1,2}").pure())
{1}+{1,2}
>>> print(clifford("{1,2}").pure())
{1,2}

```

Definition at line 1113 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [pure\(\)](#).

Referenced by [pure\(\)](#).

8.3.2.50 quad()

```

PyClical.clifford.quad (
    self)

Quadratic form == (rev(x)*x)(0).

>>> print(clifford("1+{1}+{1,2}").quad())
3.0
>>> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
2.0

```

Definition at line 1216 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [quad\(\)](#).

Referenced by [quad\(\)](#).

8.3.2.51 reframe()

```
PyClical.clifford.reframe (
    self,
    ixt)
```

Put self into a larger frame, containing the union of self.frame() and index set ixt. This can be used to make multiplication faster, by multiplying within a common frame.

```
>>> clifford("2+3{1}").reframe(index_set({1,2,3}))
clifford("2+3{1}")
>>> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);
True
```

Definition at line 649 of file [PyClical.pyx](#).

8.3.2.52 reverse()

```
PyClical.clifford.reverse (
    self)
```

Reversion, eg. `clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}")`.

```
>>> print(clifford("{1}").reverse())
{1}
>>> print((clifford("{2}") * clifford("{1}")).reverse())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).reverse())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").reverse())
1+{1}-{1,2}
```

Definition at line 1186 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [reverse\(\)](#).

Referenced by [reverse\(\)](#).

8.3.2.53 scalar()

```
PyClical.clifford.scalar (
    self)
```

Scalar part.

```
>>> clifford("1+{1}+{1,2}").scalar()
1.0
>>> clifford("{1,2}").scalar()
0.0
```

Definition at line 1102 of file [PyClical.pyx](#).

References [instance](#), [PyClical.index_set.instance](#), and [scalar\(\)](#).

Referenced by [scalar\(\)](#).

8.3.2.54 truncated()

```
PyClicl.clifford.truncated (
    self,
    limit)
```

Remove all terms of self with relative size smaller than limit.

```
>>> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
clifford("100000000")
>>> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
clifford("10000+{1}")
```

Definition at line 1258 of file [PyClicl.pyx](#).

References [instance](#), [PyClicl.index_set.instance](#), and [truncated\(\)](#).

Referenced by [truncated\(\)](#).

8.3.2.55 vector_part()

```
PyClicl.clifford.vector_part (
    self,
    frm = None)
```

Vector part of multivector, as a Python list, with respect to frm.

```
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
[2.0, 3.0]
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
[0.0, 2.0, 3.0]
```

Definition at line 1142 of file [PyClicl.pyx](#).

References [instance](#), [PyClicl.index_set.instance](#), and [vector_part\(\)](#).

Referenced by [vector_part\(\)](#).

8.3.3 Member Data Documentation

8.3.3.1 instance

```
PyClicl.clifford.instance = new Clifford((<clifford>other).unwrap())
```

Definition at line 592 of file [PyClicl.pyx](#).

Referenced by [__call__\(\)](#), [__dealloc__\(\)](#), [__getitem__\(\)](#), [__neg__\(\)](#), [conj\(\)](#), [even\(\)](#), [frame\(\)](#), [inv\(\)](#), [involute\(\)](#), [isinf\(\)](#), [isnan\(\)](#), [max_abs\(\)](#), [norm\(\)](#), [odd\(\)](#), [outer_pow\(\)](#), [pow\(\)](#), [pure\(\)](#), [quad\(\)](#), [reverse\(\)](#), [scalar\(\)](#), [truncated\(\)](#), and [vector_part\(\)](#).

The documentation for this class was generated from the following file:

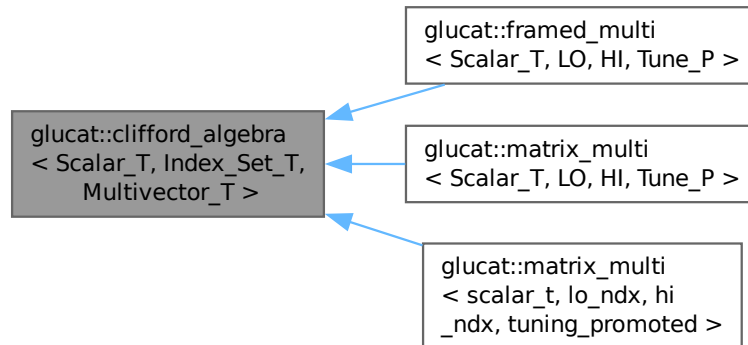
- [pyclicl/PyClicl.pyx](#)

8.4 glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T > Class Template Reference

`clifford_algebra<>` declares the operations of a [Clifford](#) algebra

```
#include <clifford_algebra.h>
```

Inheritance diagram for `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`:



Public Types

- using `scalar_t` = `Scalar_T`
- using `index_set_t` = `Index_Set_T`
- using `multivector_t` = `Multivector_T`
- using `pair_t` = `std::pair<const index_set_t, Scalar_T>`
- using `vector_t` = `std::vector<Scalar_T>`

Public Member Functions

- virtual `~clifford_algebra()` = default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0
Test for equality of multivectors.
- virtual auto `operator==` (const `Scalar_T` &scr) const -> bool=0
Test for equality of multivector and scalar.
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Geometric sum.
- virtual auto `operator+=` (const `Scalar_T` &scr) -> `multivector_t` &=0
Geometric sum of multivector and scalar.
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Geometric difference.
- virtual auto `operator-=` (const `Scalar_T` &scr) -> `multivector_t` &=0
Geometric difference of multivector and scalar.
- virtual auto `operator-` () const -> const `multivector_t`=0
Unary -.

- virtual auto `operator*=(const Scalar_T &scr) -> multivector_t &=0`
Product of multivector and scalar.
- virtual auto `operator*=(const multivector_t &rhs) -> multivector_t &=0`
Geometric product.
- virtual auto `operator%=(const multivector_t &rhs) -> multivector_t &=0`
Contraction.
- virtual auto `operator&=(const multivector_t &rhs) -> multivector_t &=0`
Inner product.
- virtual auto `operator^=(const multivector_t &rhs) -> multivector_t &=0`
Outer product.
- virtual auto `operator/=(const Scalar_T &scr) -> multivector_t &=0`
Quotient of multivector and scalar.
- virtual auto `operator/=(const multivector_t &rhs) -> multivector_t &=0`
Geometric quotient.
- virtual auto `operator|=(const multivector_t &rhs) -> multivector_t &=0`
Transformation via twisted adjoint action.
- virtual auto `inv () const -> const multivector_t=0`
Geometric multiplicative inverse.
- virtual auto `pow (int m) const -> const multivector_t=0`
**this to the m*
- virtual auto `outer_pow (int m) const -> const multivector_t=0`
Outer product power.
- virtual auto `frame () const -> const index_set_t=0`
Subalgebra generated by all generators of terms of given multivector.
- virtual auto `grade () const -> index_t=0`
Maximum of the grades of each term.
- virtual auto `operator[] (const index_set_t ist) const -> Scalar_T=0`
Subscripting: map from index set to scalar coordinate.
- virtual auto `operator() (index_t grade) const -> const multivector_t=0`
Pure grade-vector part.
- virtual auto `scalar () const -> Scalar_T=0`
Scalar part.
- virtual auto `pure () const -> const multivector_t=0`
Pure part.
- virtual auto `even () const -> const multivector_t=0`
Even part of multivector, sum of even grade terms.
- virtual auto `odd () const -> const multivector_t=0`
Odd part of multivector, sum of odd grade terms.
- virtual auto `vector_part () const -> const vector_t=0`
Vector part of multivector, as a `vector_t` with respect to `frame()`.
- virtual auto `vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t=0`
Vector part of multivector, as a `vector_t` with respect to `frm`.
- virtual auto `involute () const -> const multivector_t=0`
Main involution, each $\{i\}$ is replaced by $\{-i\}$ in each term, eg. $\{1\} \rightarrow \{-1\}$.
- virtual auto `reverse () const -> const multivector_t=0`
Reversion, eg. $\{1\}\{2\} \rightarrow \{2\}*\{1\}$.*
- virtual auto `conj () const -> const multivector_t=0`
Conjugation, reverse o involute == involute o reverse.
- virtual auto `quad () const -> Scalar_T=0`
*Scalar_T quadratic form == $(rev(x)*x)(0)$.*
- virtual auto `norm () const -> Scalar_T=0`

- Scalar_T norm == sum of norm of coordinates.*
- virtual auto [max_abs](#) () const -> Scalar_T=0
Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto [truncated](#) (const Scalar_T &limit=[default_truncation](#)) const -> const [multivector_t](#)=0
Remove all terms with relative size smaller than limit.
- virtual auto [isinf](#) () const -> bool=0
Check if a multivector contains any infinite values.
- virtual auto [isnan](#) () const -> bool=0
Check if a multivector contains any IEEE NaN values.
- virtual void [write](#) (const std::string &msg="") const =0
Write formatted multivector to output.
- virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const =0
Write formatted multivector to file.

Static Public Member Functions

- static auto [classname](#) () -> const std::string

Static Public Attributes

- static const [index_t v_lo](#) = index_set_t::v_lo
- static const [index_t v_hi](#) = index_set_t::v_hi
- static const Scalar_T [default_truncation](#)

Default for truncation.

8.4.1 Detailed Description

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
class glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >
```

[clifford_algebra<>](#) declares the operations of a [Clifford](#) algebra

Definition at line [45](#) of file [clifford_algebra.h](#).

8.4.2 Member Typedef Documentation

8.4.2.1 index_set_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::index_set_t = Index_↔
Set_T
```

Definition at line [49](#) of file [clifford_algebra.h](#).

8.4.2.2 multivector_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::multivector_t = Multivector<←
_T
```

Definition at line 52 of file [clifford_algebra.h](#).

8.4.2.3 pair_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::pair_t = std::pair<const
index_set_t, Scalar_T>
```

Definition at line 53 of file [clifford_algebra.h](#).

8.4.2.4 scalar_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar_t = Scalar_T
```

Definition at line 48 of file [clifford_algebra.h](#).

8.4.2.5 vector_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_t = std::←
vector<Scalar_T>
```

Definition at line 54 of file [clifford_algebra.h](#).

8.4.3 Constructor & Destructor Documentation

8.4.3.1 ~clifford_algebra()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::~~clifford_algebra
() [virtual], [default]
```

8.4.4 Member Function Documentation

8.4.4.1 classname()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::classname () -> const
std::string [static]
```

Definition at line 66 of file [clifford_algebra_imp.h](#).

8.4.4.2 conj()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::conj () const
-> const multivector_t [pure virtual]
```

Conjugation, reverse o involute == involute o reverse.

References [conj\(\)](#).

Referenced by [conj\(\)](#).

8.4.4.3 even()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::even () const
-> const multivector_t [pure virtual]
```

Even part of multivector, sum of even grade terms.

References [even\(\)](#).

Referenced by [even\(\)](#).

8.4.4.4 frame()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame () const
-> const index_set_t [pure virtual]
```

Subalgebra generated by all generators of terms of given multivector.

References [frame\(\)](#).

Referenced by [frame\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), and [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#).

8.4.4.5 grade()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::grade () const
-> index_t [pure virtual]
```

Maximum of the grades of each term.

References [grade\(\)](#).

Referenced by [grade\(\)](#), and [operator\(\)\(\)](#).

8.4.4.6 inv()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::inv () const
-> const multivector\_t [pure virtual]
```

Geometric multiplicative inverse.

References [inv\(\)](#).

Referenced by [inv\(\)](#).

8.4.4.7 involute()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::involute ()
const -> const multivector\_t [pure virtual]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

References [involute\(\)](#).

Referenced by [involute\(\)](#).

8.4.4.8 isinf()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isinf () const
-> bool [pure virtual]
```

Check if a multivector contains any infinite values.

References [isinf\(\)](#).

Referenced by [isinf\(\)](#).

8.4.4.9 isnan()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan () const
-> bool [pure virtual]
```

Check if a multivector contains any IEEE NaN values.

References [isnan\(\)](#).

Referenced by [glucat::cascade_log\(\)](#), [glucat::exp\(\)](#), [isnan\(\)](#), [glucat::log\(\)](#), [glucat::matrix_log\(\)](#), [glucat::matrix_sqrt\(\)](#), [glucat::pade_log\(\)](#), and [glucat::sqrt\(\)](#).

8.4.4.10 max_abs()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::max_abs ()
const -> Scalar_T [pure virtual]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

References [max_abs\(\)](#).

Referenced by [max_abs\(\)](#).

8.4.4.11 norm()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::norm () const
-> Scalar_T [pure virtual]
```

Scalar_T norm == sum of norm of coordinates.

References [norm\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), and [norm\(\)](#).

8.4.4.12 odd()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::odd () const
-> const multivector_t [pure virtual]
```

Odd part of multivector, sum of odd grade terms.

References [odd\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast\(\)](#), and [odd\(\)](#).

8.4.4.13 operator%=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator%= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Contraction.

8.4.4.14 operator&=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator&= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Inner product.

8.4.4.15 operator()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator() (
    index\_t grade) const -> const multivector\_t [pure virtual]
```

Pure grade-vector part.

References [grade\(\)](#).

8.4.4.16 operator*=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Geometric product.

8.4.4.17 operator*=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const Scalar_T & scr) -> multivector\_t & [pure virtual]
```

Product of multivector and scalar.

8.4.4.18 operator+=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Geometric sum.

8.4.4.19 operator+=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const Scalar_T & scr) -> multivector\_t & [pure virtual]
```

Geometric sum of multivector and scalar.

8.4.4.20 operator-()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator- (
    const -> const multivector\_t [pure virtual]
```

Unary -.

8.4.4.21 operator-=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Geometric difference.

8.4.4.22 operator-=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const Scalar_T & scr) -> multivector_t & [pure virtual]
```

Geometric difference of multivector and scalar.

8.4.4.23 operator/=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const multivector_t & rhs) -> multivector_t & [pure virtual]
```

Geometric quotient.

8.4.4.24 operator/=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const Scalar_T & scr) -> multivector_t & [pure virtual]
```

Quotient of multivector and scalar.

8.4.4.25 operator==() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator==(
    const multivector_t & val) const -> bool [pure virtual]
```

Test for equality of multivectors.

8.4.4.26 operator==() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator==(
    const Scalar_T & scr) const -> bool [pure virtual]
```

Test for equality of multivector and scalar.

8.4.4.27 operator[]()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator[] (
    const index\_set\_t ist) const -> Scalar_T [pure virtual]
```

Subscripting: map from index set to scalar coordinate.

8.4.4.28 operator^=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator^= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Outer product.

8.4.4.29 operator" |=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator|= (
    const multivector\_t & rhs) -> multivector\_t & [pure virtual]
```

Transformation via twisted adjoint action.

8.4.4.30 outer_pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::outer_pow (
    int m) const -> const multivector\_t [pure virtual]
```

Outer product power.

References [outer_pow\(\)](#).

Referenced by [outer_pow\(\)](#).

8.4.4.31 pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::pow (
    int m) const -> const multivector\_t [pure virtual]
```

*this to the m

References [pow\(\)](#).

Referenced by [pow\(\)](#).

8.4.4.32 pure()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pure () const
-> const multivector_t [pure virtual]
```

Pure part.

References [pure\(\)](#).

Referenced by [pure\(\)](#).

8.4.4.33 quad()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::quad () const
-> Scalar_T [pure virtual]
```

Scalar_T quadratic form == (rev(x)*x)(0).

References [quad\(\)](#).

Referenced by [quad\(\)](#).

8.4.4.34 reverse()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::reverse ()
const -> const multivector_t [pure virtual]
```

Reversion, eg. {1}*{2} -> {2}*{1}.

References [reverse\(\)](#).

Referenced by [reverse\(\)](#).

8.4.4.35 scalar()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar () const
-> Scalar_T [pure virtual]
```

Scalar part.

References [scalar\(\)](#).

Referenced by [glucat::exp\(\)](#), [glucat::matrix_log\(\)](#), [glucat::matrix_sqrt\(\)](#), and [scalar\(\)](#).

8.4.4.36 truncated()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated (
    const Scalar_T & limit = default\_truncation) const -> const multivector\_t [pure
virtual]
```

Remove all terms with relative size smaller than limit.

References [default_truncation](#), and [truncated\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::operator<<\(\)](#), and [truncated\(\)](#).

8.4.4.37 vector_part() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
    const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector_t](#) with respect to [frame\(\)](#).

References [vector_part\(\)](#).

Referenced by [vector_part\(\)](#), and [vector_part\(\)](#).

8.4.4.38 vector_part() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
    const index\_set\_t frm,
    const bool prechecked) const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector_t](#) with respect to *frm*.

References [vector_part\(\)](#).

8.4.4.39 write() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    const std::string & msg = "") const [pure virtual]
```

Write formatted multivector to output.

References [write\(\)](#).

Referenced by [write\(\)](#), and [write\(\)](#).

8.4.4.40 `write()` [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    std::ofstream & ofile,
    const std::string & msg = "") const [pure virtual]
```

Write formatted multivector to file.

References [write\(\)](#).

8.4.5 Member Data Documentation

8.4.5.1 `default_truncation`

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↵
truncation [static]
```

Default for truncation.

Definition at line 59 of file [clifford_algebra.h](#).

Referenced by [truncated\(\)](#).

8.4.5.2 `v_hi`

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index\_t glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_hi = index↵
_set_t::v_hi [static]
```

Definition at line 51 of file [clifford_algebra.h](#).

8.4.5.3 `v_lo`

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index\_t glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_lo = index↵
_set_t::v_lo [static]
```

Definition at line 50 of file [clifford_algebra.h](#).

The documentation for this class was generated from the following files:

- [glucat/clifford_algebra.h](#)
- [glucat/clifford_algebra_imp.h](#)

8.5 `glucat::compare_types< LHS_T, RHS_T >` Class Template Reference

Type comparison.

```
#include <global.h>
```

Public Types

- [enum](#) { [are_same](#) = false }

8.5.1 Detailed Description

```
template<typename LHS_T, typename RHS_T>
class glucat::compare_types< LHS_T, RHS_T >
```

Type comparison.

Definition at line 54 of file [global.h](#).

8.5.2 Member Enumeration Documentation

8.5.2.1 anonymous enum

```
template<typename LHS_T, typename RHS_T>
anonymous enum
```

Enumerator

are_same	
--------------------------	--

Definition at line 57 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

8.6 glucat::compare_types< T, T > Class Template Reference

```
#include <global.h>
```

Public Types

- [enum](#) { [are_same](#) = true }
- [enum](#)

8.6.1 Detailed Description

```
template<typename T>
class glucat::compare_types< T, T >
```

Definition at line 60 of file [global.h](#).

8.6.2 Member Enumeration Documentation

8.6.2.1 anonymous enum

anonymous enum

Definition at line 57 of file [global.h](#).

8.6.2.2 anonymous enum

```
template<typename T>
anonymous enum
```

Enumerator

are_same	
----------	--

Definition at line 63 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

8.7 glucat::control_t Class Reference

Parameters to control tests.

```
#include <control.h>
```

Public Member Functions

- int [call](#) ([intfn](#) f) const
Call a function that returns int.
- int [call](#) ([intintfn](#) f, int arg) const
Call a function of int that returns int.

Static Public Member Functions

- static const [control_t](#) & [control](#) (int argc, char **argv)
- static bool [verbose](#) ()
Produce more detailed output from tests.

Private Member Functions

- bool [valid](#) () const
- bool [catch_exceptions](#) () const
- [control_t](#) (int argc, char **argv)
Constructor from program arguments.
- [control_t](#) ()=default
- [~control_t](#) ()=default
- [control_t](#) (const [control_t](#) &)=delete
- [control_t](#) & [operator=](#) (const [control_t](#) &)=delete

Private Attributes

- bool [m_valid](#)
Test parameters are valid.
- bool [m_catch_exceptions](#)
Catch exceptions.

Static Private Attributes

- static bool [m_verbose_output](#) = false
Produce more detailed output from tests.

Friends

- class [friend_for_private_destructor](#)

8.7.1 Detailed Description

Parameters to control tests.

Definition at line 39 of file [control.h](#).

8.7.2 Constructor & Destructor Documentation

8.7.2.1 [control_t](#)() [1/3]

```
glucat::control_t::control_t (
    int argc,
    char ** argv) [private]
```

Constructor from program arguments.

Test control constructor from program arguments.

Definition at line 88 of file [control.h](#).

References [GLUCAT_PACKAGE_NAME](#), [GLUCAT_VERSION](#), [m_catch_exceptions](#), [m_valid](#), [m_verbose_output](#), and [valid\(\)](#).

Referenced by [control\(\)](#), [control_t\(\)](#), and [operator=\(\)](#).

8.7.2.2 control_t() [2/3]

```
glucat::control_t::control_t () [private], [default]
```

8.7.2.3 ~control_t()

```
glucat::control_t::~~control_t () [private], [default]
```

8.7.2.4 control_t() [3/3]

```
glucat::control_t::control_t (
    const control_t & ) [private], [delete]
```

References [control_t\(\)](#).

8.7.3 Member Function Documentation**8.7.3.1 call()** [1/2]

```
int glucat::control_t::call (
    intfn f) const [inline]
```

Call a function that returns int.

Definition at line 136 of file [control.h](#).

References [catch_exceptions\(\)](#), [glucat::try_catch\(\)](#), and [valid\(\)](#).

8.7.3.2 call() [2/2]

```
int glucat::control_t::call (
    intintfn f,
    int arg) const [inline]
```

Call a function of int that returns int.

Definition at line 150 of file [control.h](#).

References [catch_exceptions\(\)](#), [glucat::try_catch\(\)](#), and [valid\(\)](#).

8.7.3.3 catch_exceptions()

```
bool glucat::control_t::catch_exceptions () const [inline], [private]
```

Definition at line 49 of file [control.h](#).

References [m_catch_exceptions](#).

Referenced by [call\(\)](#), and [call\(\)](#).

8.7.3.4 control()

```
const control\_t & glucat::control_t::control (
    int argc,
    char ** argv) [inline], [static]
```

Single instance Ref: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.

Definition at line 71 of file [control.h](#).

References [control_t\(\)](#).

8.7.3.5 operator=()

```
control\_t & glucat::control_t::operator= (
    const control\_t & ) [private], [delete]
```

References [control_t\(\)](#).

8.7.3.6 valid()

```
bool glucat::control_t::valid () const [inline], [private]
```

Definition at line 44 of file [control.h](#).

References [m_valid](#).

Referenced by [call\(\)](#), [call\(\)](#), and [control_t\(\)](#).

8.7.3.7 verbose()

```
bool glucat::control_t::verbose () [inline], [static]
```

Produce more detailed output from tests.

Definition at line 80 of file [control.h](#).

References [m_verbose_output](#).

8.7.4 Friends And Related Symbol Documentation

8.7.4.1 friend_for_private_destructor

```
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 67 of file [control.h](#).

References [friend_for_private_destructor](#).

Referenced by [friend_for_private_destructor](#).

8.7.5 Member Data Documentation

8.7.5.1 m_catch_exceptions

```
bool glucat::control_t::m_catch_exceptions [private]
```

Catch exceptions.

Definition at line 48 of file [control.h](#).

Referenced by [catch_exceptions\(\)](#), and [control_t\(\)](#).

8.7.5.2 m_valid

```
bool glucat::control_t::m_valid [private]
```

Test parameters are valid.

Definition at line 43 of file [control.h](#).

Referenced by [control_t\(\)](#), and [valid\(\)](#).

8.7.5.3 m_verbose_output

```
bool glucat::control_t::m_verbose_output = false [static], [private]
```

Produce more detailed output from tests.

Definition at line 53 of file [control.h](#).

Referenced by [control_t\(\)](#), and [verbose\(\)](#).

The documentation for this class was generated from the following file:

- test/[control.h](#)

8.8 glucat::CTAssertion< bool > Struct Template Reference

Compile time assertion.

8.8.1 Detailed Description

```
template<bool>
struct glucat::CTAssertion< bool >
```

Compile time assertion.

Definition at line 46 of file [global.h](#).

The documentation for this struct was generated from the following file:

- glucat/[global.h](#)

8.9 `glucat::CTAssertion< true >` Struct Reference

```
#include <global.h>
```

8.9.1 Detailed Description

Definition at line 47 of file [global.h](#).

The documentation for this struct was generated from the following file:

- [glucat/global.h](#)

8.10 `glucat::numeric_traits< Scalar_T >::demoted` Struct Reference

Demoted type for long double.

```
#include <promotion.h>
```

Public Types

- using `type` = float
- using `type` = float

8.10.1 Detailed Description

```
template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::demoted
```

Demoted type for long double.

Demoted type.

Definition at line 76 of file [promotion.h](#).

8.10.2 Member Typedef Documentation

8.10.2.1 `type` [1/2]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 78 of file [promotion.h](#).

8.10.2.2 type [2/2]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 148 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

8.11 glucat::matrix::eig_genus< Matrix_T > Struct Template Reference

Structure containing classification of eigenvalues.

```
#include <matrix.h>
```

Public Types

- using [Scalar_T](#) = typename Matrix_T::value_type

Public Attributes

- bool [m_is_singular](#) = false
Is the matrix singular?
- [eig_case_t](#) [m_eig_case](#) = safe_eigs
What kind of eigenvalues does the matrix contain?
- [Scalar_T](#) [m_safe_arg](#) = [Scalar_T](#)(0)
Argument such that $\exp(\pi m_safe_arg)$ lies between arguments of eigenvalues.

8.11.1 Detailed Description

```
template<typename Matrix_T>
struct glucat::matrix::eig_genus< Matrix_T >
```

Structure containing classification of eigenvalues.

Definition at line 140 of file [matrix.h](#).

8.11.2 Member Typedef Documentation

8.11.2.1 Scalar_T

```
template<typename Matrix_T>
using glucat::matrix::eig_genus< Matrix_T >::Scalar_T = typename Matrix_T::value_type
```

Definition at line 142 of file [matrix.h](#).

8.11.3 Member Data Documentation

8.11.3.1 m_eig_case

```
template<typename Matrix_T>
eig_case_t glucat::matrix::eig_genus< Matrix_T >::m_eig_case = safe_eigs
```

What kind of eigenvalues does the matrix contain?

Definition at line 146 of file [matrix.h](#).

Referenced by [glucat::matrix::classify_eigenvalues\(\)](#).

8.11.3.2 m_is_singular

```
template<typename Matrix_T>
bool glucat::matrix::eig_genus< Matrix_T >::m_is_singular = false
```

Is the matrix singular?

Definition at line 144 of file [matrix.h](#).

Referenced by [glucat::matrix::classify_eigenvalues\(\)](#).

8.11.3.3 m_safe_arg

```
template<typename Matrix_T>
Scalar_T glucat::matrix::eig_genus< Matrix_T >::m_safe_arg = Scalar_T(0)
```

Argument such that $\exp(\pi \cdot m_safe_arg)$ lies between arguments of eigenvalues.

Definition at line 148 of file [matrix.h](#).

Referenced by [glucat::matrix::classify_eigenvalues\(\)](#).

The documentation for this struct was generated from the following file:

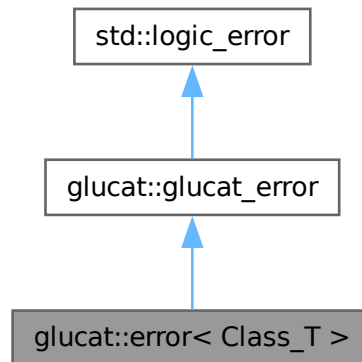
- [glucat/matrix.h](#)

8.12 glucat::error< Class_T > Class Template Reference

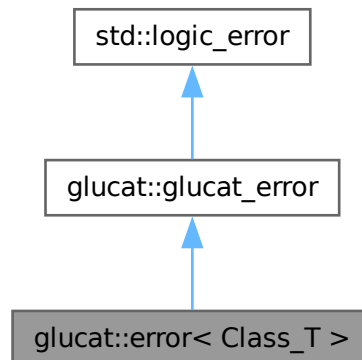
Specific exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::error< Class_T >:



Collaboration diagram for glucat::error< Class_T >:



Public Member Functions

- [error](#) (const std::string &msg)
Specific exception class.
- [error](#) (const std::string &context, const std::string &msg)
- auto [heading](#) () const noexcept -> const std::string override
- auto [classname](#) () const noexcept -> const std::string override
- void [print_error_msg](#) () const override

Public Member Functions inherited from [glucat::glucat_error](#)

- [glucat_error](#) (const std::string &context, const std::string &msg)
- [~glucat_error](#) () noexcept override=default

Additional Inherited Members

Public Attributes inherited from [glucat::glucat_error](#)

- std::string [name](#)

8.12.1 Detailed Description

```
template<class Class_T>
class glucat::error< Class_T >
```

Specific exception class.

Definition at line 56 of file [errors.h](#).

8.12.2 Constructor & Destructor Documentation

8.12.2.1 [error\(\)](#) [1/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
    const std::string & msg)
```

Specific exception class.

Definition at line 44 of file [errors_imp.h](#).

References [classname\(\)](#), and [glucat::glucat_error::glucat_error\(\)](#).

8.12.2.2 [error\(\)](#) [2/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
    const std::string & context,
    const std::string & msg)
```

Definition at line 50 of file [errors_imp.h](#).

References [glucat::glucat_error::glucat_error\(\)](#).

8.12.3 Member Function Documentation

8.12.3.1 classname()

```
template<class Class_T>
auto glucat::error< Class_T >::classname () const -> const std::string [override], [virtual],
[noexcept]
```

Implements [glucat::glucat_error](#).

Definition at line 63 of file [errors_imp.h](#).

References [glucat::glucat_error::name](#).

Referenced by [error\(\)](#), and [print_error_msg\(\)](#).

8.12.3.2 heading()

```
template<class Class_T>
auto glucat::error< Class_T >::heading () const -> const std::string [override], [virtual],
[noexcept]
```

Implements [glucat::glucat_error](#).

Definition at line 57 of file [errors_imp.h](#).

Referenced by [print_error_msg\(\)](#).

8.12.3.3 print_error_msg()

```
template<class Class_T>
void glucat::error< Class_T >::print_error_msg () const [override], [virtual]
```

Implements [glucat::glucat_error](#).

Definition at line 69 of file [errors_imp.h](#).

References [classname\(\)](#), and [heading\(\)](#).

The documentation for this class was generated from the following files:

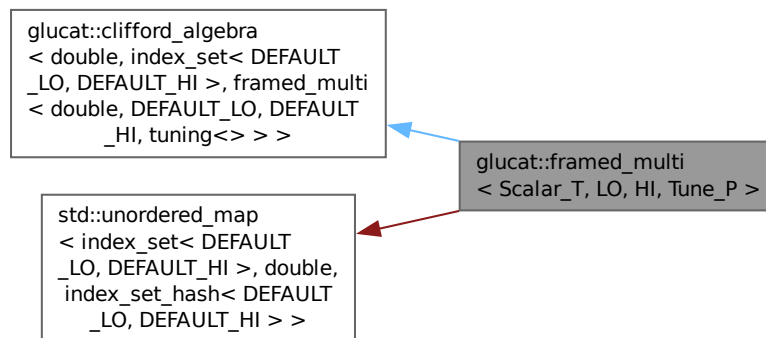
- [glucat/errors.h](#)
- [glucat/errors_imp.h](#)

8.13 `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >` Class Template Reference

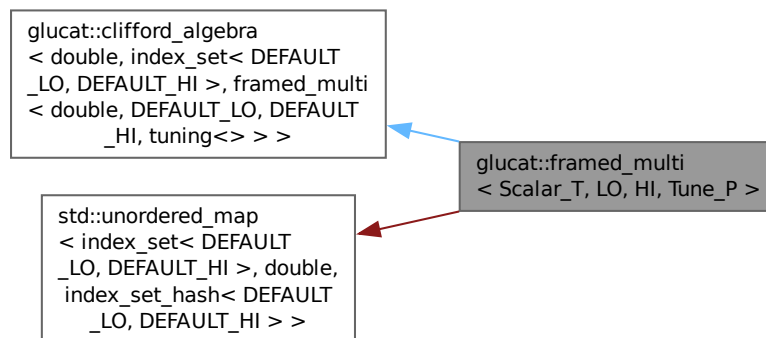
A `framed_multi<Scalar_T,LO,HI,Tune_P>` is a framed approximation to a multivector.

```
#include <framed_multi.h>
```

Inheritance diagram for `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`:



Collaboration diagram for `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`:



Classes

- class `hash_size_t`
- class `var_term`

Variable term.

Public Types

- using `multivector_t` = `framed_multi`
- using `framed_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set`<LO, HI>
- using `term_t` = `std::pair`<const `index_set_t`, `Scalar_T`>
- using `vector_t` = `std::vector`<`Scalar_T`>
- using `error_t` = `error`<`multivector_t`>
- using `matrix_multi_t` = `matrix_multi`<`Scalar_T`,LO,HI,`Tune_P` >

Public Types inherited from `glucat::clifford_algebra`< `double`, `index_set`< `DEFAULT_LO`, `DEFAULT_HI` >, `framed_multi`< `double`, `DEFAULT_LO`, `DEFAULT_HI`, `tuning`<> > >

- using `scalar_t`
- using `index_set_t`
- using `multivector_t`
- using `pair_t`
- using `vector_t`

Public Member Functions

- `~framed_multi` () override=default
Destructor.
- `framed_multi` ()
Default constructor.
- `template<typename Other_Scalar_T>`
`framed_multi` (const `framed_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val)
Construct a multivector from a multivector with a different scalar type.
- `template<typename Other_Scalar_T>`
`framed_multi` (const `framed_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given multivector.
- `framed_multi` (const `framed_multi_t` &val, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given multivector.
- `framed_multi` (const `index_set_t` ist, const `Scalar_T` &crd=`Scalar_T`(1))
Construct a multivector from an index set and a scalar coordinate.
- `framed_multi` (const `index_set_t` ist, const `Scalar_T` &crd, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from an index set and a scalar coordinate.
- `framed_multi` (const `Scalar_T` &scr, const `index_set_t` frm=`index_set_t`())
Construct a multivector from a scalar (within a frame, if given).
- `framed_multi` (const int scr, const `index_set_t` frm=`index_set_t`())
Construct a multivector from an int (within a frame, if given).
- `framed_multi` (const `vector_t` &vec, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given vector.
- `framed_multi` (const `std::string` &str)
Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- `framed_multi` (const `std::string` &str, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- `framed_multi` (const char *str)

- Construct a multivector from a `char*`: eg: `"3+2{1,2}-6.1e-2{2,3}"`.
- `framed_multi` (const `char *str`, const `index_set_t` `frm`, const bool `prechecked=false`)
Construct a multivector, within a given frame, from a `char*`: eg: `"3+2{1,2}-6.1e-2{2,3}"`.
- template<typename Other_Scalar_T>
`framed_multi` (const `matrix_multi`< Other_Scalar_T, LO, HI, Tune_P > &val)
Construct a multivector from a `matrix_multi_t`.
- template<typename Other_Scalar_T>
auto `fast_matrix_multi` (const `index_set_t` `frm`) const -> const `matrix_multi`< Other_Scalar_T, LO, HI, Tune_P >
Use generalized FFT to construct a `matrix_multi_t`.
- auto `fast_framed_multi` () const -> const `framed_multi_t`
Use inverse generalized FFT to construct a `framed_multi_t`.
- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `nbr_terms` () const -> unsigned long
Number of terms.
- auto `operator+=` (const `term_t` &term) -> `multivector_t` &
Add a term, if non-zero.

Public Member Functions inherited from `glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>`

- virtual `~clifford_algebra` ()=default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0
Test for equality of multivectors.
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Geometric sum.
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Geometric difference.
- virtual auto `operator-` () const -> const `multivector_t`=0
Unary -.
- virtual auto `operator*=` (const double &scr) -> `multivector_t` &=0
Product of multivector and scalar.
- virtual auto `operator%=-` (const `multivector_t` &rhs) -> `multivector_t` &=0
Contraction.
- virtual auto `operator&=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Inner product.
- virtual auto `operator^=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Outer product.
- virtual auto `operator/=` (const double &scr) -> `multivector_t` &=0
Quotient of multivector and scalar.
- virtual auto `operator|=` (const `multivector_t` &rhs) -> `multivector_t` &=0
Transformation via twisted adjoint action.
- virtual auto `inv` () const -> const `multivector_t`=0
Geometric multiplicative inverse.
- virtual auto `pow` (int m) const -> const `multivector_t`=0
*this to the m
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0
Outer product power.
- virtual auto `frame` () const -> const `index_set_t`=0
Subalgebra generated by all generators of terms of given multivector.

- virtual auto `grade` () const -> `index_t=0`
Maximum of the grades of each term.
- virtual auto `operator[]` (const `index_set_t` ist) const -> `double=0`
Subscripting: map from index set to scalar coordinate.
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t=0`
Pure grade-vector part.
- virtual auto `scalar` () const -> `double=0`
Scalar part.
- virtual auto `pure` () const -> const `multivector_t=0`
Pure part.
- virtual auto `even` () const -> const `multivector_t=0`
Even part of multivector, sum of even grade terms.
- virtual auto `odd` () const -> const `multivector_t=0`
Odd part of multivector, sum of odd grade terms.
- virtual auto `vector_part` () const -> const `vector_t=0`
Vector part of multivector, as a `vector_t` with respect to `frame()`.
- virtual auto `involute` () const -> const `multivector_t=0`
Main involution, each $\{i\}$ is replaced by $-\{i\}$ in each term, eg. $\{1\} \rightarrow -\{1\}$.
- virtual auto `reverse` () const -> const `multivector_t=0`
Reversion, eg. $\{1\}\{2\} \rightarrow \{2\}*\{1\}$.*
- virtual auto `conj` () const -> const `multivector_t=0`
Conjugation, reverse o involute == involute o reverse.
- virtual auto `quad` () const -> `double=0`
*Scalar_T quadratic form == $(\text{rev}(x)*x)(0)$.*
- virtual auto `norm` () const -> `double=0`
Scalar_T norm == sum of norm of coordinates.
- virtual auto `max_abs` () const -> `double=0`
Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto `truncated` (const `double` &limit=`default_truncation`) const -> const `multivector_t=0`
Remove all terms with relative size smaller than limit.
- virtual auto `isinf` () const -> `bool=0`
Check if a multivector contains any infinite values.
- virtual auto `isnan` () const -> `bool=0`
Check if a multivector contains any IEEE NaN values.
- virtual void `write` (const `std::string` &msg="") const=0
Write formatted multivector to output.

Static Public Member Functions

- static auto `classname` () -> const `std::string`
Class name used in messages.
- static auto `random` (const `index_set_t` frm, `Scalar_T` fill=`Scalar_T(1)`) -> const `multivector_t`
Random multivector within a frame.

Static Public Member Functions inherited from `glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >`

- static auto `classname` () -> const `std::string`

Private Types

- using `var_term_t` = class `var_term`
- using `matrix_t` = typename `matrix_multi_t::matrix_t`
- using `sorted_map_t` = `std::map< index_set_t, Scalar_T, std::less<const index_set_t> >`
- using `map_t` = `std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>>`
- using `framed_pair_t` = `std::pair<const multivector_t, const multivector_t>`
- using `size_type` = typename `map_t::size_type`
- using `iterator` = typename `map_t::iterator`
- using `const_iterator` = typename `map_t::const_iterator`

Private Member Functions

- `framed_multi` (const `hash_size_t` &hash_size)
Private constructor using hash_size.
- auto `fold` (const `index_set_t` frm) const -> `multivector_t`
Subalgebra isomorphism: fold each term within the given frame.
- auto `unfold` (const `index_set_t` frm) const -> `multivector_t`
Subalgebra isomorphism: unfold each term within the given frame.
- auto `centre_pm4_qp4` (`index_t` &p, `index_t` &q) -> `multivector_t` &
Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{p-4,q+4\}}$.
- auto `centre_pp4_qm4` (`index_t` &p, `index_t` &q) -> `multivector_t` &
Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{p+4,q-4\}}$.
- auto `centre_qp1_pm1` (`index_t` &p, `index_t` &q) -> `multivector_t` &
Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{q+1,p-1\}}$.
- auto `divide` (const `index_set_t` ist) const -> const `framed_pair_t`
Divide multivector into part divisible by `index_set` and remainder.
- auto `fast` (const `index_t` level, const bool odd) const -> const `matrix_t`
Generalized FFT from `multivector_t` to `matrix_t`.

Friends

- template<typename Other_Scalar_T, const `index_t` Other_LO, const `index_t` Other_HI, typename Other_Tune_P>
class `matrix_multi`
- template<typename Other_Scalar_T, const `index_t` Other_LO, const `index_t` Other_HI, typename Other_Tune_P>
class `framed_multi`
- auto `operator*` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator^` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator&` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator%` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `star` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> `Scalar_T`
- auto `operator/` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator|` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator>>` (std::istream &s, `multivector_t` &val) -> std::istream &
- auto `operator<<` (std::ostream &os, const `multivector_t` &val) -> std::ostream &
- auto `operator<<` (std::ostream &os, const `term_t` &term) -> std::ostream &
- auto `exp` (const `multivector_t` &val) -> const `multivector_t`

Additional Inherited Members

Static Public Attributes inherited from [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>](#)

- static const [index_t v_lo](#)
- static const [index_t v_hi](#)
- static const double [default_truncation](#)

Default for truncation.

8.13.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >
```

A [framed_multi<Scalar_T,LO,HI,Tune_P>](#) is a framed approximation to a multivector.

Definition at line 126 of file [framed_multi.h](#).

8.13.2 Member Typedef Documentation

8.13.2.1 const_iterator

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::const_iterator = typename map_t::const_iterator [private]
```

Definition at line 167 of file [framed_multi.h](#).

8.13.2.2 error_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector\_t>
```

Definition at line 138 of file [framed_multi.h](#).

8.13.2.3 framed_multi_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = multivector\_t
```

Definition at line 132 of file [framed_multi.h](#).

8.13.2.4 framed_pair_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_pair_t = std::pair<const multivector_t,
const multivector_t> [private]
```

Definition at line 164 of file [framed_multi.h](#).

8.13.2.5 index_set_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 135 of file [framed_multi.h](#).

8.13.2.6 iterator

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::iterator = typename map_t::iterator
[private]
```

Definition at line 166 of file [framed_multi.h](#).

8.13.2.7 map_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::map_t = std::unordered_map<index_set_t,
Scalar_T, index_set_hash<LO, HI>> [private]
```

Definition at line 150 of file [framed_multi.h](#).

8.13.2.8 matrix_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = matrix_multi<Scalar_T, LO, HI, Tune_P >
```

Definition at line 139 of file [framed_multi.h](#).

8.13.2.9 matrix_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = typename matrix_multi_t::matrix_t
[private]
```

Definition at line 148 of file [framed_multi.h](#).

8.13.2.10 `multivector_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = framed\_multi
```

Definition at line 131 of file [framed_multi.h](#).

8.13.2.11 `scalar_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 133 of file [framed_multi.h](#).

8.13.2.12 `size_type`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::size_type = typename map_t::size_type
[private]
```

Definition at line 165 of file [framed_multi.h](#).

8.13.2.13 `sorted_map_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::sorted_map_t = std::map< index\_set\_t,
Scalar_T, std::less<const index\_set\_t> > [private]
```

Definition at line 149 of file [framed_multi.h](#).

8.13.2.14 `term_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index\_set\_t,
Scalar_T>
```

Definition at line 136 of file [framed_multi.h](#).

8.13.2.15 `tune_p`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 134 of file [framed_multi.h](#).

8.13.2.16 var_term_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term_t = class var_term [private]
```

Definition at line 147 of file [framed_multi.h](#).

8.13.2.17 vector_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 137 of file [framed_multi.h](#).

8.13.3 Constructor & Destructor Documentation

8.13.3.1 ~framed_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::~~framed_multi () [override], [default]
```

Destructor.

8.13.3.2 framed_multi() [1/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi ()
```

Default constructor.

Definition at line 59 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.3 framed_multi() [2/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const hash_size_t & hash_size) [private]
```

Private constructor using hash_size.

Definition at line 66 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.4 `framed_multi()` [3/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 74 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#), [framed_multi](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

8.13.3.5 `framed_multi()` [4/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 85 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame\(\)](#), [framed_multi](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

8.13.3.6 `framed_multi()` [5/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 98 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#), and [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame\(\)](#).

8.13.3.7 `framed_multi()` [6/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 111 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.8 framed_multi() [7/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 121 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.9 framed_multi() [8/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from a scalar (within a frame, if given).

Definition at line 134 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.10 framed_multi() [9/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from an int (within a frame, if given).

Definition at line 144 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.11 framed_multi() [10/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 154 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#), [glucat::index_set< LO, HI >::count\(\)](#), [glucat::index_set< LO, HI >::max\(\)](#), and [glucat::index_set< LO, HI >::min\(\)](#).

8.13.3.12 framed_multi() [11/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 176 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.13 framed_multi() [12/15]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 192 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#).

8.13.3.14 framed_multi() [13/15]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str) [inline]
```

Construct a multivector from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 209 of file [framed_multi.h](#).

8.13.3.15 framed_multi() [14/15]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str,
    const index\_set\_t frm,
    const bool prechecked = false) [inline]
```

Construct a multivector, within a given frame, from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 212 of file [framed_multi.h](#).

8.13.3.16 framed_multi() [15/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a [matrix_multi_t](#).

Definition at line 205 of file [framed_multi_imp.h](#).

References [_GLUCAT_HASH_N](#), [_GLUCAT_HASH_SIZE_T](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#), [fast_framed_multi\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame\(\)](#), [glucat::matrix::inner\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix](#), [matrix_multi](#), [glucat::matrix::nnz\(\)](#), [glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated\(\)](#), and [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>::truncated\(\)](#).

8.13.4 Member Function Documentation

8.13.4.1 centre_pm4_qp4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4 (
    index_t & p,
    index_t & q) -> multivector_t & [private]
```

Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{p-4,q+4\}}$.

Definition at line 1464 of file [framed_multi_imp.h](#).

References [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>::frame\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#).

8.13.4.2 centre_pp4_qm4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pp4_qm4 (
    index_t & p,
    index_t & q) -> multivector_t & [private]
```

Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{p+4,q-4\}}$.

Definition at line 1506 of file [framed_multi_imp.h](#).

References [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>::frame\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#).

8.13.4.3 centre_qp1_pm1()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1 (
    index_t & p,
    index_t & q) -> multivector_t & [private]
```

Subalgebra isomorphism: $R_{\{p,q\}}$ to $R_{\{q+1,p-1\}}$.

Definition at line 1548 of file [framed_multi_imp.h](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#).

8.13.4.4 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 50 of file [framed_multi_imp.h](#).

8.13.4.5 divide()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::divide (
    const index_set_t ist) const -> const framed_pair_t [private]
```

Divide multivector into part divisible by [index_set](#) and remainder.

Divide multivector into quotient with terms divisible by index set, and remainder.

Definition at line 1581 of file [framed_multi_imp.h](#).

Referenced by [fast\(\)](#).

8.13.4.6 fast()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast (
    const index_t level,
    const bool odd) const -> const matrix_t [private]
```

Generalized FFT from [multivector_t](#) to [matrix_t](#).

Definition at line 1597 of file [framed_multi_imp.h](#).

References [divide\(\)](#), [glucat::matrix::kron\(\)](#), [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::odd\(\)](#), [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::scalar\(\)](#), and [glucat::matrix::unit\(\)](#).

8.13.4.7 fast_framed_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi_t [inline]
```

Use inverse generalized FFT to construct a [framed_multi_t](#).

Definition at line 1695 of file [framed_multi_imp.h](#).

Referenced by [framed_multi\(\)](#).

8.13.4.8 fast_matrix_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm) const -> const matrix_multi< Other_Scalar_T, LO, HI, Tune_P >
```

Use generalized FFT to construct a [matrix_multi_t](#).

Definition at line 1663 of file [framed_multi_imp.h](#).

References [fold\(\)](#), [matrix_multi](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi](#), and [glucat::pos_mod\(\)](#).

8.13.4.9 fold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fold (
    const index_set_t frm) const -> multivector_t [private]
```

Subalgebra isomorphism: fold each term within the given frame.

Definition at line 1429 of file [framed_multi_imp.h](#).

Referenced by [fast_matrix_multi\(\)](#).

8.13.4.10 nbr_terms()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::nbr_terms () const -> unsigned long
```

Number of terms.

Definition at line 1351 of file [framed_multi_imp.h](#).

8.13.4.11 operator+=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & term) -> multivector_t & [inline]
```

Add a term, if non-zero.

Insert a term into a multivector, add terms with same index set.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 295 of file [framed_multi_imp.h](#).

8.13.4.12 random()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index_set_t frm,
    Scalar_T fill = Scalar_T(1)) -> const multivector_t [static]
```

Random multivector within a frame.

Definition at line 1054 of file [framed_multi_imp.h](#).

References [framed_multi](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::random\(\)](#).

8.13.4.13 unfold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold (
    const index_set_t frm) const -> multivector_t [private]
```

Subalgebra isomorphism: unfold each term within the given frame.

Definition at line 1446 of file [framed_multi_imp.h](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi\(\)](#).

8.13.5 Friends And Related Symbol Documentation

8.13.5.1 exp

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto exp (
    const multivector_t & val) -> const multivector_t [friend]
```

8.13.5.2 framed_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
friend class framed_multi [friend]
```

Definition at line 143 of file [framed_multi.h](#).

Referenced by [framed_multi\(\)](#), [framed_multi\(\)](#), [framed_multi\(\)](#), [framed_multi\(\)](#), and [random\(\)](#).

8.13.5.3 matrix_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
friend class matrix_multi [friend]
```

Definition at line 141 of file [framed_multi.h](#).

Referenced by [fast_matrix_multi\(\)](#), and [framed_multi\(\)](#).

8.13.5.4 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.5 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.6 operator*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.7 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.8 operator<< [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val) -> std::ostream & [friend]
```

8.13.5.9 operator<< [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const term_t & term) -> std::ostream & [friend]
```

8.13.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val) -> std::istream & [friend]
```

8.13.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.12 operator" |

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const multivector_t & lhs,
    const multivector_t & rhs) -> const multivector_t [friend]
```

8.13.5.13 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const multivector_t & lhs,
    const multivector_t & rhs) -> Scalar_T [friend]
```

The documentation for this class was generated from the following files:

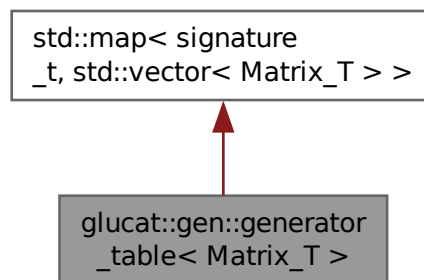
- [glucat/framed_multi.h](#)
- [glucat/framed_multi_imp.h](#)

8.14 glucat::gen::generator_table< Matrix_T > Class Template Reference

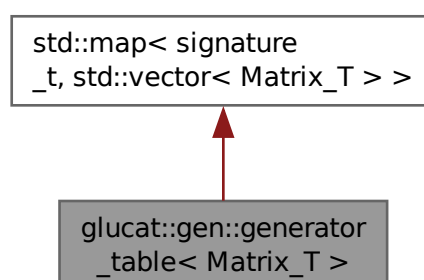
Table of generators for specific signatures.

```
#include <generation.h>
```

Inheritance diagram for glucat::gen::generator_table< Matrix_T >:



Collaboration diagram for glucat::gen::generator_table< Matrix_T >:



Public Member Functions

- auto [operator\(\)](#) (const [index_t](#) p, const [index_t](#) q) -> const Matrix_T *
Pointer to generators for a specific signature.
- [generator_table](#) (const generator_table &)=delete
- auto [operator=](#) (const [generator_table](#) &) -> [generator_table](#) &=delete

Static Public Member Functions

- static auto [generator](#) () -> [generator_table](#)< Matrix_T > &
Single instance of generator table.

Private Member Functions

- auto [gen_vector](#) (const [index_t](#) p, const [index_t](#) q) -> const std::vector< Matrix_T > &
Construct a vector of generators for a specific signature.
- void [gen_from_pm1_qm1](#) (const std::vector< Matrix_T > &old, const [signature_t](#) sig)
Construct generators for p,q given generators for p-1,q-1.
- void [gen_from_pm4_qp4](#) (const std::vector< Matrix_T > &old, const [signature_t](#) sig)
Construct generators for p,q given generators for p-4,q+4.
- void [gen_from_pp4_qm4](#) (const std::vector< Matrix_T > &old, const [signature_t](#) sig)
Construct generators for p,q given generators for p+4,q-4.
- void [gen_from_qp1_pm1](#) (const std::vector< Matrix_T > &old, const [signature_t](#) sig)
Construct generators for p,q given generators for q+1,p-1.
- [generator_table](#) ()=default
- [~generator_table](#) ()=default

Friends

- class [friend_for_private_destructor](#)

8.14.1 Detailed Description

```
template<class Matrix_T>
class glucat::gen::generator_table< Matrix_T >
```

Table of generators for specific signatures.

Definition at line 52 of file [generation.h](#).

8.14.2 Constructor & Destructor Documentation

8.14.2.1 generator_table() [1/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table () [private], [default]
```

Referenced by [generator\(\)](#), [generator_table\(\)](#), and [operator=\(\)](#).

8.14.2.2 `~generator_table()`

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::~~generator_table () [private], [default]
```

8.14.2.3 `generator_table()` [2/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table (
    const generator_table< Matrix_T > & ) [delete]
```

References [generator_table\(\)](#).

8.14.3 Member Function Documentation

8.14.3.1 `gen_from_pm1_qm1()`

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p-1,q-1.

Definition at line 127 of file [generation_imp.h](#).

References [glucat::matrix::mono_kron\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [gen_vector\(\)](#).

8.14.3.2 `gen_from_pm4_qp4()`

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm4_qp4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p-4,q+4.

Definition at line 165 of file [generation_imp.h](#).

References [glucat::matrix::mono_prod\(\)](#).

Referenced by [gen_vector\(\)](#).

8.14.3.3 gen_from_pp4_qm4()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for p+4,q-4.

Definition at line 198 of file [generation_imp.h](#).

References [glucat::matrix::mono_prod\(\)](#).

Referenced by [gen_vector\(\)](#).

8.14.3.4 gen_from_qp1_pm1()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig) [private]
```

Construct generators for p,q given generators for q+1,p-1.

Definition at line 231 of file [generation_imp.h](#).

References [glucat::matrix::mono_prod\(\)](#).

Referenced by [gen_vector\(\)](#).

8.14.3.5 gen_vector()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::gen_vector (
    const index_t p,
    const index_t q) -> const std::vector< Matrix_T > & [private]
```

Construct a vector of generators for a specific signature.

Definition at line 79 of file [generation_imp.h](#).

References [gen_from_pm1_qm1\(\)](#), [gen_from_pm4_qp4\(\)](#), [gen_from_pp4_qm4\(\)](#), [gen_from_qp1_pm1\(\)](#), [gen_vector\(\)](#), [glucat::pos_mod\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [gen_vector\(\)](#), and [operator>\(\)\(\)](#).

8.14.3.6 generator()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::generator () -> generator_table< Matrix_T > &
[static]
```

Single instance of generator table.

Definition at line 49 of file [generation_imp.h](#).

References [generator_table\(\)](#).

Referenced by [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element\(\)](#).

8.14.3.7 operator()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator() (
    const index_t p,
    const index_t q) -> const Matrix_T * [inline]
```

Pointer to generators for a specific signature.

Definition at line 58 of file [generation_imp.h](#).

References [gen_vector\(\)](#), and [glucat::pos_mod\(\)](#).

8.14.3.8 operator=()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator= (
    const generator_table< Matrix_T > & ) -> generator_table &=delete [delete]
```

References [generator_table\(\)](#).

8.14.4 Friends And Related Symbol Documentation

8.14.4.1 friend_for_private_destructor

```
template<class Matrix_T>
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 75 of file [generation.h](#).

References [friend_for_private_destructor](#).

Referenced by [friend_for_private_destructor](#).

The documentation for this class was generated from the following files:

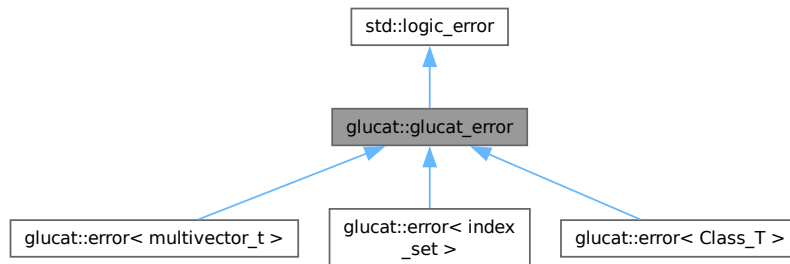
- [glucat/generation.h](#)
- [glucat/generation_imp.h](#)

8.15 glucat::glucat_error Class Reference

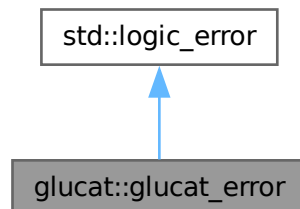
Abstract exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::glucat_error:



Collaboration diagram for glucat::glucat_error:



Public Member Functions

- `glucat_error` (const std::string &context, const std::string &msg)
- `~glucat_error` () noexcept override=default
- virtual auto `heading` () const noexcept -> const std::string=0
- virtual auto `classname` () const noexcept -> const std::string=0
- virtual void `print_error_msg` () const =0

Public Attributes

- std::string `name`

8.15.1 Detailed Description

Abstract exception class.

Definition at line 41 of file [errors.h](#).

8.15.2 Constructor & Destructor Documentation

8.15.2.1 `glucat_error()`

```
glucat::glucat_error::glucat_error (
    const std::string & context,
    const std::string & msg) [inline]
```

Definition at line 44 of file [errors.h](#).

References [name](#).

Referenced by [glucat::error< Class_T >::error\(\)](#), and [glucat::error< Class_T >::error\(\)](#).

8.15.2.2 `~glucat_error()`

```
glucat::glucat_error::~~glucat_error () [override], [default], [noexcept]
```

8.15.3 Member Function Documentation

8.15.3.1 `classname()`

```
virtual auto glucat::glucat_error::classname () const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class_T >](#), [glucat::error< index_set >](#), and [glucat::error< multivector_t >](#).

References [classname\(\)](#).

Referenced by [classname\(\)](#).

8.15.3.2 `heading()`

```
virtual auto glucat::glucat_error::heading () const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class_T >](#), [glucat::error< index_set >](#), and [glucat::error< multivector_t >](#).

References [heading\(\)](#).

Referenced by [heading\(\)](#).

8.15.3.3 `print_error_msg()`

```
virtual void glucat::glucat_error::print_error_msg () const [pure virtual]
```

Implemented in [glucat::error< Class_T >](#), [glucat::error< index_set >](#), and [glucat::error< multivector_t >](#).

References [print_error_msg\(\)](#).

Referenced by [print_error_msg\(\)](#).

8.15.4 Member Data Documentation

8.15.4.1 `name`

```
std::string glucat::glucat_error::name
```

Definition at line 51 of file [errors.h](#).

Referenced by [glucat::error< Class_T >::classname\(\)](#), and [glucat_error\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/errors.h](#)

8.16 `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t` Class Reference

Public Member Functions

- [hash_size_t](#) (size_t hash_size)
- auto [operator\(\)](#) () const -> size_t

Private Attributes

- size_t [n](#)

8.16.1 Detailed Description

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t
```

Definition at line 152 of file [framed_multi.h](#).

8.16.2 Constructor & Destructor Documentation

8.16.2.1 hash_size_t()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::hash_size_t (
    size_t hash_size) [inline]
```

Definition at line 155 of file [framed_multi.h](#).

References [n](#).

8.16.3 Member Function Documentation

8.16.3.1 operator()()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::operator() () const ->
size_t [inline]
```

Definition at line 158 of file [framed_multi.h](#).

References [n](#).

8.16.4 Member Data Documentation

8.16.4.1 n

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
size_t glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::n [private]
```

Definition at line 161 of file [framed_multi.h](#).

Referenced by [hash_size_t\(\)](#), and [operator\(\)\(\)](#).

The documentation for this class was generated from the following file:

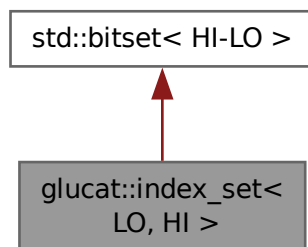
- [glucat/framed_multi.h](#)

8.17 glucat::index_set< LO, HI > Class Template Reference

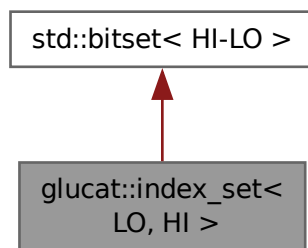
Index set class based on std::bitset<> in Gnu standard C++ library.

```
#include <index_set.h>
```

Inheritance diagram for glucat::index_set< LO, HI >:



Collaboration diagram for glucat::index_set< LO, HI >:



Classes

- class [reference](#)
Index set member reference.

Public Types

- using [index_set_t](#) = [index_set](#)
- using [index_pair_t](#) = std::pair<[index_t](#), [index_t](#)>

Public Member Functions

- [index_set](#) ()=default
Default constructor creates an empty set.
- [index_set](#) (const [bitset_t](#) bst)
Constructor from [bitset_t](#).
- [index_set](#) (const [index_t](#) idx)
Constructor from index.
- [index_set](#) (const [set_value_t](#) folded_val, const [index_set_t](#) frm, const bool prechecked=false)
Constructor from set value of an index set folded within the given frame.
- [index_set](#) (const [index_pair_t](#) &range, const bool prechecked=false)
Constructor from range of indices from range.first to range.second.
- [index_set](#) (const std::string &str)
Constructor from string.
- auto [operator==](#) (const [index_set_t](#) rhs) const -> bool
Equality.
- auto [operator!=](#) (const [index_set_t](#) rhs) const -> bool
Inequality.
- auto [operator~](#) () const -> [index_set_t](#)
Set complement: not.
- auto [operator^](#) = (const [index_set_t](#) rhs) -> [index_set_t](#) &
Symmetric set difference: exclusive or.
- auto [operator&](#) = (const [index_set_t](#) rhs) -> [index_set_t](#) &
Set intersection: and.
- auto [operator|](#) = (const [index_set_t](#) rhs) -> [index_set_t](#) &
Set union: or.
- auto [operator\[\]](#) (const [index_t](#) idx) const -> bool
Subscripting: Test idx for membership: test value of bit idx.
- auto [test](#) (const [index_t](#) idx) const -> bool
Test idx for membership: test value of bit idx.
- auto [set](#) () -> [index_set_t](#) &
Include all indices except 0: set all bits except 0.
- auto [set](#) (const [index_t](#) idx) -> [index_set_t](#) &
Include idx: Set bit at idx if idx != 0.
- auto [set](#) (const [index_t](#) idx, const int val) -> [index_set_t](#) &
Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.
- auto [reset](#) () -> [index_set_t](#) &
Make set empty: Set all bits to 0.
- auto [reset](#) (const [index_t](#) idx) -> [index_set_t](#) &
Exclude idx: Set bit at idx to 0.
- auto [flip](#) () -> [index_set_t](#) &
Set complement, except 0: flip all bits, except 0.
- auto [flip](#) (const [index_t](#) idx) -> [index_set_t](#) &
Complement membership of idx if idx != 0: flip bit at idx if idx != 0.
- auto [count](#) () const -> [index_t](#)
Cardinality: Number of indices included in set.
- auto [count_neg](#) () const -> [index_t](#)
Number of negative indices included in set.
- auto [count_pos](#) () const -> [index_t](#)
Number of positive indices included in set.
- auto [min](#) () const -> [index_t](#)

Minimum member.

- auto `max` () const -> `index_t`

Maximum member.

- auto `operator<` (const `index_set_t` rhs) const -> bool

Less than operator used for comparisons, map, etc.

- auto `is_contiguous` () const -> bool

Determine if the index set is contiguous, ie. has no gaps.

- auto `fold` () const -> const `index_set_t`

Fold this index set within itself as a frame.

- auto `fold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`

Fold this index set within the given frame.

- auto `unfold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`

Unfold this index set within the given frame.

- auto `value_of_fold` (const `index_set_t` frm) const -> `set_value_t`

The set value of the fold of this index set within the given frame.

- auto `sign_of_mult` (const `index_set_t` ist) const -> int

*Sign of geometric product of two *Clifford* basis elements.*

- auto `sign_of_square` () const -> int

*Sign of geometric square of a *Clifford* basis element.*

- auto `hash_fn` () const -> `size_t`

Hash function.

- auto `operator[]` (`index_t` idx) -> `reference`

Subscripting: Element access.

Static Public Member Functions

- static auto `classname` () -> const std::string

Static Public Attributes

- static const `index_t v_lo` = LO
- static const `index_t v_hi` = HI

Private Types

- using `bitset_t` = std::bitset<HI - LO>
- using `error_t` = `error<index_set>`

Private Member Functions

- `BOOST_STATIC_ASSERT` ((LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< _GLUCAT_BITS_PER_ULONG) &&(HI< _GLUCAT_BITS_PER_ULONG) &&(HI-LO<= _GLUCAT_BITS_PER_ULONG))
 - auto `lex_less_than` (const `index_set_t` rhs) const -> bool
- Lexicographic ordering of two sets: *this < rhs.*

Friends

- class [reference](#)
- auto [operator^](#) (const [index_set_t](#) &lhs, const [index_set_t](#) &rhs) -> const [index_set_t](#)
- auto [operator&](#) (const [index_set_t](#) &lhs, const [index_set_t](#) &rhs) -> const [index_set_t](#)
- auto [operator|](#) (const [index_set_t](#) &lhs, const [index_set_t](#) &rhs) -> const [index_set_t](#)
- auto [compare](#) (const [index_set_t](#) &lhs, const [index_set_t](#) &rhs) -> int

8.17.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >
```

Index set class based on `std::bitset<>` in Gnu standard C++ library.

Definition at line 73 of file [index_set.h](#).

8.17.2 Member Typedef Documentation

8.17.2.1 [bitset_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::bitset_t = std::bitset<HI - LO> [private]
```

Definition at line 81 of file [index_set.h](#).

8.17.2.2 [error_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::error_t = error<index\_set> [private]
```

Definition at line 82 of file [index_set.h](#).

8.17.2.3 [index_pair_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::index_pair_t = std::pair<index\_t, index\_t>
```

Definition at line 85 of file [index_set.h](#).

8.17.2.4 [index_set_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set< LO, HI >::index_set_t = index\_set
```

Definition at line 84 of file [index_set.h](#).

8.17.3 Constructor & Destructor Documentation

8.17.3.1 index_set() [1/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set () [default]
```

Default constructor creates an empty set.

Referenced by [glucat::operator>>\(\)](#).

8.17.3.2 index_set() [2/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const bitset_t bst)
```

Constructor from [bitset_t](#).

Definition at line 61 of file [index_set_imp.h](#).

8.17.3.3 index_set() [3/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_t idx)
```

Constructor from index.

Constructor from index value.

Definition at line 55 of file [index_set_imp.h](#).

References [set\(\)](#).

8.17.3.4 index_set() [4/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const set_value_t folded_val,
    const index_set_t frm,
    const bool prechecked = false)
```

Constructor from set value of an index set folded within the given frame.

Definition at line 68 of file [index_set_imp.h](#).

References [count\(\)](#), [fold\(\)](#), [min\(\)](#), and [unfold\(\)](#).

8.17.3.5 `index_set()` [5/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_pair_t & range,
    const bool prechecked = false)
```

Constructor from range of indices from `range.first` to `range.second`.

Definition at line 82 of file [index_set_imp.h](#).

8.17.3.6 `index_set()` [6/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const std::string & str)
```

Constructor from string.

Definition at line 102 of file [index_set_imp.h](#).

8.17.4 Member Function Documentation

8.17.4.1 `BOOST_STATIC_ASSERT()`

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::BOOST_STATIC_ASSERT (
    (LO<=0) && (0<=HI) && (LO< HI) && (-LO< _GLUCAT_BITS_PER_ULONG) && (HI< _GLUCAT_BITS_PER_ULONG) && (HI-LO<=_GLUCAT_BITS_PER_ULONG) ) [private]
```

8.17.4.2 `classname()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::classname () -> const std::string [inline], [static]
```

Definition at line 49 of file [index_set_imp.h](#).

8.17.4.3 `count()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count () const -> index_t [inline]
```

Cardinality: Number of indices included in set.

Definition at line 344 of file [index_set_imp.h](#).

Referenced by [count_neg\(\)](#), [count_pos\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [index_set\(\)](#), [is_contiguous\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [operator<\(\)](#), and [sign_of_square\(\)](#).

8.17.4.4 count_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_neg () const -> index_t [inline]
```

Number of negative indices included in set.

Definition at line 364 of file [index_set_imp.h](#).

References [count\(\)](#).

Referenced by [sign_of_square\(\)](#).

8.17.4.5 count_pos()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_pos () const -> index_t [inline]
```

Number of positive indices included in set.

Definition at line 376 of file [index_set_imp.h](#).

References [count\(\)](#).

8.17.4.6 flip() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip () -> index_set_t &
```

Set complement, except 0: flip all bits, except 0.

8.17.4.7 flip() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip (
    const index_t idx) -> index_set_t & [inline]
```

Complement membership of idx if idx != 0: flip bit at idx if idx != 0.

Definition at line 330 of file [index_set_imp.h](#).

8.17.4.8 fold() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold () const -> const index_set_t
```

Fold this index set within itself as a frame.

Referenced by [index_set\(\)](#), and [value_of_fold\(\)](#).

8.17.4.9 fold() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold (
    const index_set_t frm,
    const bool prechecked = false) const -> const index_set_t
```

Fold this index set within the given frame.

8.17.4.10 hash_fn()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::hash_fn () const -> size_t [inline]
```

Hash function.

Definition at line 950 of file [index_set_imp.h](#).

8.17.4.11 is_contiguous()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::is_contiguous () const -> bool [inline]
```

Determine if the index set is contiguous, ie. has no gaps.

Determine if the index set is contiguous, ie. has no gaps when 0 is included.

Definition at line 732 of file [index_set_imp.h](#).

References [count\(\)](#), [max\(\)](#), and [min\(\)](#).

8.17.4.12 lex_less_than()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::lex_less_than (
    const index_set_t rhs) const -> bool [inline], [private]
```

Lexicographic ordering of two sets: *this < rhs.

Definition at line 588 of file [index_set_imp.h](#).

Referenced by [operator<\(\)](#).

8.17.4.13 max()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::max () const -> index_t
```

Maximum member.

Maximum member, or 0 if none.

Definition at line 550 of file [index_set_imp.h](#).

References [test\(\)](#).

Referenced by [PyClical.index_set::__iter__\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [is_contiguous\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), and [unfold\(\)](#).

8.17.4.14 min()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::min () const -> index_t
```

Minimum member.

Minimum member, or 0 if none.

Definition at line 461 of file [index_set_imp.h](#).

References [test\(\)](#).

Referenced by [PyClical.index_set::__iter__\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [index_set\(\)](#), [is_contiguous\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), and [unfold\(\)](#).

8.17.4.15 operator"!="()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator!= (
    const index_set_t rhs) const -> bool [inline]
```

Inequality.

Definition at line 130 of file [index_set_imp.h](#).

8.17.4.16 operator&=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator&= (
    const index_set_t rhs) -> index_set_t & [inline]
```

Set intersection: and.

Definition at line 174 of file [index_set_imp.h](#).

8.17.4.17 operator<()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator< (
    const index_set_t rhs) const -> bool [inline]
```

Less than operator used for comparisons, map, etc.

Definition at line 596 of file [index_set_imp.h](#).

References [count\(\)](#), and [lex_less_than\(\)](#).

8.17.4.18 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator== (
    const index_set_t rhs) const -> bool [inline]
```

Equality.

Definition at line 119 of file [index_set_imp.h](#).

8.17.4.19 operator[]() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    const index_t idx) const -> bool [inline]
```

Subscripting: Test idx for membership: test value of bit idx.

Definition at line 232 of file [index_set_imp.h](#).

References [test\(\)](#).

8.17.4.20 operator[]() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    index_t idx) -> reference [inline]
```

Subscripting: Element access.

Definition at line 224 of file [index_set_imp.h](#).

8.17.4.21 operator^=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator^= (
    const index_set_t rhs) -> index_set_t & [inline]
```

Symmetric set difference: exclusive or.

Definition at line 149 of file [index_set_imp.h](#).

8.17.4.22 operator" |=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator|= (
    const index_set_t rhs) -> index_set_t & [inline]
```

Set union: or.

Definition at line 199 of file [index_set_imp.h](#).

8.17.4.23 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator~ () const -> index_set_t [inline]
```

Set complement: not.

Definition at line 141 of file [index_set_imp.h](#).

8.17.4.24 reset() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset () -> index_set_t & [inline]
```

Make set empty: Set all bits to 0.

Definition at line 294 of file [index_set_imp.h](#).

8.17.4.25 reset() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset (
    const index_t idx) -> index_set_t & [inline]
```

Exclude idx: Set bit at idx to 0.

Definition at line 305 of file [index_set_imp.h](#).

8.17.4.26 set() [1/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set () -> index_set_t & [inline]
```

Include all indices except 0: set all bits except 0.

Definition at line 255 of file [index_set_imp.h](#).

Referenced by [index_set\(\)](#).

8.17.4.27 set() [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx) -> index_set_t & [inline]
```

Include idx: Set bit at idx if idx != 0.

Definition at line 266 of file [index_set_imp.h](#).

8.17.4.28 set() [3/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx,
    const int val) -> index_set_t & [inline]
```

Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.

Definition at line 280 of file [index_set_imp.h](#).

8.17.4.29 sign_of_mult()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_mult (
    const index_set_t ist) const -> int
```

Sign of geometric product of two [Clifford](#) basis elements.

Definition at line 880 of file [index_set_imp.h](#).

References [glucat::inverse_gray\(\)](#), and [glucat::inverse_reversed_gray\(\)](#).

8.17.4.30 sign_of_square()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_square () const -> int [inline]
```

Sign of geometric square of a [Clifford](#) basis element.

Definition at line 930 of file [index_set_imp.h](#).

References [count\(\)](#), and [count_neg\(\)](#).

8.17.4.31 test()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::test (
    const index_t idx) const -> bool [inline]
```

Test idx for membership: test value of bit idx.

Definition at line 240 of file [index_set_imp.h](#).

Referenced by [max\(\)](#), [min\(\)](#), [operator\[\]\(\)](#), and [unfold\(\)](#).

8.17.4.32 unfold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::unfold (
    const index_set_t frm,
    const bool prechecked = false) const -> const index_set_t
```

Unfold this index set within the given frame.

Definition at line 794 of file [index_set_imp.h](#).

References [max\(\)](#), [min\(\)](#), and [test\(\)](#).

Referenced by [index_set\(\)](#).

8.17.4.33 value_of_fold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::value_of_fold (
    const index_set_t frm) const -> set_value_t [inline]
```

The set value of the fold of this index set within the given frame.

Definition at line 829 of file [index_set_imp.h](#).

References [fold\(\)](#).

8.17.5 Friends And Related Symbol Documentation

8.17.5.1 compare

```
template<const index_t LO, const index_t HI>
auto compare (
    const index_set_t & lhs,
    const index_set_t & rhs) -> int [friend]
```

8.17.5.2 operator&

```
template<const index_t LO, const index_t HI>
auto operator& (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

8.17.5.3 operator^

```
template<const index_t LO, const index_t HI>
auto operator^ (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

8.17.5.4 operator" |

```
template<const index_t LO, const index_t HI>
auto operator| (
    const index_set_t & lhs,
    const index_set_t & rhs) -> const index_set_t [friend]
```

8.17.5.5 reference

```
template<const index_t LO, const index_t HI>
friend class reference [friend]
```

Definition at line 174 of file [index_set.h](#).

8.17.6 Member Data Documentation

8.17.6.1 v_hi

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_hi = HI [static]
```

Definition at line 88 of file [index_set.h](#).

8.17.6.2 v_lo

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_lo = LO [static]
```

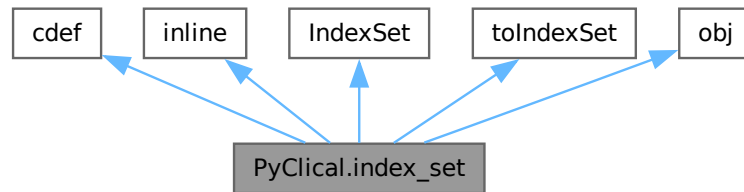
Definition at line 87 of file [index_set.h](#).

The documentation for this class was generated from the following files:

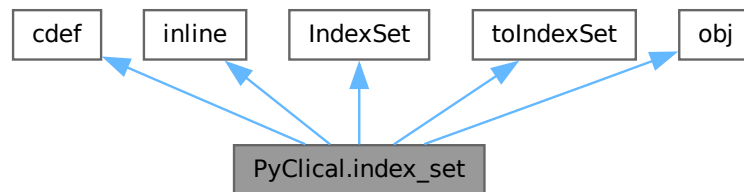
- [glucat/index_set.h](#)
- [glucat/index_set_imp.h](#)

8.18 PyClical.index_set Class Reference

Inheritance diagram for PyClical.index_set:



Collaboration diagram for PyClical.index_set:



Public Member Functions

- [__cinit__](#) (self, other=0)
- [__dealloc__](#) (self)
- [__richcmp__](#) (lhs, rhs, int, op)
- [__setitem__](#) (self, idx, val)
- [__getitem__](#) (self, idx)
- [__contains__](#) (self, idx)
- [__iter__](#) (self)
- [__invert__](#) (self)
- [__xor__](#) (lhs, rhs)
- [__ixor__](#) (self, rhs)
- [__and__](#) (lhs, rhs)
- [__iand__](#) (self, rhs)
- [__or__](#) (lhs, rhs)
- [__ior__](#) (self, rhs)
- [count](#) (self)
- [count_neg](#) (self)
- [count_pos](#) (self)
- [min](#) (self)

- [max](#) (self)
- [hash_fn](#) (self)
- [sign_of_mult](#) (self, rhs)
- [sign_of_square](#) (self)
- [__repr__](#) (self)
- [__str__](#) (self)

Public Attributes

- [instance](#) = new [IndexSet](#)((<[index_set](#)>other).unwrap())

8.18.1 Detailed Description

Return the C++ `IndexSet` instance wrapped by `index_set(obj)`.

Python class `index_set` wraps C++ class `IndexSet`.

Definition at line 38 of file [PyClical.pyx](#).

8.18.2 Member Function Documentation

8.18.2.1 `__and__()`

```
PyClical.index_set.__and__ (
    lhs,
    rhs)
```

Set intersection: `and`.

```
>>> print(index_set({1}) & index_set({2}))
{}
>>> print(index_set({1,2}) & index_set({2}))
{2}
```

Definition at line 271 of file [PyClical.pyx](#).

8.18.2.2 `__cinit__()`

```
PyClical.index_set.__cinit__ (
    self,
    other = 0)
```

Construct an object of type `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set("{1,2,1}"))
{1,2}
>>> print(index_set(""))
{}
```

Definition at line 74 of file [PyClical.pyx](#).

8.18.2.3 `__contains__()`

```
PyClical.index_set.__contains__ (
    self,
    idx)
```

Check that an `index_set` object contains the index `idx`: `idx in self`.

```
>>> 1 in index_set({1})
True
>>> 2 in index_set({1})
False
>>> -1 in index_set({2})
False
>>> 1 in index_set({2})
False
>>> 2 in index_set({2})
True
>>> 33 in index_set({2})
False
```

Definition at line 210 of file [PyClical.pyx](#).

References [instance](#).

8.18.2.4 `__dealloc__()`

```
PyClical.index_set.__dealloc__ (
    self)
```

Clean up by deallocating the instance of C++ class `IndexSet`.

Definition at line 116 of file [PyClical.pyx](#).

References [instance](#).

8.18.2.5 `__getitem__()`

```
PyClical.index_set.__getitem__ (
    self,
    idx)
```

Get the value of an `index_set` object at an index.

```
>>> index_set({1})[1]
True
>>> index_set({1})[2]
False
>>> index_set({2})[-1]
False
>>> index_set({2})[1]
False
>>> index_set({2})[2]
True
>>> index_set({2})[33]
False
```

Definition at line 191 of file [PyClical.pyx](#).

References [instance](#).

8.18.2.6 `__iand__()`

```
PyClical.index_set.__iand__ (
    self,
    rhs)
```

Set intersection: and.

```
>>> x = index_set({1}); x &= index_set({2}); print(x)
{}
>>> x = index_set({1,2}); x &= index_set({2}); print(x)
{2}
```

Definition at line 282 of file [PyClical.pyx](#).

8.18.2.7 `__invert__()`

```
PyClical.index_set.__invert__ (
    self)
```

Set complement: not.

```
>>> print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,
```

Definition at line 240 of file [PyClical.pyx](#).

References [instance](#).

8.18.2.8 `__ior__()`

```
PyClical.index_set.__ior__ (
    self,
    rhs)
```

Set union: or.

```
>>> x = index_set({1}); x |= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x |= index_set({2}); print(x)
{1,2}
```

Definition at line 304 of file [PyClical.pyx](#).

8.18.2.9 `__iter__()`

```
PyClical.index_set.__iter__ (
    self)
```

Iterate over the indices of an `index_set`.

```
>>> for i in index_set({-3,4,7}):print(i, end=",")
-3,4,7,
```

Definition at line 229 of file [PyClical.pyx](#).

References [glucat::index_set< LO, HI >.max\(\)](#), [glucat::index_set< lo_ndx, hi_ndx >.max\(\)](#), [max\(\)](#), [glucat::index_set< LO, HI >.min\(\)](#), [glucat::index_set< lo_ndx, hi_ndx >.min\(\)](#), and [min\(\)](#).

8.18.2.10 `__ixor__()`

```
PyClical.index_set.__ixor__ (
    self,
    rhs)
```

Symmetric set difference: exclusive or.

```
>>> x = index_set({1}); x ^= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x ^= index_set({2}); print(x)
{1}
```

Definition at line 260 of file [PyClical.pyx](#).

8.18.2.11 `__or__()`

```
PyClical.index_set.__or__ (
    lhs,
    rhs)
```

Set union: or.

```
>>> print(index_set({1}) | index_set({2}))
{1,2}
>>> print(index_set({1,2}) | index_set({2}))
{1,2}
```

Definition at line 293 of file [PyClical.pyx](#).

8.18.2.12 `__repr__()`

```
PyClical.index_set.__repr__ (
    self)
```

The “official” string representation of self.

```
>>> index_set({1,2}).__repr__()
'index_set({1,2})'
>>> repr(index_set({1,2}))
'index_set({1,2})'
```

Definition at line 384 of file [PyClical.pyx](#).

References [index_set_to_repr\(\)](#).

8.18.2.13 `__richcmp__()`

```
PyClical.index_set.__richcmp__ (
    lhs,
    rhs,
    int,
    op)
```

Compare two objects of class `index_set`.

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
```

Definition at line 122 of file [PyClical.pyx](#).

8.18.2.14 `__setitem__()`

```
PyClical.index_set.__setitem__ (
    self,
    idx,
    val)
```

Set the value of an `index_set` object at index `idx` to value `val`.

```
>>> s=index_set({1}); s[2] = True; print(s)
{1,2}
>>> s=index_set({1,2}); s[1] = False; print(s)
{2}
```

Definition at line 179 of file [PyClical.pyx](#).

References [instance](#).

8.18.2.15 `__str__()`

```
PyClical.index_set.__str__ (
    self)
```

The “informal” string representation of `self`.

```
>>> index_set({1,2}).__str__()
'{1,2}'
>>> str(index_set({1,2}))
'{1,2}'
```

Definition at line 395 of file [PyClical.pyx](#).

References [index_set_to_str\(\)](#).

8.18.2.16 `__xor__()`

```
PyClical.index_set.__xor__ (  
    lhs,  
    rhs)
```

Symmetric set difference: exclusive or.

```
>>> print(index_set({1}) ^ index_set({2}))  
{1,2}  
>>> print(index_set({1,2}) ^ index_set({2}))  
{1}
```

Definition at line 249 of file [PyClical.pyx](#).

8.18.2.17 `count()`

```
PyClical.index_set.count (  
    self)
```

Cardinality: Number of indices included in set.

```
>>> index_set({-1,1,2}).count()  
3
```

Definition at line 315 of file [PyClical.pyx](#).

References [count\(\)](#), and [instance](#).

Referenced by [count\(\)](#).

8.18.2.18 `count_neg()`

```
PyClical.index_set.count_neg (  
    self)
```

Number of negative indices included in set.

```
>>> index_set({-1,1,2}).count_neg()  
1
```

Definition at line 324 of file [PyClical.pyx](#).

References [count_neg\(\)](#), and [instance](#).

Referenced by [count_neg\(\)](#).

8.18.2.19 count_pos()

```
PyClical.index_set.count_pos (  
    self)
```

Number of positive indices included in set.

```
>>> index_set({-1,1,2}).count_pos()  
2
```

Definition at line 333 of file [PyClical.pyx](#).

References [count_pos\(\)](#), and [instance](#).

Referenced by [count_pos\(\)](#).

8.18.2.20 hash_fn()

```
PyClical.index_set.hash_fn (  
    self)
```

Hash function.

Definition at line 360 of file [PyClical.pyx](#).

References [hash_fn\(\)](#), and [instance](#).

Referenced by [hash_fn\(\)](#).

8.18.2.21 max()

```
PyClical.index_set.max (  
    self)
```

Maximum member.

```
>>> index_set({-1,1,2}).max()  
2
```

Definition at line 351 of file [PyClical.pyx](#).

References [instance](#), and [max\(\)](#).

Referenced by [__iter__\(\)](#), and [max\(\)](#).

8.18.2.22 min()

```
PyClical.index_set.min (  
    self)
```

Minimum member.

```
>>> index_set({-1,1,2}).min()  
-1
```

Definition at line 342 of file [PyClical.pyx](#).

References [instance](#), and [min\(\)](#).

Referenced by [__iter__\(\)](#), and [min\(\)](#).

8.18.2.23 sign_of_mult()

```
PyClical.index_set.sign_of_mult (  
    self,  
    rhs)
```

Sign of geometric product of two Clifford basis elements.

```
>>> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)  
1
```

Definition at line 366 of file [PyClical.pyx](#).

References [instance](#), and [sign_of_mult\(\)](#).

Referenced by [sign_of_mult\(\)](#).

8.18.2.24 sign_of_square()

```
PyClical.index_set.sign_of_square (  
    self)
```

Sign of geometric square of a Clifford basis element.

```
>>> s = index_set({1,2}); s.sign_of_square()  
-1
```

Definition at line 375 of file [PyClical.pyx](#).

References [instance](#), and [sign_of_square\(\)](#).

Referenced by [sign_of_square\(\)](#).

8.18.3 Member Data Documentation

8.18.3.1 instance

`PyClical.index_set.instance = new IndexSet((<index_set>other).unwrap())`

Definition at line 95 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.__call__\(\)](#), [__contains__\(\)](#), [PyClical.clifford.__dealloc__\(\)](#), [__dealloc__\(\)](#), [PyClical.clifford.__getitem__\(\)](#), [__getitem__\(\)](#), [__invert__\(\)](#), [PyClical.clifford.__neg__\(\)](#), [__setitem__\(\)](#), [PyClical.clifford.conj\(\)](#), [count\(\)](#), [count_neg\(\)](#), [count_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [hash_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [max\(\)](#), [PyClical.clifford.max_abs\(\)](#), [min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [sign_of_mult\(\)](#), [sign_of_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)

8.19 glucat::index_set_hash< LO, HI > Class Template Reference

```
#include <framed_multi.h>
```

Public Types

- using [index_set_t](#) = [index_set](#)<LO, HI>

Public Member Functions

- auto [operator\(\)](#) ([index_set_t](#) val) const -> [size_t](#)

8.19.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set_hash< LO, HI >
```

Definition at line 117 of file [framed_multi.h](#).

8.19.2 Member Typedef Documentation

8.19.2.1 index_set_t

```
template<const index\_t LO, const index\_t HI>
using glucat::index\_set\_hash< LO, HI >::index_set_t = index\_set<LO, HI>
```

Definition at line 120 of file [framed_multi.h](#).

8.19.3 Member Function Documentation

8.19.3.1 operator()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set_hash< LO, HI >::operator() (
    index_set_t val) const -> size_t [inline]
```

Definition at line 121 of file [framed_multi.h](#).

The documentation for this class was generated from the following file:

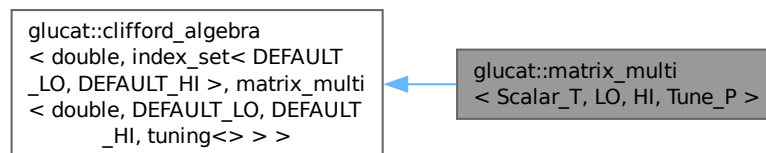
- [glucat/framed_multi.h](#)

8.20 glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > Class Template Reference

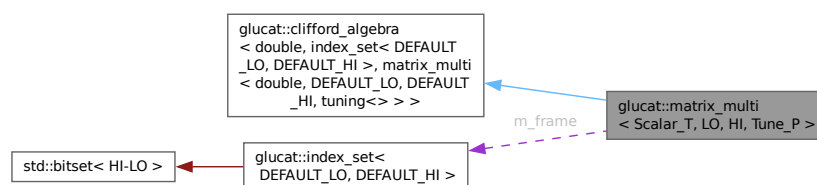
A [matrix_multi<Scalar_T,LO,HI,Tune_P>](#) is a matrix approximation to a multivector.

```
#include <matrix_multi.h>
```

Inheritance diagram for glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >:



Collaboration diagram for glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >:



Public Types

- using `multivector_t` = `matrix_multi`
- using `matrix_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set`<LO, HI>
- using `term_t` = `std::pair`<const `index_set_t`, `Scalar_T`>
- using `vector_t` = `std::vector`<`Scalar_T`>
- using `error_t` = `error`<`multivector_t`>
- using `framed_multi_t` = `framed_multi`<`Scalar_T`,LO,HI,`Tune_P`>

Public Types inherited from `glucat::clifford_algebra`< `double`, `index_set`< `DEFAULT_LO`, `DEFAULT_HI` >, `matrix_multi`< `double`, `DEFAULT_LO`, `DEFAULT_HI`, `tuning`<> > >

- using `scalar_t`
- using `index_set_t`
- using `multivector_t`
- using `pair_t`
- using `vector_t`

Public Member Functions

- `~matrix_multi` () override=default
Destructor.
- `matrix_multi` ()
Default constructor.
- `template<typename Other_Scalar_T>`
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val)
Construct a multivector from a multivector with a different scalar type.
- `template<typename Other_Scalar_T>`
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given multivector.
- `matrix_multi` (const `multivector_t` &val, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given multivector.
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd=`Scalar_T`(1))
Construct a multivector from an index set and a scalar coordinate.
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from an index set and a scalar coordinate.
- `matrix_multi` (const `Scalar_T` &scr, const `index_set_t` frm=`index_set_t`())
Construct a multivector from a scalar (within a frame, if given).
- `matrix_multi` (const int scr, const `index_set_t` frm=`index_set_t`())
Construct a multivector from an int (within a frame, if given).
- `matrix_multi` (const `vector_t` &vec, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a given vector.
- `matrix_multi` (const `std::string` &str)
Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- `matrix_multi` (const `std::string` &str, const `index_set_t` frm, const bool prechecked=false)
Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- `matrix_multi` (const char *str)

- Construct a multivector from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".
- `matrix_multi` (const char *str, const `index_set_t` frm, const bool prechecked=false)
 - Construct a multivector, within a given frame, from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".
- template<typename Other_Scalar_T>
 `matrix_multi` (const `framed_multi`< Other_Scalar_T, LO, HI, Tune_P > &val)
 - Construct a multivector from a `framed_multi_t`.
- template<typename Other_Scalar_T>
 `matrix_multi` (const `framed_multi`< Other_Scalar_T, LO, HI, Tune_P > &val, const `index_set_t` frm, const bool prechecked=false)
 - Construct a multivector, within a given frame, from a `framed_multi_t`.
- auto `fast_matrix_multi` (const `index_set_t` frm) const -> const `matrix_multi_t`
 - Use generalized FFT to construct a `matrix_multi_t`.
- template<typename Other_Scalar_T>
 auto `fast_framed_multi` () const -> const `framed_multi`< Other_Scalar_T, LO, HI, Tune_P >
 - Use inverse generalized FFT to construct a `framed_multi_t`.
- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `operator=` (const `multivector_t` &rhs) -> `multivector_t` &
 - Assignment operator.
- auto `operator+=` (const `term_t` &rhs) -> `multivector_t` &
 - Add a term, if non-zero.

Public Member Functions inherited from `glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>`

- virtual `~clifford_algebra` ()=default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0
 - Test for equality of multivectors.
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Geometric sum.
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Geometric difference.
- virtual auto `operator-` () const -> const `multivector_t`=0
 - Unary -.
- virtual auto `operator*=` (const double &scr) -> `multivector_t` &=0
 - Product of multivector and scalar.
- virtual auto `operator%=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Contraction.
- virtual auto `operator&=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Inner product.
- virtual auto `operator^=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Outer product.
- virtual auto `operator/=` (const double &scr) -> `multivector_t` &=0
 - Quotient of multivector and scalar.
- virtual auto `operator|=` (const `multivector_t` &rhs) -> `multivector_t` &=0
 - Transformation via twisted adjoint action.
- virtual auto `inv` () const -> const `multivector_t`=0
 - Geometric multiplicative inverse.
- virtual auto `pow` (int m) const -> const `multivector_t`=0
 - *this to the m
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0

- *Outer product power.*
- virtual auto `frame` () const -> const `index_set_t`=0
Subalgebra generated by all generators of terms of given multivector.
- virtual auto `grade` () const -> `index_t`=0
Maximum of the grades of each term.
- virtual auto `operator[]` (const `index_set_t` ist) const -> double=0
Subscripting: map from index set to scalar coordinate.
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t`=0
Pure grade-vector part.
- virtual auto `scalar` () const -> double=0
Scalar part.
- virtual auto `pure` () const -> const `multivector_t`=0
Pure part.
- virtual auto `even` () const -> const `multivector_t`=0
Even part of multivector, sum of even grade terms.
- virtual auto `odd` () const -> const `multivector_t`=0
Odd part of multivector, sum of odd grade terms.
- virtual auto `vector_part` () const -> const `vector_t`=0
Vector part of multivector, as a `vector_t` with respect to `frame()`.
- virtual auto `involute` () const -> const `multivector_t`=0
Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.
- virtual auto `reverse` () const -> const `multivector_t`=0
Reversion, eg. {1}{2} -> {2}*{1}.*
- virtual auto `conj` () const -> const `multivector_t`=0
Conjugation, reverse o involute == involute o reverse.
- virtual auto `quad` () const -> double=0
*Scalar_T quadratic form == (rev(x)*x)(0).*
- virtual auto `norm` () const -> double=0
Scalar_T norm == sum of norm of coordinates.
- virtual auto `max_abs` () const -> double=0
Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto `truncated` (const double &limit=`default_truncation`) const -> const `multivector_t`=0
Remove all terms with relative size smaller than limit.
- virtual auto `isinf` () const -> bool=0
Check if a multivector contains any infinite values.
- virtual auto `isnan` () const -> bool=0
Check if a multivector contains any IEEE NaN values.
- virtual void `write` (const std::string &msg="") const=0
Write formatted multivector to output.

Static Public Member Functions

- static auto `classname` () -> const std::string
Class name used in messages.
- static auto `random` (const `index_set_t` frm, Scalar_T fill=Scalar_T(1)) -> const `matrix_multi_t`
Random multivector within a frame.

Static Public Member Functions inherited from [glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >](#)

- static auto [classname](#) () -> const std::string

Private Types

- using [orientation_t](#) = ublas::row_major
- using [basis_matrix_t](#) = ublas::compressed_matrix<int, [orientation_t](#)>
- using [matrix_t](#) = ublas::matrix<Scalar_T, [orientation_t](#)>
- using [matrix_index_t](#) = typename matrix_t::size_type

Private Member Functions

- template<typename Matrix_T>
[matrix_multi](#) (const Matrix_T &mtx, const [index_set_t](#) frm)
Construct a multivector within a given frame from a given matrix.
- [matrix_multi](#) (const [matrix_t](#) &mtx, const [index_set_t](#) frm)
Construct a multivector within a given frame from a given matrix.
- auto [basis_element](#) (const [index_set](#)< LO, HI > &ist) const -> const [basis_matrix_t](#)
Create a basis element matrix within the current frame.

Private Attributes

- [index_set_t m_frame](#)
Index set representing the frame for the subalgebra which contains the multivector.
- [matrix_t m_matrix](#)
Matrix value representing the multivector within the folded frame.

Friends

- template<typename Other_Scalar_T, const [index_t](#) Other_LO, const [index_t](#) Other_HI, typename Other_Tune_P>
class [framed_multi](#)
- template<typename Other_Scalar_T, const [index_t](#) Other_LO, const [index_t](#) Other_HI, typename Other_Tune_P>
class [matrix_multi](#)
- auto [operator*](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [operator^](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [operator&](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [operator%](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [star](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> Scalar_T
- auto [operator/](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [operator|](#) (const [matrix_multi_t](#) &lhs, const [matrix_multi_t](#) &rhs) -> const [matrix_multi_t](#)
- auto [operator>>](#) (std::istream &s, [multivector_t](#) &val) -> std::istream &
- auto [operator<<](#) (std::ostream &os, const [multivector_t](#) &val) -> std::ostream &
- template<typename Other_Scalar_T, const [index_t](#) Other_LO, const [index_t](#) Other_HI, typename Other_Tune_P>
auto [reframe](#) (const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs, const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs, [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs_reframed, [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs_reframed) -> const [index_set](#)< Other_LO, Other_HI >
- template<typename Other_Scalar_T, const [index_t](#) Other_LO, const [index_t](#) Other_HI, typename Other_Tune_P>
auto [matrix_sqrt](#) (const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const [index_t](#) level) -> const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >
- template<typename Other_Scalar_T, const [index_t](#) Other_LO, const [index_t](#) Other_HI, typename Other_Tune_P>
auto [matrix_log](#) (const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const [index_t](#) level) -> const [matrix_multi](#)< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >

Additional Inherited Members

Static Public Attributes inherited from `glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<>>>`

- static const `index_t v_lo`
- static const `index_t v_hi`
- static const double `default_truncation`

Default for truncation.

8.20.1 Detailed Description

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >
```

A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.

Definition at line 137 of file `matrix_multi.h`.

8.20.2 Member Typedef Documentation

8.20.2.1 basis_matrix_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_matrix_t = ublas::compressed_matrix<int, orientation_t> [private]
```

Definition at line 157 of file `matrix_multi.h`.

8.20.2.2 error_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 148 of file `matrix_multi.h`.

8.20.2.3 framed_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = framed_multi<Scalar_T, LO, HI, Tune_P>
```

Definition at line 149 of file `matrix_multi.h`.

8.20.2.4 `index_set_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index\_set<LO, HI>
```

Definition at line 145 of file [matrix_multi.h](#).

8.20.2.5 `matrix_index_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_index_t = typename matrix\_t::size_type [private]
```

Definition at line 159 of file [matrix_multi.h](#).

8.20.2.6 `matrix_multi_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = multivector\_t
```

Definition at line 142 of file [matrix_multi.h](#).

8.20.2.7 `matrix_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = ublas::matrix<Scalar_T, orientation\_t> [private]
```

Definition at line 158 of file [matrix_multi.h](#).

8.20.2.8 `multivector_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = matrix\_multi
```

Definition at line 141 of file [matrix_multi.h](#).

8.20.2.9 `orientation_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::orientation_t = ublas::row\_major [private]
```

Definition at line 156 of file [matrix_multi.h](#).

8.20.2.10 scalar_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 143 of file [matrix_multi.h](#).

8.20.2.11 term_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index\_set\_t, Scalar_T>
```

Definition at line 146 of file [matrix_multi.h](#).

8.20.2.12 tune_p

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 144 of file [matrix_multi.h](#).

8.20.2.13 vector_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 147 of file [matrix_multi.h](#).

8.20.3 Constructor & Destructor Documentation

8.20.3.1 ~matrix_multi()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::~~matrix_multi () [override], [default]
```

Destructor.

8.20.3.2 matrix_multi() [1/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi ()
```

Default constructor.

Definition at line 106 of file [matrix_multi_imp.h](#).

References [m_frame](#), and [m_matrix](#).

8.20.3.3 matrix_multi() [2/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 115 of file [matrix_multi_imp.h](#).

References [m_frame](#), [m_matrix](#), [matrix_multi](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

8.20.3.4 matrix_multi() [3/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 134 of file [matrix_multi_imp.h](#).

References [glucat::folded_dim\(\)](#), [m_frame](#), [m_matrix](#), [matrix_multi](#), and [glucat::numeric_traits< Scalar_T >::to_scalar_t\(\)](#).

8.20.3.5 matrix_multi() [4/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const multivector\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 159 of file [matrix_multi_imp.h](#).

References [m_frame](#), and [m_matrix](#).

8.20.3.6 matrix_multi() [5/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 171 of file [matrix_multi_imp.h](#).

References [glucat::folded_dim\(\)](#), [m_frame](#), and [m_matrix](#).

8.20.3.7 matrix_multi() [6/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 183 of file [matrix_multi_imp.h](#).

References [glucat::folded_dim\(\)](#), [m_frame](#), and [m_matrix](#).

8.20.3.8 matrix_multi() [7/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from a scalar (within a frame, if given).

Definition at line 197 of file [matrix_multi_imp.h](#).

References [glucat::folded_dim\(\)](#), [m_frame](#), and [m_matrix](#).

8.20.3.9 matrix_multi() [8/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from an int (within a frame, if given).

Definition at line 209 of file [matrix_multi_imp.h](#).

8.20.3.10 matrix_multi() [9/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 215 of file [matrix_multi_imp.h](#).

References [glucat::index_set< LO, HI >::count\(\)](#), [glucat::folded_dim\(\)](#), [m_frame](#), [m_matrix](#), [glucat::index_set< LO, HI >::max\(\)](#), and [glucat::index_set< LO, HI >::min\(\)](#).

8.20.3.11 matrix_multi() [10/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 240 of file [matrix_multi_imp.h](#).

8.20.3.12 matrix_multi() [11/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 246 of file [matrix_multi_imp.h](#).

8.20.3.13 matrix_multi() [12/17]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str) [inline]
```

Construct a multivector from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 196 of file [matrix_multi.h](#).

8.20.3.14 matrix_multi() [13/17]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str,
    const index\_set\_t frm,
    const bool prechecked = false) [inline]
```

Construct a multivector, within a given frame, from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 199 of file [matrix_multi.h](#).

8.20.3.15 matrix_multi() [14/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a [framed_multi_t](#).

Definition at line 253 of file [matrix_multi_imp.h](#).

References [glucat::folded_dim\(\)](#), [glucat::clifford_algebra](#)< double, [index_set](#)< DEFAULT_LO, DEFAULT_HI >, [matrix_multi](#)< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::frame(), [framed_multi](#), [m_frame](#), and [m_matrix](#).

8.20.3.16 matrix_multi() [15/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false)
```

Construct a multivector, within a given frame, from a [framed_multi_t](#).

Definition at line 276 of file [matrix_multi_imp.h](#).

References [fast_matrix_multi\(\)](#), [glucat::folded_dim\(\)](#), [framed_multi](#), [m_frame](#), [m_matrix](#), and [glucat::clifford_algebra](#)< Scalar_T, [Index](#)

8.20.3.17 matrix_multi() [16/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
template<typename Matrix_T>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Matrix_T & mtx,
    const index\_set\_t frm) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 301 of file [matrix_multi_imp.h](#).

References [m_frame](#), [m_matrix](#), and [glucat::numeric_traits](#)< Scalar_T >::to_scalar_t().

8.20.3.18 matrix_multi() [17/17]

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_t & mtx,
    const index\_set\_t frm) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 320 of file [matrix_multi_imp.h](#).

References [m_frame](#), and [m_matrix](#).

8.20.4 Member Function Documentation

8.20.4.1 basis_element()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element (
    const index_set< LO, HI > & ist) const -> const basis_matrix_t [private]
```

Create a basis element matrix within the current frame.

Definition at line 1183 of file [matrix_multi_imp.h](#).

References [glucat::gen::generator_table< Matrix_T >::generator\(\)](#), [m_frame](#), [glucat::matrix::mono_prod\(\)](#), [glucat::offset_level\(\)](#), and [glucat::matrix::unit\(\)](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#).

8.20.4.2 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 78 of file [matrix_multi_imp.h](#).

8.20.4.3 fast_framed_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi< Other_Scalar_T, LO, HI, Tune_P >
```

Use inverse generalized FFT to construct a [framed_multi_t](#).

Definition at line 1106 of file [matrix_multi_imp.h](#).

References [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1\(\)](#), [glucat::fast\(\)](#), [framed_multi](#), [m_frame](#), [m_matrix](#), [glucat::pos_mod\(\)](#), and [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold\(\)](#).

Referenced by [fast_matrix_multi\(\)](#).

8.20.4.4 fast_matrix_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm) const -> const matrix_multi_t [inline]
```

Use generalized FFT to construct a [matrix_multi_t](#).

Definition at line 1093 of file [matrix_multi_imp.h](#).

References [fast_framed_multi\(\)](#), [fast_matrix_multi\(\)](#), and [m_frame](#).

Referenced by [fast_matrix_multi\(\)](#), and [matrix_multi\(\)](#).

8.20.4.5 operator+=()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term\_t & rhs) -> multivector\_t & [inline]
```

Add a term, if non-zero.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line [414](#) of file [matrix_multi_imp.h](#).

8.20.4.6 operator=()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::operator= (
    const multivector\_t & rhs) -> multivector\_t &
```

Assignment operator.

Definition at line [328](#) of file [matrix_multi_imp.h](#).

References [m_frame](#), and [m_matrix](#).

8.20.4.7 random()

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index\_set\_t frm,
    Scalar_T fill = Scalar_T(1)) -> const matrix\_multi\_t [static]
```

Random multivector within a frame.

Definition at line [923](#) of file [matrix_multi_imp.h](#).

References [glucat::framed_multi](#)< [Scalar_T](#), [LO](#), [HI](#), [Tune_P](#) >::random().

8.20.5 Friends And Related Symbol Documentation

8.20.5.1 framed_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P>
friend class framed_multi [friend]
```

Definition at line [151](#) of file [matrix_multi.h](#).

Referenced by [fast_framed_multi\(\)](#), [matrix_multi\(\)](#), and [matrix_multi\(\)](#).

8.20.5.2 matrix_log

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
auto matrix_log (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI,
Other_Tune_P > [friend]
```

8.20.5.3 matrix_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
friend class matrix_multi [friend]
```

Definition at line 153 of file [matrix_multi.h](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), and [matrix_multi\(\)](#).

8.20.5.4 matrix_sqrt

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
auto matrix_sqrt (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI,
Other_Tune_P > [friend]
```

8.20.5.5 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.6 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.7 operator*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.8 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.9 operator<<

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val) -> std::ostream & [friend]
```

8.20.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val) -> std::istream & [friend]
```

8.20.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.12 operator"|

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs) -> const matrix_multi_t [friend]
```

8.20.5.13 `reframe`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P>
auto reframe (
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs,
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs,
    matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs_reframed,
    matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs_reframed)
-> const index\_set< Other_LO, Other_HI > [friend]
```

8.20.5.14 `star`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const matrix\_multi\_t & lhs,
    const matrix\_multi\_t & rhs) -> Scalar_T [friend]
```

8.20.6 Member Data Documentation

8.20.6.1 `m_frame`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
index\_set\_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_frame [private]
```

Index set representing the frame for the subalgebra which contains the multivector.

Definition at line 278 of file [matrix_multi.h](#).

Referenced by [basis_element\(\)](#), [fast_framed_multi\(\)](#), [fast_matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), and [operator=\(\)](#).

8.20.6.2 `m_matrix`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
matrix\_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix [private]
```

Matrix value representing the multivector within the folded frame.

Definition at line 280 of file [matrix_multi.h](#).

Referenced by [fast_framed_multi\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), [matrix_multi\(\)](#), and [operator=\(\)](#).

The documentation for this class was generated from the following files:

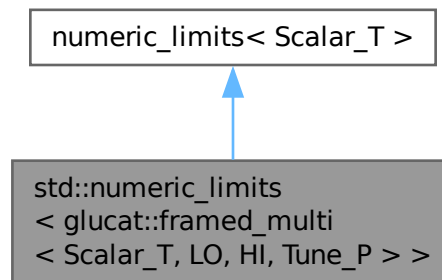
- [glucat/framed_multi.h](#)
- [glucat/matrix_multi.h](#)
- [glucat/matrix_multi_imp.h](#)

8.21 `std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >` Struct Template Reference

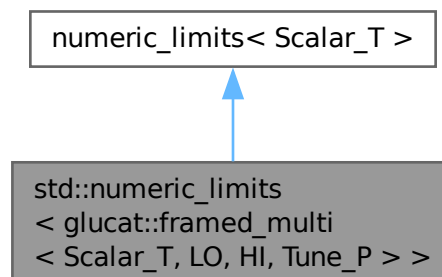
Numeric limits for `framed_multi` inherit limits for the corresponding scalar type.

```
#include <framed_multi.h>
```

Inheritance diagram for `std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >`:



Collaboration diagram for `std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >`:



8.21.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for `framed_multi` inherit limits for the corresponding scalar type.

Definition at line 345 of file [framed_multi.h](#).

The documentation for this struct was generated from the following file:

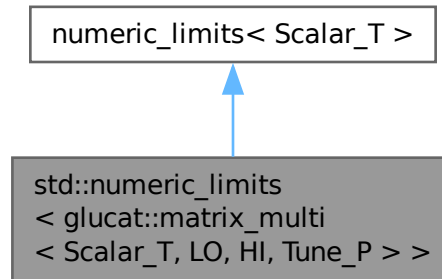
- [glucat/framed_multi.h](#)

8.22 std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > > Struct Template Reference

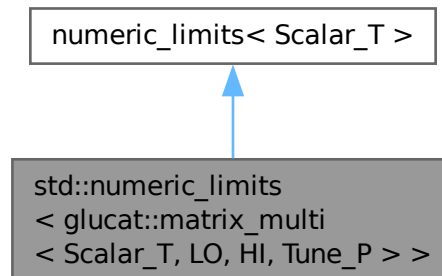
Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

```
#include <matrix_multi.h>
```

Inheritance diagram for std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >:



Collaboration diagram for std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >:



8.22.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

Definition at line 296 of file [matrix_multi.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi.h](#)

8.23 `glucat::numeric_traits< Scalar_T >` Class Template Reference

Extra traits which extend numeric limits.

```
#include <scalar.h>
```

Classes

- struct [promoted](#)
Extra traits which extend numeric limits.
- struct [demoted](#)
Demoted type for long double.

Public Member Functions

- auto [pi](#) () -> long double
Pi for long double.
- auto [ln_2](#) () -> long double
log(2) for long double
- auto [to_scalar_t](#) (const Other_Scalar_T &val) -> float
Extra traits which extend numeric limits.
- auto [to_scalar_t](#) (const Other_Scalar_T &val) -> double
Cast to double.
- auto [to_scalar_t](#) (const dd_real &val) -> long double
Cast to long double.
- auto [to_scalar_t](#) (const qd_real &val) -> long double
Cast to long double.
- auto [to_scalar_t](#) (const long double &val) -> dd_real
Cast to dd_real.
- auto [to_scalar_t](#) (const qd_real &val) -> dd_real
Cast to dd_real.
- auto [to_scalar_t](#) (const long double &val) -> qd_real
Cast to qd_real.
- auto [to_scalar_t](#) (const dd_real &val) -> qd_real
Cast to qd_real.

Static Public Member Functions

- static auto [isInf](#) (const Scalar_T &val) -> bool
Smart isinf.
- static auto [isNaN](#) (const Scalar_T &val) -> bool
Smart isnan.
- static auto [isNaN_or_isInf](#) (const Scalar_T &val) -> bool
Smart isnan or isinf.
- static auto [NaN](#) () -> Scalar_T
Smart NaN.
- static auto [to_int](#) (const Scalar_T &val) -> int
Cast to int.
- static auto [to_double](#) (const Scalar_T &val) -> double

Cast to double.

- `template<typename Other_Scalar_T>`
`static auto to_scalar_t (const Other_Scalar_T &val) -> Scalar_T`

Cast to Scalar_T.

- `static auto fmod (const Scalar_T &lhs, const Scalar_T &rhs) -> Scalar_T`

Modulo function for scalar.

- `static auto conj (const Scalar_T &val) -> Scalar_T`

Complex conjugate of scalar.

- `static auto real (const Scalar_T &val) -> Scalar_T`

Real part of scalar.

- `static auto imag (const Scalar_T &val) -> Scalar_T`

Imaginary part of scalar.

- `static auto abs (const Scalar_T &val) -> Scalar_T`

Absolute value of scalar.

- `static auto pi () -> Scalar_T`

Pi.

- `static auto ln_2 () -> Scalar_T`

log(2)

- `static auto pow (const Scalar_T &val, int n) -> Scalar_T`

Integer power.

- `static auto sqrt (const Scalar_T &val) -> Scalar_T`

Square root of scalar.

- `static auto exp (const Scalar_T &val) -> Scalar_T`

Exponential.

- `static auto log (const Scalar_T &val) -> Scalar_T`

Logarithm of scalar.

- `static auto log2 (const Scalar_T &val) -> Scalar_T`

Log base 2.

- `static auto cos (const Scalar_T &val) -> Scalar_T`

Cosine of scalar.

- `static auto acos (const Scalar_T &val) -> Scalar_T`

Inverse cosine of scalar.

- `static auto cosh (const Scalar_T &val) -> Scalar_T`

Hyperbolic cosine of scalar.

- `static auto sin (const Scalar_T &val) -> Scalar_T`

Sine of scalar.

- `static auto asin (const Scalar_T &val) -> Scalar_T`

Inverse sine of scalar.

- `static auto sinh (const Scalar_T &val) -> Scalar_T`

Hyperbolic sine of scalar.

- `static auto tan (const Scalar_T &val) -> Scalar_T`

Tangent of scalar.

- `static auto atan (const Scalar_T &val) -> Scalar_T`

Inverse tangent of scalar.

- `static auto tanh (const Scalar_T &val) -> Scalar_T`

Hyperbolic tangent of scalar.

Static Private Member Functions

- static auto `isInf` (const Scalar_T &val, `bool_to_type`< false >) -> bool
Smart isinf specialised for Scalar_T without infinity.
- static auto `isInf` (const Scalar_T &val, `bool_to_type`< true >) -> bool
Smart isinf specialised for Scalar_T with infinity.
- static auto `isNaN` (const Scalar_T &val, `bool_to_type`< false >) -> bool
Smart isnan specialised for Scalar_T without quiet NaN.
- static auto `isNaN` (const Scalar_T &val, `bool_to_type`< true >) -> bool
Smart isnan specialised for Scalar_T with quiet NaN.

8.23.1 Detailed Description

```
template<typename Scalar_T>
class glucat::numeric_traits< Scalar_T >
```

Extra traits which extend numeric limits.

Definition at line 47 of file [scalar.h](#).

8.23.2 Member Function Documentation

8.23.2.1 `abs()`

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::abs (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Absolute value of scalar.

Definition at line 182 of file [scalar.h](#).

8.23.2.2 `acos()`

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::acos (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Inverse cosine of scalar.

Definition at line 245 of file [scalar.h](#).

8.23.2.3 `asin()`

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::asin (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Inverse sine of scalar.

Definition at line 266 of file [scalar.h](#).

8.23.2.4 atan()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::atan (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Inverse tangent of scalar.

Definition at line 287 of file [scalar.h](#).

8.23.2.5 conj()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::conj (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Complex conjugate of scalar.

Definition at line 161 of file [scalar.h](#).

8.23.2.6 cos()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::cos (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Cosine of scalar.

Definition at line 238 of file [scalar.h](#).

8.23.2.7 cosh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::cosh (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Hyperbolic cosine of scalar.

Definition at line 252 of file [scalar.h](#).

8.23.2.8 exp()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::exp (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Exponential.

Definition at line 217 of file [scalar.h](#).

8.23.2.9 fmod()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::fmod (
    const Scalar_T & lhs,
    const Scalar_T & rhs) -> Scalar_T [inline], [static]
```

Modulo function for scalar.

Definition at line 154 of file [scalar.h](#).

8.23.2.10 imag()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::imag (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Imaginary part of scalar.

Definition at line 175 of file [scalar.h](#).

8.23.2.11 isInf() [1/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val) -> bool [inline], [static]
```

Smart isinf.

Definition at line 83 of file [scalar.h](#).

References [isInf\(\)](#).

8.23.2.12 isInf() [2/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool [inline], [static], [private]
```

Smart isinf specialised for Scalar_T without infinity.

Definition at line 54 of file [scalar.h](#).

Referenced by [isInf\(\)](#), [glucat::matrix::isinf\(\)](#), and [isNaN_or_isInf\(\)](#).

8.23.2.13 isInf() [3/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool [inline], [static], [private]
```

Smart isinf specialised for Scalar_T with infinity.

Definition at line 61 of file [scalar.h](#).

References [_GLUCAT_ISINF](#).

8.23.2.14 isNaN() [1/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val) -> bool [inline], [static]
```

Smart isnan.

Definition at line 93 of file [scalar.h](#).

References [isNaN\(\)](#).

8.23.2.15 isNaN() [2/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool [inline], [static], [private]
```

Smart isnan specialised for Scalar_T without quiet NaN.

Definition at line 68 of file [scalar.h](#).

Referenced by [isNaN\(\)](#), [glucat::matrix::isnan\(\)](#), [isNaN_or_isInf\(\)](#), [glucat::matrix::norm_frob2\(\)](#), and [glucat::matrix::trace\(\)](#).

8.23.2.16 isNaN() [3/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool [inline], [static], [private]
```

Smart isnan specialised for Scalar_T with quiet NaN.

Definition at line 75 of file [scalar.h](#).

References [_GLUCAT_ISNAN](#).

8.23.2.17 isNaN_or_isInf()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN_or_isInf (
    const Scalar_T & val) -> bool [inline], [static]
```

Smart isnan or isinf.

Definition at line 103 of file [scalar.h](#).

References [isInf\(\)](#), and [isNaN\(\)](#).

8.23.2.18 ln_2() [1/2]

```
auto glucat::numeric_traits< longdouble >::ln_2 () -> long double [inline]
```

log(2) for long double

Definition at line 59 of file [long_double.h](#).

References [glucat::!_ln2](#).

8.23.2.19 ln_2() [2/2]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::ln_2 () -> Scalar_T [inline], [static]
```

log(2)

Definition at line 196 of file [scalar.h](#).

Referenced by [log2\(\)](#).

8.23.2.20 log()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::log (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Logarithm of scalar.

Definition at line 224 of file [scalar.h](#).

Referenced by [log2\(\)](#).

8.23.2.21 log2()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::log2 (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Log base 2.

Definition at line 231 of file [scalar.h](#).

References [ln_2\(\)](#), and [log\(\)](#).

Referenced by [glucat::log2\(\)](#).

8.23.2.22 NaN()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::NaN () -> Scalar_T [inline], [static]
```

Smart NaN.

Definition at line 115 of file [scalar.h](#).

Referenced by [glucat::cr_sqrt\(\)](#), [glucat::db_sqrt\(\)](#), [glucat::matrix::norm_frob2\(\)](#), [glucat::operator*\(\)](#), and [glucat::matrix::trace\(\)](#).

8.23.2.23 pi() [1/2]

```
auto glucat::numeric_traits< longdouble >::pi () -> long double [inline]
```

Pi for long double.

Definition at line 51 of file [long_double.h](#).

References [glucat::l_pi](#).

8.23.2.24 pi() [2/2]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::pi () -> Scalar_T [inline], [static]
```

Pi.

Definition at line 189 of file [scalar.h](#).

Referenced by [glucat::matrix::classify_eigenvalues\(\)](#).

8.23.2.25 pow()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::pow (
    const Scalar_T & val,
    int n) -> Scalar_T [inline], [static]
```

Integer power.

Definition at line 203 of file [scalar.h](#).

Referenced by [glucat::error_squared_tol\(\)](#).

8.23.2.26 real()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::real (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Real part of scalar.

Definition at line 168 of file [scalar.h](#).

8.23.2.27 sin()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sin (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Sine of scalar.

Definition at line 259 of file [scalar.h](#).

8.23.2.28 sinh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sinh (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Hyperbolic sine of scalar.

Definition at line 273 of file [scalar.h](#).

8.23.2.29 sqrt()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sqrt (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Square root of scalar.

Definition at line 210 of file [scalar.h](#).

Referenced by [glucat::abs\(\)](#).

8.23.2.30 tan()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::tan (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Tangent of scalar.

Definition at line 280 of file [scalar.h](#).

8.23.2.31 tanh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::tanh (
    const Scalar_T & val) -> Scalar_T [inline], [static]
```

Hyperbolic tangent of scalar.

Definition at line 294 of file [scalar.h](#).

8.23.2.32 to_double()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_double (
    const Scalar_T & val) -> double [inline], [static]
```

Cast to double.

Definition at line 133 of file [scalar.h](#).

Referenced by [glucat::matrix::classify_eigenvalues\(\)](#), [glucat::operator<<\(\)](#), [PyFloat_FromDouble\(\)](#), [to_scalar_t\(\)](#), and [to_scalar_t\(\)](#).

8.23.2.33 to_int()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_int (
    const Scalar_T & val) -> int [inline], [static]
```

Cast to int.

Definition at line 126 of file [scalar.h](#).

8.23.2.34 to_scalar_t() [1/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const dd_real & val) -> long double [inline]
```

Cast to long double.

Definition at line 71 of file [scalar_imp.h](#).

8.23.2.35 to_scalar_t() [2/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const dd_real & val) -> qd_real [inline]
```

Cast to qd_real.

Definition at line 116 of file [scalar_imp.h](#).

8.23.2.36 to_scalar_t() [3/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const long double & val) -> dd_real [inline]
```

Cast to dd_real.

Definition at line 89 of file [scalar_imp.h](#).

8.23.2.37 to_scalar_t() [4/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const long double & val) -> qd_real [inline]
```

Cast to qd_real.

Definition at line 107 of file [scalar_imp.h](#).

8.23.2.38 to_scalar_t() [5/9]

```
auto glucat::numeric_traits< double >::to_scalar_t (
    const Other_Scalar_T & val) -> double [inline]
```

Cast to double.

Definition at line 61 of file [scalar_imp.h](#).

References [to_double\(\)](#).

8.23.2.39 to_scalar_t() [6/9]

```
auto glucat::numeric_traits< float >::to_scalar_t (
    const Other_Scalar_T & val) -> float [inline]
```

Extra traits which extend numeric limits.

Cast to float

Definition at line 52 of file [scalar_imp.h](#).

References [to_double\(\)](#).

8.23.2.40 `to_scalar_t()` [7/9]

```
template<typename Scalar_T>
template<typename Other_Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_scalar_t (
    const Other_Scalar_T & val) -> Scalar_T [inline], [static]
```

Cast to `Scalar_T`.

Definition at line 141 of file [scalar.h](#).

Referenced by [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi\(\)](#), [glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi\(\)](#), [glucat::matrix::nork_range\(\)](#), [glucat::to_demote\(\)](#), and [glucat::to_promote\(\)](#).

8.23.2.41 `to_scalar_t()` [8/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const qd_real & val) -> dd_real [inline]
```

Cast to `dd_real`.

Definition at line 98 of file [scalar_imp.h](#).

8.23.2.42 `to_scalar_t()` [9/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const qd_real & val) -> long double [inline]
```

Cast to long double.

Definition at line 80 of file [scalar_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/scalar.h](#)

8.24 `pade::pade_log_denom< Scalar_T >` Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by `Pade1(log(1+x),x,n,n)`.

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = `std::array<Scalar_T, 14>`

Static Public Attributes

- static const [array](#) [denom](#)

8.24.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by `Pade1(log(1+x),x,n,n)`.

Definition at line [1721](#) of file [matrix_multi_imp.h](#).

8.24.2 Member Typedef Documentation

8.24.2.1 array

```
template<typename Scalar_T>
using pade::pade_log_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line [1723](#) of file [matrix_multi_imp.h](#).

8.24.3 Member Data Documentation

8.24.3.1 denom

```
template<typename Scalar_T>
const array pade::pade_log_denom< Scalar_T >::denom [static]
```

Definition at line [1724](#) of file [matrix_multi_imp.h](#).

Referenced by [glucat::pade_log\(\)](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.25 pade::pade_log_denom< dd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = `std::array<dd_real, 22>`

Static Public Attributes

- static const [array](#) `denom`

8.25.1 Detailed Description

Definition at line 1810 of file [matrix_multi_imp.h](#).

8.25.2 Member Typedef Documentation

8.25.2.1 `array`

```
using pade::pade\_log\_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1812 of file [matrix_multi_imp.h](#).

8.25.3 Member Data Documentation

8.25.3.1 `denom`

```
const array pade::pade\_log\_denom< dd_real >::denom [static]
```

Definition at line 1813 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.26 `pade::pade_log_denom< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<float, 10>

Static Public Attributes

- static const [array](#) `denom`

8.26.1 Detailed Description

Definition at line 1748 of file [matrix_multi_imp.h](#).

8.26.2 Member Typedef Documentation

8.26.2.1 array

```
using pade::pade_log_denom< float >::array = std::array<float, 10>
```

Definition at line 1750 of file [matrix_multi_imp.h](#).

8.26.3 Member Data Documentation

8.26.3.1 denom

```
const array pade::pade_log_denom< float >::denom [static]
```

Definition at line 1751 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.27 pade::pade_log_denom< long double > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<long double, 18>

Static Public Attributes

- static const [array](#) [denom](#)

8.27.1 Detailed Description

Definition at line 1775 of file [matrix_multi_imp.h](#).

8.27.2 Member Typedef Documentation

8.27.2.1 array

```
using pade::pade_log_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1777 of file [matrix_multi_imp.h](#).

8.27.3 Member Data Documentation

8.27.3.1 denom

```
const array pade::pade_log_denom< long double >::denom [static]
```

Definition at line 1778 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.28 pade::pade_log_denom< qd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<qd_real, 34>

Static Public Attributes

- static const [array](#) [denom](#)

8.28.1 Detailed Description

Definition at line 1857 of file [matrix_multi_imp.h](#).

8.28.2 Member Typedef Documentation

8.28.2.1 array

```
using pade::pade_log_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1859 of file [matrix_multi_imp.h](#).

8.28.3 Member Data Documentation

8.28.3.1 denom

```
const array pade::pade_log_denom< qd_real >::denom [static]
```

Definition at line 1860 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.29 pade::pade_log_numer< Scalar_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<Scalar_T, 14>

Static Public Attributes

- static const [array numer](#)

8.29.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

Definition at line 1704 of file [matrix_multi_imp.h](#).

8.29.2 Member Typedef Documentation

8.29.2.1 array

```
template<typename Scalar_T>
using pade::pade\_log\_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1706 of file [matrix_multi_imp.h](#).

8.29.3 Member Data Documentation

8.29.3.1 numer

```
template<typename Scalar_T>
const array pade::pade\_log\_numer< Scalar_T >::numer [static]
```

Definition at line 1707 of file [matrix_multi_imp.h](#).

Referenced by [glucat::pade_log\(\)](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.30 `pade::pade_log_numer< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<dd_real, 22>`

Static Public Attributes

- static const `array numer`

8.30.1 Detailed Description

Definition at line 1790 of file `matrix_multi_imp.h`.

8.30.2 Member Typedef Documentation

8.30.2.1 `array`

```
using pade::pade_log_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1792 of file `matrix_multi_imp.h`.

8.30.3 Member Data Documentation

8.30.3.1 `numer`

```
const array pade::pade_log_numer< dd_real >::numer [static]
```

Definition at line 1793 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.31 `pade::pade_log_numer< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<float, 10>`

Static Public Attributes

- static const [array](#) [number](#)

8.31.1 Detailed Description

Definition at line [1736](#) of file [matrix_multi_imp.h](#).

8.31.2 Member Typedef Documentation

8.31.2.1 array

```
using pade::pade\_log\_number< float >::array = std::array<float, 10>
```

Definition at line [1738](#) of file [matrix_multi_imp.h](#).

8.31.3 Member Data Documentation

8.31.3.1 numer

```
const array pade::pade\_log\_number< float >::numer [static]
```

Definition at line [1739](#) of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.32 [pade::pade_log_number< long double >](#) Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<long double, 18>

Static Public Attributes

- static const [array](#) [number](#)

8.32.1 Detailed Description

Definition at line [1761](#) of file [matrix_multi_imp.h](#).

8.32.2 Member Typedef Documentation

8.32.2.1 array

using `pade::pade_log_number< long double >::array` = `std::array<long double, 18>`

Definition at line 1763 of file `matrix_multi_imp.h`.

8.32.3 Member Data Documentation

8.32.3.1 numer

const `array pade::pade_log_number< long double >::numer` [static]

Definition at line 1764 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.33 `pade::pade_log_number< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<qd_real, 34>`

Static Public Attributes

- static const `array numer`

8.33.1 Detailed Description

Definition at line 1831 of file `matrix_multi_imp.h`.

8.33.2 Member Typedef Documentation

8.33.2.1 array

using `pade::pade_log_number< qd_real >::array` = `std::array<qd_real, 34>`

Definition at line 1833 of file `matrix_multi_imp.h`.

8.33.3 Member Data Documentation

8.33.3.1 numer

```
const array pade::pade_log_numer< qd_real >::numer [static]
```

Definition at line 1834 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.34 pade::pade_sqrt_denom< Scalar_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<Scalar_T, 14>

Static Public Attributes

- static const [array](#) [denom](#)

8.34.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

Definition at line 1393 of file [matrix_multi_imp.h](#).

8.34.2 Member Typedef Documentation

8.34.2.1 array

```
template<typename Scalar_T>
using pade::pade_sqrt_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1395 of file [matrix_multi_imp.h](#).

8.34.3 Member Data Documentation

8.34.3.1 `denom`

```
template<typename Scalar_T>
const array pade::pade_sqrt_denom< Scalar_T >::denom [static]
```

Definition at line 1396 of file `matrix_multi_imp.h`.

Referenced by `glucat::matrix_sqrt()`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.35 `pade::pade_sqrt_denom< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<dd_real, 22>`

Static Public Attributes

- static const `array denom`

8.35.1 Detailed Description

Definition at line 1483 of file `matrix_multi_imp.h`.

8.35.2 Member Typedef Documentation

8.35.2.1 `array`

```
using pade::pade_sqrt_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1485 of file `matrix_multi_imp.h`.

8.35.3 Member Data Documentation

8.35.3.1 `denom`

```
const array pade::pade_sqrt_denom< dd_real >::denom [static]
```

Definition at line 1486 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.36 `pade::pade_sqrt_denom< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<float, 10>`

Static Public Attributes

- static const `array` `denom`

8.36.1 Detailed Description

Definition at line 1420 of file `matrix_multi_imp.h`.

8.36.2 Member Typedef Documentation

8.36.2.1 `array`

```
using pade::pade_sqrt_denom< float >::array = std::array<float, 10>
```

Definition at line 1422 of file `matrix_multi_imp.h`.

8.36.3 Member Data Documentation

8.36.3.1 `denom`

```
const array pade::pade_sqrt_denom< float >::denom [static]
```

Definition at line 1423 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.37 `pade::pade_sqrt_denom< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<long double, 18>`

Static Public Attributes

- static const [array](#) `denom`

8.37.1 Detailed Description

Definition at line 1447 of file [matrix_multi_imp.h](#).

8.37.2 Member Typedef Documentation

8.37.2.1 `array`

```
using pade::pade\_sqrt\_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1449 of file [matrix_multi_imp.h](#).

8.37.3 Member Data Documentation

8.37.3.1 `denom`

```
const array pade::pade\_sqrt\_denom< long double >::denom [static]
```

Definition at line 1450 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.38 `pade::pade_sqrt_denom< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<qd_real, 34>

Static Public Attributes

- static const [array](#) `denom`

8.38.1 Detailed Description

Definition at line 1530 of file [matrix_multi_imp.h](#).

8.38.2 Member Typedef Documentation

8.38.2.1 array

```
using pade::pade_sqrt_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1532 of file [matrix_multi_imp.h](#).

8.38.3 Member Data Documentation

8.38.3.1 denom

```
const array pade::pade_sqrt_denom< qd_real >::denom [static]
```

Definition at line 1533 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.39 pade::pade_sqrt_numer< Scalar_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<Scalar_T, 14>

Static Public Attributes

- static const [array numer](#)

8.39.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

Definition at line 1376 of file [matrix_multi_imp.h](#).

8.39.2 Member Typedef Documentation

8.39.2.1 array

```
template<typename Scalar_T>
using pade::pade_sqrt_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1378 of file `matrix_multi_imp.h`.

8.39.3 Member Data Documentation

8.39.3.1 numer

```
template<typename Scalar_T>
const array pade::pade_sqrt_numer< Scalar_T >::numer [static]
```

Definition at line 1379 of file `matrix_multi_imp.h`.

Referenced by `glucat::matrix_sqrt()`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.40 `pade::pade_sqrt_numer< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = std::array<dd_real, 22>

Static Public Attributes

- static const `array` `numer`

8.40.1 Detailed Description

Definition at line 1463 of file `matrix_multi_imp.h`.

8.40.2 Member Typedef Documentation

8.40.2.1 array

```
using pade::pade_sqrt_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1465 of file `matrix_multi_imp.h`.

8.40.3 Member Data Documentation

8.40.3.1 numer

```
const array pade::pade_sqrt_numer< dd_real >::numer [static]
```

Definition at line 1466 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.41 pade::pade_sqrt_numer< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using [array](#) = std::array<float, 10>

Static Public Attributes

- static const [array](#) numer

8.41.1 Detailed Description

Definition at line 1408 of file [matrix_multi_imp.h](#).

8.41.2 Member Typedef Documentation

8.41.2.1 array

```
using pade::pade_sqrt_numer< float >::array = std::array<float, 10>
```

Definition at line 1410 of file [matrix_multi_imp.h](#).

8.41.3 Member Data Documentation

8.41.3.1 numer

```
const array pade::pade_sqrt_numer< float >::numer [static]
```

Definition at line 1411 of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.42 `pade::pade_sqrt_numer< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<long double, 18>`

Static Public Attributes

- static const `array numer`

8.42.1 Detailed Description

Definition at line 1433 of file `matrix_multi_imp.h`.

8.42.2 Member Typedef Documentation

8.42.2.1 `array`

```
using pade::pade_sqrt_numer< long double >::array = std::array<long double, 18>
```

Definition at line 1435 of file `matrix_multi_imp.h`.

8.42.3 Member Data Documentation

8.42.3.1 `numer`

```
const array pade::pade_sqrt_numer< long double >::numer [static]
```

Definition at line 1436 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

8.43 `pade::pade_sqrt_numer< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using `array` = `std::array<qd_real, 34>`

Static Public Attributes

- static const [array](#) [number](#)

8.43.1 Detailed Description

Definition at line [1504](#) of file [matrix_multi_imp.h](#).

8.43.2 Member Typedef Documentation

8.43.2.1 array

```
using pade::pade\_sqrt\_numer< qd\_real >::array = std::array<qd\_real, 34>
```

Definition at line [1506](#) of file [matrix_multi_imp.h](#).

8.43.3 Member Data Documentation

8.43.3.1 numer

```
const array pade::pade\_sqrt\_numer< qd\_real >::numer [static]
```

Definition at line [1507](#) of file [matrix_multi_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix_multi_imp.h](#)

8.44 [glucat::numeric_traits< Scalar_T >::promoted](#) Struct Reference

Extra traits which extend numeric limits.

```
#include <promotion.h>
```

Public Types

- using [type](#) = double
- using [type](#) = long double
- using [type](#) = double

8.44.1 Detailed Description

```
template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::promoted
```

Extra traits which extend numeric limits.

Promoted type.

Promoted type for long double.

Promoted type for double

Definition at line 70 of file [promotion.h](#).

8.44.2 Member Typedef Documentation

8.44.2.1 `type` [1/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = long double
```

Definition at line 86 of file [promotion.h](#).

8.44.2.2 `type` [2/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 72 of file [promotion.h](#).

8.44.2.3 `type` [3/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 145 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

8.45 `glucat::random_generator< Scalar_T >` Class Template Reference

Random number generator with single instance per `Scalar_T`.

```
#include <random.h>
```

Public Member Functions

- [random_generator](#) (const random_generator &)=delete
- auto [operator=](#) (const [random_generator](#) &) -> [random_generator](#) &=delete
- auto [uniform](#) () -> Scalar_T
- auto [normal](#) () -> Scalar_T

Static Public Member Functions

- static auto [generator](#) () -> [random_generator](#) &
Single instance of Random number generator.

Private Member Functions

- [random_generator](#) ()
- [~random_generator](#) ()=default

Private Attributes

- std::mt19937 [uint_gen](#)
- std::uniform_real_distribution< double > [uniform_dist](#)
- std::normal_distribution< double > [normal_dist](#)

Static Private Attributes

- static const unsigned long [seed](#) = 19590921UL

Friends

- class [friend_for_private_destructor](#)

8.45.1 Detailed Description

```
template<typename Scalar_T>
class glucat::random_generator< Scalar_T >
```

Random number generator with single instance per Scalar_T.

Definition at line 42 of file [random.h](#).

8.45.2 Constructor & Destructor Documentation

8.45.2.1 random_generator() [1/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator (
    const random_generator< Scalar_T > & ) [delete]
```

References [random_generator\(\)](#).

Referenced by [generator\(\)](#), [operator=\(\)](#), and [random_generator\(\)](#).

8.45.2.2 `random_generator()` [2/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator () [inline], [private]
```

Definition at line 61 of file [random.h](#).

References [normal_dist](#), [seed](#), [uint_gen](#), and [uniform_dist](#).

8.45.2.3 `~random_generator()`

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::~~random_generator () [private], [default]
```

8.45.3 Member Function Documentation

8.45.3.1 `generator()`

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::generator () -> random_generator & [inline],
[static]
```

Single instance of Random number generator.

Definition at line 51 of file [random.h](#).

References [random_generator\(\)](#).

8.45.3.2 `normal()`

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::normal () -> Scalar_T [inline]
```

Definition at line 70 of file [random.h](#).

8.45.3.3 `operator=()`

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::operator= (
    const random_generator< Scalar_T > & ) -> random_generator &=delete [delete]
```

References [random_generator\(\)](#).

8.45.3.4 `uniform()`

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::uniform () -> Scalar_T [inline]
```

Definition at line 68 of file [random.h](#).

8.45.4 Friends And Related Symbol Documentation

8.45.4.1 friend_for_private_destructor

```
template<typename Scalar_T>
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 48 of file [random.h](#).

References [friend_for_private_destructor](#).

Referenced by [friend_for_private_destructor](#).

8.45.5 Member Data Documentation

8.45.5.1 normal_dist

```
template<typename Scalar_T>
std::normal_distribution<double> glucat::random_generator< Scalar_T >::normal_dist [private]
```

Definition at line 59 of file [random.h](#).

Referenced by [random_generator\(\)](#).

8.45.5.2 seed

```
template<typename Scalar_T>
const unsigned long glucat::random_generator< Scalar_T >::seed = 19590921UL [static], [private]
```

Definition at line 55 of file [random.h](#).

Referenced by [random_generator\(\)](#).

8.45.5.3 uint_gen

```
template<typename Scalar_T>
std::mt19937 glucat::random_generator< Scalar_T >::uint_gen [private]
```

Definition at line 57 of file [random.h](#).

Referenced by [random_generator\(\)](#).

8.45.5.4 uniform_dist

```
template<typename Scalar_T>
std::uniform_real_distribution<double> glucat::random_generator< Scalar_T >::uniform_dist
[private]
```

Definition at line 58 of file [random.h](#).

Referenced by [random_generator\(\)](#).

The documentation for this class was generated from the following file:

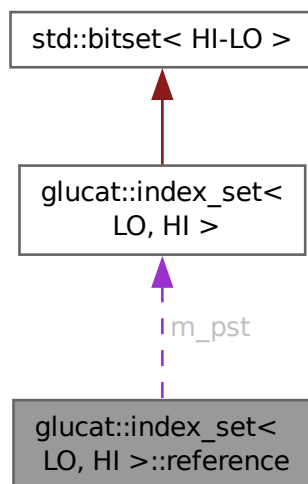
- [glucat/random.h](#)

8.46 glucat::index_set< LO, HI >::reference Class Reference

Index set member reference.

```
#include <index_set.h>
```

Collaboration diagram for glucat::index_set< LO, HI >::reference:



Public Member Functions

- [reference](#) ()=delete
Default constructor is deleted.
- [reference](#) ([index_set_t](#) &ist, [index_t](#) idx)
[index_set](#) reference
- [~reference](#) ()=default
- auto [operator==](#) (const [reference](#) &c_j) const -> bool
for b[i] == c[j];
- auto [operator=](#) (const bool x) -> [reference](#) &
for b[i] = x;
- auto [operator=](#) (const [reference](#) &c_j) -> [reference](#) &
for b[i] = c[j];
- auto [operator~](#) () const -> bool
Flips a bit.
- [operator bool](#) () const
for x = b[i];
- auto [flip](#) () -> [reference](#) &
for b[i].flip();

Private Attributes

- [index_set_t](#) * m_pst
- [index_t](#) m_idx

Friends

- class [index_set](#)

8.46.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >::reference
```

Index set member reference.

Definition at line 177 of file [index_set.h](#).

8.46.2 Constructor & Destructor Documentation

8.46.2.1 [reference\(\)](#) [1/2]

```
template<const index\_t LO, const index\_t HI>
glucat::index_set< LO, HI >::reference::reference () [delete]
```

Default constructor is deleted.

Referenced by [flip\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), and [~reference\(\)](#).

8.46.2.2 reference() [2/2]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::reference (
    index_set_t & ist,
    index_t idx) [inline]
```

[index_set](#) [reference](#)

Definition at line 985 of file [index_set_imp.h](#).

References [m_idx](#), and [m_pst](#).

8.46.2.3 ~reference()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::~reference () [default]
```

References [flip\(\)](#), and [reference\(\)](#).

8.46.3 Member Function Documentation

8.46.3.1 flip()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::flip () -> reference & [inline]
```

for [b\[i\].flip\(\)](#);

Definition at line 1049 of file [index_set_imp.h](#).

References [m_idx](#), [m_pst](#), and [reference\(\)](#).

Referenced by [~reference\(\)](#).

8.46.3.2 operator bool()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::operator bool () const [inline]
```

for [x = b\[i\]](#);

Definition at line 1041 of file [index_set_imp.h](#).

References [m_idx](#), and [m_pst](#).

8.46.3.3 operator=() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const bool x) -> reference & [inline]
```

for b[i] = x;

Definition at line 1003 of file [index_set_imp.h](#).

References [m_idx](#), [m_pst](#), and [reference\(\)](#).

8.46.3.4 operator=() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const reference & c_j) -> reference & [inline]
```

for b[i] = c[j];

Definition at line 1017 of file [index_set_imp.h](#).

References [m_idx](#), [m_pst](#), and [reference\(\)](#).

8.46.3.5 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator== (
    const reference & c_j) const -> bool [inline]
```

for b[i] == c[j];

Definition at line 995 of file [index_set_imp.h](#).

References [m_idx](#), [m_pst](#), and [reference\(\)](#).

8.46.3.6 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator~ () const -> bool [inline]
```

Flips a bit.

flips the bit

Definition at line 1034 of file [index_set_imp.h](#).

References [m_idx](#), and [m_pst](#).

8.46.4 Friends And Related Symbol Documentation

8.46.4.1 index_set

```
template<const index_t LO, const index_t HI>
friend class index_set [friend]
```

Definition at line 178 of file [index_set.h](#).

References [index_set](#).

Referenced by [index_set](#).

8.46.5 Member Data Documentation

8.46.5.1 m_idx

```
template<const index_t LO, const index_t HI>
index_t glucat::index_set< LO, HI >::reference::m_idx [private]
```

Definition at line 200 of file [index_set.h](#).

Referenced by [flip\(\)](#), [operator bool\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), [operator~\(\)](#), and [reference\(\)](#).

8.46.5.2 m_pst

```
template<const index_t LO, const index_t HI>
index_set_t* glucat::index_set< LO, HI >::reference::m_pst [private]
```

Definition at line 199 of file [index_set.h](#).

Referenced by [flip\(\)](#), [operator bool\(\)](#), [operator=\(\)](#), [operator=\(\)](#), [operator==\(\)](#), [operator~\(\)](#), and [reference\(\)](#).

The documentation for this class was generated from the following files:

- [glucat/index_set.h](#)
- [glucat/index_set_imp.h](#)

8.47 glucat::sorted_range< Map_T, Sorted_Map_T > Class Template Reference

Sorted range for use with output.

```
#include <framed_multi_imp.h>
```

Public Types

- using `map_t` = `Map_T`
- using `sorted_map_t` = `Sorted_Map_T`
- using `sorted_iterator` = `typename Sorted_Map_T::const_iterator`

Public Member Functions

- `sorted_range` (`Sorted_Map_T &sorted_val, const Map_T &val`)

Public Attributes

- `sorted_iterator sorted_begin`
- `sorted_iterator sorted_end`

8.47.1 Detailed Description

```
template<typename Map_T, typename Sorted_Map_T>
class glucat::sorted_range< Map_T, Sorted_Map_T >
```

Sorted range for use with output.

Definition at line 1108 of file [framed_multi_imp.h](#).

8.47.2 Member Typedef Documentation

8.47.2.1 `map_t`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::map_t = Map_T
```

Definition at line 1111 of file [framed_multi_imp.h](#).

8.47.2.2 `sorted_iterator`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T::↵
const_iterator
```

Definition at line 1113 of file [framed_multi_imp.h](#).

8.47.2.3 `sorted_map_t`

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1112 of file [framed_multi_imp.h](#).

8.47.3 Constructor & Destructor Documentation

8.47.3.1 `sorted_range()`

```
template<typename Map_T, typename Sorted_Map_T>
glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Map_T & val) [inline]
```

Definition at line 1115 of file [framed_multi_imp.h](#).

References [sorted_begin](#), and [sorted_end](#).

Referenced by [glucat::operator<<\(\)](#).

8.47.4 Member Data Documentation

8.47.4.1 `sorted_begin`

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1122 of file [framed_multi_imp.h](#).

Referenced by [sorted_range\(\)](#).

8.47.4.2 `sorted_end`

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1123 of file [framed_multi_imp.h](#).

Referenced by [sorted_range\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/framed_multi_imp.h](#)

8.48 `glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >` Class Template Reference

```
#include <framed_multi_imp.h>
```

Public Types

- using [map_t](#) = `Sorted_Map_T`
- using [sorted_map_t](#) = `Sorted_Map_T`
- using [sorted_iterator](#) = `typename Sorted_Map_T::const_iterator`

Public Member Functions

- [sorted_range](#) (Sorted_Map_T &sorted_val, const Sorted_Map_T &val)

Public Attributes

- [sorted_iterator](#) [sorted_begin](#)
- [sorted_iterator](#) [sorted_end](#)

8.48.1 Detailed Description

```
template<typename Sorted_Map_T>
class glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >
```

Definition at line 1127 of file [framed_multi_imp.h](#).

8.48.2 Member Typedef Documentation

8.48.2.1 map_t

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::map_t = Sorted_Map_T
```

Definition at line 1130 of file [framed_multi_imp.h](#).

8.48.2.2 sorted_iterator

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T::const_iterator
```

Definition at line 1132 of file [framed_multi_imp.h](#).

8.48.2.3 sorted_map_t

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1131 of file [framed_multi_imp.h](#).

8.48.3 Constructor & Destructor Documentation

8.48.3.1 sorted_range()

```
template<typename Sorted_Map_T>
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Sorted_Map_T & val) [inline]
```

Definition at line 1134 of file [framed_multi_imp.h](#).

References [sorted_begin](#), and [sorted_end](#).

8.48.4 Member Data Documentation

8.48.4.1 sorted_begin

```
template<typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1138 of file [framed_multi_imp.h](#).

Referenced by [sorted_range\(\)](#).

8.48.4.2 sorted_end

```
template<typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1139 of file [framed_multi_imp.h](#).

Referenced by [sorted_range\(\)](#).

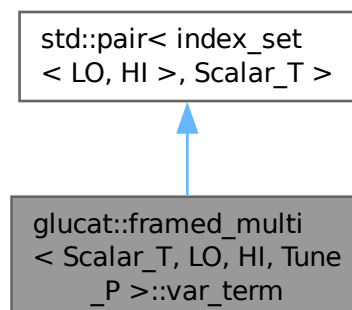
The documentation for this class was generated from the following file:

- [glucat/framed_multi_imp.h](#)

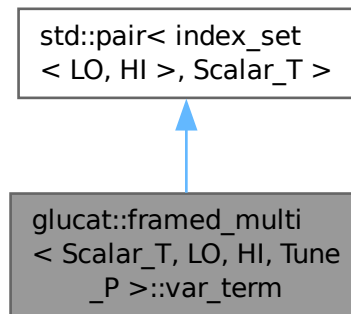
8.49 glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term Class Reference

Variable term.

Inheritance diagram for glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term:



Collaboration diagram for `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term`:



Public Types

- using `var_pair_t` = `std::pair<index_set<LO, HI>, Scalar_T>`

Public Member Functions

- `~var_term()` = default
Destructor.
- `var_term()`
Default constructor.
- `var_term(const index_set_t ist, const Scalar_T &crd=Scalar_T(1))`
Construct a variable term from an index set and a scalar coordinate.
- `auto operator*= (const term_t &rhs) -> var_term_t &`
Product of variable term and term.

Static Public Member Functions

- `static auto classname() -> const std::string`
Class name used in messages.

8.49.1 Detailed Description

```

template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term

```

Variable term.

Definition at line 279 of file `framed_multi.h`.

8.49.2 Member Typedef Documentation

8.49.2.1 var_pair_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>
```

Definition at line 283 of file [framed_multi.h](#).

8.49.3 Constructor & Destructor Documentation

8.49.3.1 ~var_term()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::~~var_term () [default]
```

Destructor.

8.49.3.2 var_term() [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term () [inline]
```

Default constructor.

Definition at line 291 of file [framed_multi.h](#).

8.49.3.3 var_term() [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term (
    const index_set_t ist,
    const Scalar_T & crd = Scalar_T(1)) [inline]
```

Construct a variable term from an index set and a scalar coordinate.

Definition at line 295 of file [framed_multi.h](#).

8.49.4 Member Function Documentation

8.49.4.1 classname()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::classname () -> const std::string [inline], [static]
```

Class name used in messages.

Definition at line 286 of file [framed_multi.h](#).

8.49.4.2 operator*=()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::operator*= (
    const term\_t & rhs) -> var\_term\_t & [inline]
```

Product of variable term and term.

Definition at line 299 of file [framed_multi.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed_multi.h](#)

Chapter 9

File Documentation

9.1 glucat/clifford_algebra.h File Reference

```
#include "glucat/global.h"
```

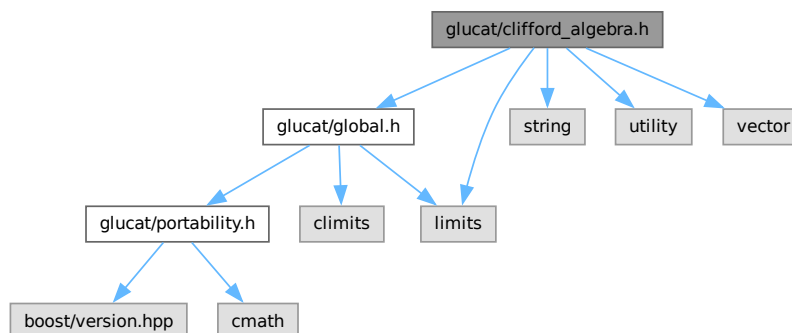
```
#include <limits>
```

```
#include <string>
```

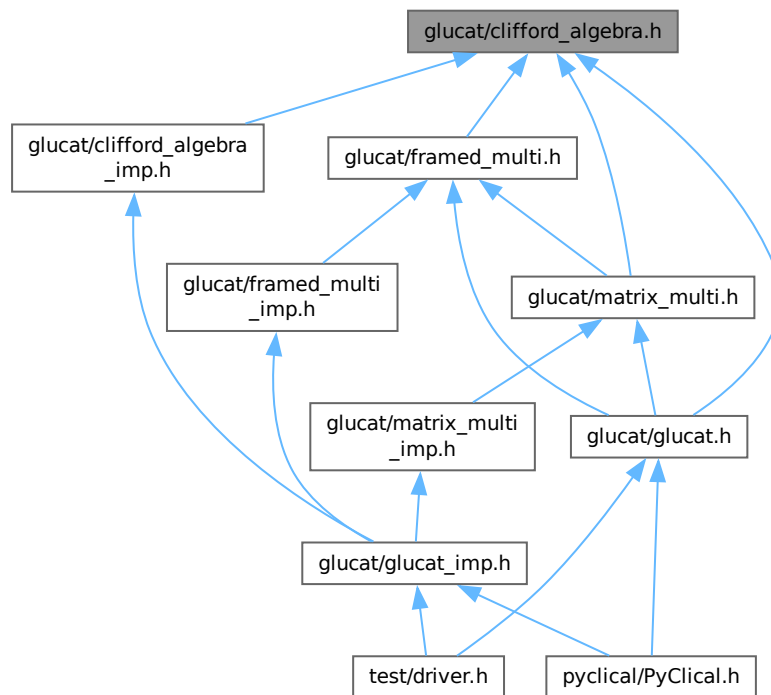
```
#include <utility>
```

```
#include <vector>
```

Include dependency graph for clifford_algebra.h:



This graph shows which files directly or indirectly include this file:



Classes

- class `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`
clifford_algebra<> declares the operations of a *Clifford algebra*

Namespaces

- namespace `glucat`

Macros

- `#define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS`

Functions

- template<template< typename, const `index_t`, const `index_t`, typename > class Multivector, template< typename, const `index_t`, const `index_t`, typename > class RHS, typename Scalar_T, const `index_t` LO, const `index_t` HI, typename Tune_P>
 auto `glucat::operator!=` (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool
Test for inequality of multivectors.
- template<template< typename, const `index_t`, const `index_t`, typename > class Multivector, typename Scalar_T, const `index_t` LO, const `index_t` HI, typename Tune_P>
 auto `glucat::operator!=` (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool

Test for inequality of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`

Test for inequality of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Quadratic norm error tolerance relative to a specific multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`

Relative or absolute error using the quadratic norm.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`

Test for approximate equality of multivectors.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`

Test for approximate equality of multivectors.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric sum.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference of multivector and scalar.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference of scalar and multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Geometric difference.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Geometric multiplicative inverse.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Integer power of multivector.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Multivector power of multivector.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Outer product power of multivector.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Scalar part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Real part: synonym for scalar part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`
Imaginary part: deprecated (always 0).
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Pure part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Even part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
Odd part.
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`
Vector part of multivector, as a vector_t with respect to frame().
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Main involution, each $\{i\}$ is replaced by $-\{i\}$ in each term, eg. $\{1\}\{2\} \rightarrow (-\{2\})*(-\{1\})$.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Reversion, eg. $\{1\}\{2\} \rightarrow \{2\}*\{1\}$.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Conjugation, $rev \circ invo == invo \circ rev$.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar_T quadratic form == $(rev(x)*x)(0)$.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Scalar_T norm == sum of norm of coordinates.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Absolute value == $\sqrt{\text{norm}}$.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

Maximum of absolute values of components of multivector: multivector infinity norm.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of -1 which commutes with all members of the frame of the given multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Square root of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Exponential of multivector.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector with specified complexifier.

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

Natural logarithm of multivector.

- ```

• template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO,
const index_t HI, typename Tune_P>
auto glucatt::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI,
Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

```

*Cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Sine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucatt::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Sine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



## 9.1.1 Macro Definition Documentation

### 9.1.1.1 \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

#define \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

Definition at line 145 of file clifford\_algebra.h.

Referenced by `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element()`, and `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::basis_element()`.

## 9.2 clifford\_algebra.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 clifford_algebra.h : Declare the operations of a Clifford algebra
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <limits>
00037 #include <string>
00038 #include <utility>
00039 #include <vector>
00040
00041 namespace glucat
00042 {
00043 template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00044 class clifford_algebra
00045 {
00046 public:
00047 using scalar_t = Scalar_T;
00048 using index_set_t = Index_Set_T;
00049 static const index_t v_lo = index_set_t::v_lo;
00050 static const index_t v_hi = index_set_t::v_hi;
00051 using multivector_t = Multivector_T;
00052 using pair_t = std::pair<const index_set_t, Scalar_T>;
00053 using vector_t = std::vector<Scalar_T>;
00054
00055 static auto classname() -> const std::string;
00056
00057 static const Scalar_T default_truncation;
00058
00059 virtual ~clifford_algebra() = default;
00060
00061 // clifford_algebra operations
00062 virtual auto operator==(const multivector_t& val) const -> bool = 0;

```

```

00067 virtual auto operator== (const Scalar_T& scr) const -> bool = 0;
00069 virtual auto operator+= (const multivector_t& rhs) -> multivector_t& = 0;
00071 virtual auto operator+= (const Scalar_T& scr) -> multivector_t& = 0;
00073 virtual auto operator-= (const multivector_t& rhs) -> multivector_t& = 0;
00075 virtual auto operator-= (const Scalar_T& scr) -> multivector_t& = 0;
00077 virtual auto operator- () const -> const multivector_t = 0;
00079 virtual auto operator*= (const Scalar_T& scr) -> multivector_t& = 0;
00081 virtual auto operator*= (const multivector_t& rhs) -> multivector_t& = 0;
00083 virtual auto operator%= (const multivector_t& rhs) -> multivector_t& = 0;
00085 virtual auto operator&= (const multivector_t& rhs) -> multivector_t& = 0;
00087 virtual auto operator^= (const multivector_t& rhs) -> multivector_t& = 0;
00089 virtual auto operator/= (const Scalar_T& scr) -> multivector_t& = 0;
00091 virtual auto operator/= (const multivector_t& rhs) -> multivector_t& = 0;
00093 virtual auto operator|= (const multivector_t& rhs) -> multivector_t& = 0;
00095 virtual auto inv () const -> const multivector_t = 0;
00097 virtual auto pow (int m) const -> const multivector_t = 0;
00099 virtual auto outer_pow (int m) const -> const multivector_t = 0;
00101 virtual auto frame () const -> const index_set_t = 0;
00103 virtual auto grade () const -> index_t = 0;
00105 virtual auto operator[] (const index_set_t ist) const -> Scalar_T = 0;
00107 virtual auto operator() (index_t grade) const -> const multivector_t = 0;
00109 virtual auto scalar () const -> Scalar_T = 0;
00111 virtual auto pure () const -> const multivector_t = 0;
00113 virtual auto even () const -> const multivector_t = 0;
00115 virtual auto odd () const -> const multivector_t = 0;
00117 virtual auto vector_part () const -> const vector_t = 0;
00119 virtual auto vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t =
0;
00121 virtual auto involute () const -> const multivector_t = 0;
00123 virtual auto reverse () const -> const multivector_t = 0;
00125 virtual auto conj () const -> const multivector_t = 0;
00127 virtual auto quad () const -> Scalar_T = 0;
00129 virtual auto norm () const -> Scalar_T = 0;
00131 virtual auto max_abs () const -> Scalar_T = 0;
00133 virtual auto truncated (const Scalar_T& limit = default_truncation) const -> const multivector_t
= 0;
00135 virtual auto isinf () const -> bool = 0;
00137 virtual auto isnan () const -> bool = 0;
00139 virtual void write (const std::string& msg="") const = 0;
00141 virtual void write (std::ofstream& ofile, const std::string& msg="") const = 0;
00142 };
00143
00144 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00145 #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS \
00146 auto operator== (const multivector_t& val) const -> bool override;\
00147 auto operator== (const Scalar_T& scr) const -> bool override;\
00148 auto operator+= (const multivector_t& rhs) -> multivector_t& override;\
00149 auto operator+= (const Scalar_T& scr) -> multivector_t& override;\
00150 auto operator-= (const multivector_t& rhs) -> multivector_t& override;\
00151 auto operator-= (const Scalar_T& scr) -> multivector_t& override;\
00152 auto operator- () const -> const multivector_t override;\
00153 auto operator*= (const Scalar_T& scr) -> multivector_t& override;\
00154 auto operator*= (const multivector_t& rhs) -> multivector_t& override;\
00155 auto operator%= (const multivector_t& rhs) -> multivector_t& override;\
00156 auto operator&= (const multivector_t& rhs) -> multivector_t& override;\
00157 auto operator^= (const multivector_t& rhs) -> multivector_t& override;\
00158 auto operator/= (const Scalar_T& scr) -> multivector_t& override;\
00159 auto operator/= (const multivector_t& rhs) -> multivector_t& override;\
00160 auto operator|= (const multivector_t& rhs) -> multivector_t& override;\
00161 auto inv () const -> const multivector_t override;\
00162 auto pow (int m) const -> const multivector_t override;\
00163 auto outer_pow (int m) const -> const multivector_t override;\
00164 auto frame () const -> const index_set_t override;\
00165 auto grade () const -> index_t override;\
00166 auto operator[] (const index_set_t ist) const -> Scalar_T override;\
00167 auto operator() (index_t grade) const -> const multivector_t override;\
00168 auto scalar () const -> Scalar_T override;\
00169 auto pure () const -> const multivector_t override;\
00170 auto even () const -> const multivector_t override;\
00171 auto odd () const -> const multivector_t override;\
00172 auto vector_part () const -> const vector_t override;\
00173 auto vector_part (const index_set_t frm, const bool prechecked = false) const \
00174 -> const vector_t override;\
00175 auto involute () const -> const multivector_t override;\
00176 auto reverse () const -> const multivector_t override;\
00177 auto conj () const -> const multivector_t override;\
00178 auto quad () const -> Scalar_T override;\
00179 auto norm () const -> Scalar_T override;\
00180 auto max_abs () const -> Scalar_T override;\
00181 auto truncated (const Scalar_T& limit = multivector_t::default_truncation) const \
00182 -> const multivector_t override;\
00183 auto isinf () const -> bool override;\
00184 auto isnan () const -> bool override;\
00185 void write (const std::string& msg="") const override;\
00186 void write (std::ofstream& ofile, const std::string& msg="") const override;\
00187 #endif // _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00188

```

```

00190 template
00191 <
00192 template<typename, const index_t, const index_t, typename> class Multivector,
00193 template<typename, const index_t, const index_t, typename> class RHS,
00194 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00195 >
00196 auto
00197 operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
bool;
00198
00200 template
00201 <
00202 template<typename, const index_t, const index_t, typename> class Multivector,
00203 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00204 >
00205 auto
00206 operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool;
00207
00209 template
00210 <
00211 template<typename, const index_t, const index_t, typename> class Multivector,
00212 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00213 >
00214 auto
00215 operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00216
00218 template
00219 <
00220 template<typename, const index_t, const index_t, typename> class Multivector,
00221 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00222 >
00223 auto
00224 error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00225
00227 template
00228 <
00229 template<typename, const index_t, const index_t, typename> class Multivector,
00230 template<typename, const index_t, const index_t, typename> class RHS,
00231 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232 >
00233 auto
00234 error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00235 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00236 const Scalar_T threshold) -> Scalar_T;
00237
00239 template
00240 <
00241 template<typename, const index_t, const index_t, typename> class Multivector,
00242 template<typename, const index_t, const index_t, typename> class RHS,
00243 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00244 >
00245 auto
00246 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00247 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00248 const Scalar_T threshold,
00249 const Scalar_T tolerance) -> bool;
00250
00252 template
00253 <
00254 template<typename, const index_t, const index_t, typename> class Multivector,
00255 template<typename, const index_t, const index_t, typename> class RHS,
00256 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00257 >
00258 auto
00259 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00260 const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00261
00263 template
00264 <
00265 template<typename, const index_t, const index_t, typename> class Multivector,
00266 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00267 >
00268 auto
00269 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00270
00272 template
00273 <
00274 template<typename, const index_t, const index_t, typename> class Multivector,
00275 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00276 >
00277 auto
00278 operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00279
00281 template
00282 <

```



```

00283 template<typename, const index_t, const index_t, typename> class Multivector,
00284 template<typename, const index_t, const index_t, typename> class RHS,
00285 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00286 >
00287 auto
00288 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00289
00291 template
00292 <
00293 template<typename, const index_t, const index_t, typename> class Multivector,
00294 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00295 >
00296 auto
00297 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00298
00300 template
00301 <
00302 template<typename, const index_t, const index_t, typename> class Multivector,
00303 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304 >
00305 auto
00306 operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00307
00309 template
00310 <
00311 template<typename, const index_t, const index_t, typename> class Multivector,
00312 template<typename, const index_t, const index_t, typename> class RHS,
00313 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00314 >
00315 auto
00316 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00317
00319 template
00320 <
00321 template<typename, const index_t, const index_t, typename> class Multivector,
00322 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00323 >
00324 auto
00325 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00326
00328 template
00329 <
00330 template<typename, const index_t, const index_t, typename> class Multivector,
00331 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00332 >
00333 auto
00334 operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00335
00337 template
00338 <
00339 template<typename, const index_t, const index_t, typename> class Multivector,
00340 template<typename, const index_t, const index_t, typename> class RHS,
00341 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00342 >
00343 auto
00344 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00345
00347 template
00348 <
00349 template<typename, const index_t, const index_t, typename> class Multivector,
00350 template<typename, const index_t, const index_t, typename> class RHS,
00351 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00352 >
00353 auto
00354 operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00355
00357 template
00358 <
00359 template<typename, const index_t, const index_t, typename> class Multivector,
00360 template<typename, const index_t, const index_t, typename> class RHS,
00361 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00362 >
00363 auto
00364 operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00365
00367 template
00368 <
00369 template<typename, const index_t, const index_t, typename> class Multivector,

```



```

00370 template<typename, const index_t, const index_t, typename> class RHS,
00371 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00372 >
00373 auto
00374 operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00375
00376 template
00377 <
00378 template<typename, const index_t, const index_t, typename> class Multivector,
00379 template<typename, const index_t, const index_t, typename> class RHS,
00380 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00381 >
00382 auto
00383 star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T;
00384
00385 template
00386 <
00387 template<typename, const index_t, const index_t, typename> class Multivector,
00388 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00389 >
00390 auto
00391 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00392
00393 template
00394 <
00395 template<typename, const index_t, const index_t, typename> class Multivector,
00396 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00397 >
00398 auto
00399 operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00400
00401 template
00402 <
00403 template<typename, const index_t, const index_t, typename> class Multivector,
00404 template<typename, const index_t, const index_t, typename> class RHS,
00405 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00406 >
00407 auto
00408 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00409
00410 template
00411 <
00412 template<typename, const index_t, const index_t, typename> class Multivector,
00413 template<typename, const index_t, const index_t, typename> class RHS,
00414 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00415 >
00416 auto
00417 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00418
00419 template
00420 <
00421 template<typename, const index_t, const index_t, typename> class Multivector,
00422 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00423 >
00424 auto
00425 inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00426
00427 template
00428 <
00429 template<typename, const index_t, const index_t, typename> class Multivector,
00430 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00431 >
00432 auto
00433 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00434
00435 template
00436 <
00437 template<typename, const index_t, const index_t, typename> class Multivector,
00438 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00439 >
00440 auto
00441 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00442
00443 template<
 template<typename, const index_t, const index_t, typename> class Multivector,
 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00444 auto
00445 outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

```

```

00457
00459 template
00460 <
00461 template<typename, const index_t, const index_t, typename> class Multivector,
00462 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00463 >
00464 auto
00465 scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00466
00468 template
00469 <
00470 template<typename, const index_t, const index_t, typename> class Multivector,
00471 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00472 >
00473 auto
00474 real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00475
00477 template
00478 <
00479 template<typename, const index_t, const index_t, typename> class Multivector,
00480 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00481 >
00482 auto
00483 imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00484
00486 template
00487 <
00488 template<typename, const index_t, const index_t, typename> class Multivector,
00489 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00490 >
00491 auto
00492 pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00493
00495 template
00496 <
00497 template<typename, const index_t, const index_t, typename> class Multivector,
00498 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00499 >
00500 auto
00501 even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00502
00504 template
00505 <
00506 template<typename, const index_t, const index_t, typename> class Multivector,
00507 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00508 >
00509 auto
00510 odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00511
00513 template
00514 <
00515 template<typename, const index_t, const index_t, typename> class Multivector,
00516 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00517 >
00518 auto
00519 vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>;
00520
00522 template
00523 <
00524 template<typename, const index_t, const index_t, typename> class Multivector,
00525 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00526 >
00527 auto
00528 involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00529
00531 template
00532 <
00533 template<typename, const index_t, const index_t, typename> class Multivector,
00534 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00535 >
00536 auto
00537 reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00538
00540 template
00541 <
00542 template<typename, const index_t, const index_t, typename> class Multivector,
00543 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00544 >
00545 auto
00546 conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00547
00549 template
00550 <
00551 template<typename, const index_t, const index_t, typename> class Multivector,
00552 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00553 >
00554 auto

```

```

00555 quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00556
00557 template
00558 <
00559 template<typename, const index_t, const index_t, typename> class Multivector,
00560 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00561 >
00562 auto
00563 norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00564
00565 template
00566 <
00567 template<typename, const index_t, const index_t, typename> class Multivector,
00568 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00569 >
00570 auto
00571 abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00572
00573 template
00574 <
00575 template<typename, const index_t, const index_t, typename> class Multivector,
00576 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00577 >
00578 auto
00579 max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00580
00581 template
00582 <
00583 template<typename, const index_t, const index_t, typename> class Multivector,
00584 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00585 >
00586 auto
00587 complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00588 Multivector<Scalar_T,LO,HI,Tune_P>;
00589
00590 template
00591 <
00592 template<typename, const index_t, const index_t, typename> class Multivector,
00593 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00594 >
00595 auto
00596 elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00597
00598 template
00599 <
00600 template<typename, const index_t, const index_t, typename> class Multivector,
00601 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00602 >
00603 auto
00604 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00605 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00606 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00607
00608 template
00609 <
00610 template<typename, const index_t, const index_t, typename> class Multivector,
00611 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00612 >
00613 auto
00614 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00615
00616 // Transcendental functions
00617
00618 template
00619 <
00620 template<typename, const index_t, const index_t, typename> class Multivector,
00621 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00622 >
00623 auto
00624 clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00625 Multivector<Scalar_T,LO,HI,Tune_P>;
00626
00627 template
00628 <
00629 template<typename, const index_t, const index_t, typename> class Multivector,
00630 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00631 >
00632 auto
00633 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00634 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00635 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00636
00637 template
00638 <
00639 template<typename, const index_t, const index_t, typename> class Multivector,
00640 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00641 >
00642 auto

```

```

00651 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00652
00653 template
00654 <
00655 template<typename, const index_t, const index_t, typename> class Multivector,
00656 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00657 >
00658 auto
00659 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00660 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00661 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00662
00663 template
00664 <
00665 template<typename, const index_t, const index_t, typename> class Multivector,
00666 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00667 >
00668 auto
00669 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00670
00671 template
00672 <
00673 template<typename, const index_t, const index_t, typename> class Multivector,
00674 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00675 >
00676 auto
00677 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00678 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00679 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00680
00681 template
00682 <
00683 template<typename, const index_t, const index_t, typename> class Multivector,
00684 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00685 >
00686 auto
00687 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00688
00689 template
00690 <
00691 template<typename, const index_t, const index_t, typename> class Multivector,
00692 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00693 >
00694 auto
00695 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00696
00697 template
00698 <
00699 template<typename, const index_t, const index_t, typename> class Multivector,
00700 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00701 >
00702 auto
00703 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00704
00705 template
00706 <
00707 template<typename, const index_t, const index_t, typename> class Multivector,
00708 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00709 >
00710 auto
00711 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00712 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00713 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00714
00715 template
00716 <
00717 template<typename, const index_t, const index_t, typename> class Multivector,
00718 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00719 >
00720 auto
00721 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00722
00723 template
00724 <
00725 template<typename, const index_t, const index_t, typename> class Multivector,
00726 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00727 >
00728 auto
00729 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00730 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00731 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00732
00733 template
00734 <
00735 template<typename, const index_t, const index_t, typename> class Multivector,
00736 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00737 >
00738 auto
00739 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00740
00741 template
00742 <
00743 template<typename, const index_t, const index_t, typename> class Multivector,
00744 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00745 >
00746 auto
00747 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;

```

```

00748 auto
00749 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00750 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00751 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00752
00753 template
00754 <
00755 template<typename, const index_t, const index_t, typename> class Multivector,
00756 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00757 >
00758 auto
00759 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00760
00761 template
00762 <
00763 template<typename, const index_t, const index_t, typename> class Multivector,
00764 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00765 >
00766 auto
00767 sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00770
00771 template
00772 <
00773 template<typename, const index_t, const index_t, typename> class Multivector,
00774 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00775 >
00776 auto
00777 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00778 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00779 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00781
00782 template
00783 <
00784 template<typename, const index_t, const index_t, typename> class Multivector,
00785 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00786 >
00787 auto
00788 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00790
00791 template
00792 <
00793 template<typename, const index_t, const index_t, typename> class Multivector,
00794 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00795 >
00796 auto
00797 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00798 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00799 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00801
00802 template
00803 <
00804 template<typename, const index_t, const index_t, typename> class Multivector,
00805 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00806 >
00807 auto
00808 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00810
00811 template
00812 <
00813 template<typename, const index_t, const index_t, typename> class Multivector,
00814 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00815 >
00816 auto
00817 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00818 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00819 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00821
00822 template
00823 <
00824 template<typename, const index_t, const index_t, typename> class Multivector,
00825 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00826 >
00827 auto
00828 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00830
00831 template
00832 <
00833 template<typename, const index_t, const index_t, typename> class Multivector,
00834 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00835 >
00836 auto
00837 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00839
00840 template
00841 <
00842 template<typename, const index_t, const index_t, typename> class Multivector,
00843 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P

```

```

00845 >
00846 auto
00847 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00848 const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00849 const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00850
00852 template
00853 <
00854 template<typename, const index_t, const index_t, typename> class Multivector,
00855 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00856 >
00857 auto
00858 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00859 }
00860 #endif // _GLUCAT_CLIFFORD_ALGEBRA_H

```

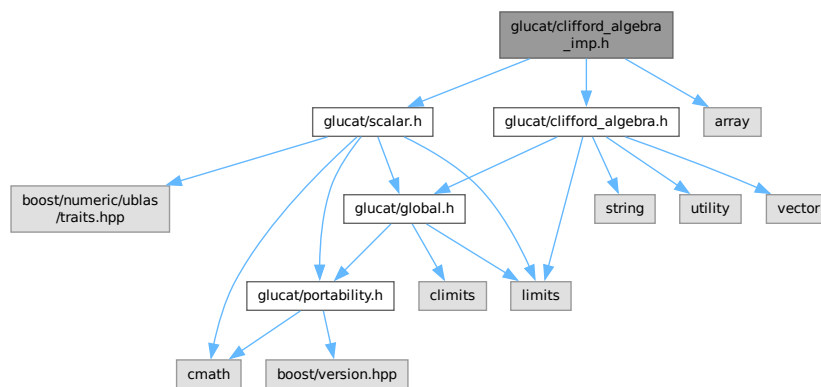
### 9.3 glucat/clifford\_algebra\_imp.h File Reference

```

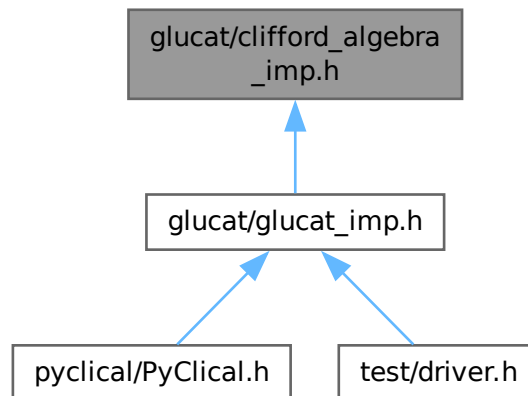
#include "glucat/clifford_algebra.h"
#include "glucat/scalar.h"
#include <array>

```

Include dependency graph for clifford\_algebra\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## Functions

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool`  
*Test for inequality of multivector and scalar.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Quadratic norm error tolerance relative to a specific multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`  
*Relative or absolute error using the quadratic norm.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`





*Outer product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric multiplicative inverse.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Integer power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Multivector power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Outer product power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Real part: synonym for scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Imaginary part: deprecated (always 0).*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Pure part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T, HI, Tune_P >`

*Vector part of multivector, as a `vector_t` with respect to frame().*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-i$  in each term, eg.  $\{1\}\{2\} \rightarrow (-\{2\})*(-\{1\})$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}\{2\} \rightarrow \{2\}\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Absolute value == sqrt(norm).*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::max\\_abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::complexifier](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::elliptic](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static void [glucat::check\\_complex](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false)

*Check that i is a valid complexifier for val.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::clifford\\_exp](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::cosh](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Hyperbolic cosine of multivector.*



- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::asin](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse sine of multivector with specified complexifier.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::asin](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse sine of multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::tanh](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Hyperbolic tangent of multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::atanh](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse hyperbolic tangent of multivector with specified complexifier.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::atanh](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse hyperbolic tangent of multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::tan](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Tangent of multivector with specified complexifier.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::tan](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Tangent of multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::atan](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse tangent of multivector with specified complexifier.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::atan](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Inverse tangent of multivector.*

## Variables

- template<typename Scalar\_T, typename Index\_Set\_T, typename Multivector\_T>  
const Scalar\_T [glucat::clifford\\_algebra](#)< Scalar\_T, Index\_Set\_T, Multivector\_T >::default\_truncation = std::numeric\_limits<Scalar\_T>::epsilon()  
*Default for truncation.*

## 9.4 clifford\_algebra\_imp.h

Go to the documentation of this file.

```

00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 clifford_algebra_imp.h : Implement common Clifford algebra functions
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 // References for algorithms:
00035 // [AS]:
00036 // Milton Abramowicz and Irene A. Stegun, "Handbook of mathematical functions",
00037 // Dover 1972, first published 1965.
00038 // [CHKL]:
00039 // Sheung Hun Cheng, Nicholas J. Higham, Charles S. Kenney and Alan J. Laub,
00040 // "Approximating the Logarithm of a Matrix to Specified Accuracy", 1999.
00041 // ftp://ftp.ma.man.ac.uk/pub/narep/narep353.ps.gz
00042 // [GL]:
00043 // Gene H. Golub and Charles F. van Loan,
00044 // "Matrix Computations", 3rd ed., Johns Hopkins UP, 1996.
00045 // [GW]:
00046 // C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis",
00047 // 6th Edition, Addison-Wesley, 1999.
00048 // [H]:
00049 // Nicholas J. Higham
00050 // "The Scaling and Squaring Method for the Matrix Exponential Revisited",
00051 // SIAM Journal on Matrix Analysis and Applications,
00052 // Vol. 26, Issue 4 (2005), pp. 1179-1193.
00053 // [Z]:
00054 // Doron Zeilberger, "PADE" (Maple code), 2002.
00055 // http://www.math.rutgers.edu/~zeilberg/tokhniot/PADE
00056
00057 #include "glucat/clifford_algebra.h"
00058 #include "glucat/scalar.h"
00059
00060 #include <array>
00061
00062 namespace glucat
00063 {
00064 template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00065 auto
00066 clifford_algebra<Scalar_T, Index_Set_T, Multivector_T>::
00067 classname() -> const std::string
00068 { return "clifford_algebra"; }
00069
00070 template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00071 const
00072 Scalar_T
00073 clifford_algebra<Scalar_T, Index_Set_T, Multivector_T>::
00074 default_truncation = std::numeric_limits<Scalar_T>::epsilon();
00075
00076 template
00077 <
00078 template<typename, const index_t, const index_t, typename> class Multivector,
00079 template<typename, const index_t, const index_t, typename> class RHS,
00080 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00081 >
00082 inline

```

```

00085 auto
00086 operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
bool
00087 { return !(lhs == rhs); }
00088
00090 template< template<typename, const index_t, const index_t, typename> class Multivector,
00091 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092 inline
00093 auto
00094 operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool
00095 { return !(lhs == scr); }
00096
00098 template< template<typename, const index_t, const index_t, typename> class Multivector,
00099 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00100 inline
00101 auto
00102 operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00103 { return !(rhs == scr); }
00104
00106 template
00107 <
00108 template<typename, const index_t, const index_t, typename> class Multivector,
00109 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00110 >
00111 auto
00112 error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00113 {
00114 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00115 static const auto scalar_eps = std::numeric_limits<Scalar_T>::epsilon();
00116 static const auto nbr_different_bits =
00117 std::numeric_limits<Scalar_T>::digits / Tune_P::denom_different_bits +
Tune_P::extra_different_bits;
00118 static const auto abs_tol = scalar_eps *
00119 numeric_traits<Scalar_T>::pow(Scalar_T(2), nbr_different_bits);
00120 using framed_multi_t = typename multivector_t::framed_multi_t;
00121 const auto nbr_terms = double(framed_multi_t(val).truncated(scalar_eps).nbr_terms());
00122 return abs_tol * abs_tol * std::max(Scalar_T(nbr_terms), Scalar_T(1));
00123 }
00124
00126 template
00127 <
00128 template<typename, const index_t, const index_t, typename> class Multivector,
00129 template<typename, const index_t, const index_t, typename> class RHS,
00130 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00131 >
00132 inline
00133 auto
00134 error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00135 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00136 const Scalar_T threshold) -> Scalar_T
00137 {
00138 const auto relative = norm(rhs) > threshold;
00139 const auto abs_norm_diff = norm(rhs-lhs);
00140 return (relative)
00141 ? abs_norm_diff/norm(rhs)
00142 : abs_norm_diff;
00143 }
00144
00146 template
00147 <
00148 template<typename, const index_t, const index_t, typename> class Multivector,
00149 template<typename, const index_t, const index_t, typename> class RHS,
00150 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00151 >
00152 inline
00153 auto
00154 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00155 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00156 const Scalar_T threshold,
00157 const Scalar_T tolerance) -> bool
00158 { return error_squared(lhs, rhs, threshold) < tolerance; }
00159
00161 template
00162 <
00163 template<typename, const index_t, const index_t, typename> class Multivector,
00164 template<typename, const index_t, const index_t, typename> class RHS,
00165 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00166 >
00167 inline
00168 auto
00169 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00170 const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00171 {
00172 const Scalar_T rhs_tol = error_squared_tol(rhs);
00173 return approx_equal(lhs, rhs, rhs_tol, rhs_tol);
00174 }
00175

```



```

00177 template< template<typename, const index_t, const index_t, typename> class Multivector,
00178 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00179 inline
00180 auto
00181 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00182 {
00183 auto result = lhs;
00184 return result += scr;
00185 }
00186
00187 template< template<typename, const index_t, const index_t, typename> class Multivector,
00188 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00189 inline
00190 auto
00191 operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00192 {
00193 return rhs + scr;
00194 }
00195
00196 template
00197 <
00198 template<typename, const index_t, const index_t, typename> class Multivector,
00199 template<typename, const index_t, const index_t, typename> class RHS,
00200 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00201 >
00202 inline
00203 auto
00204 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00205 {
00206 auto result = lhs;
00207 return result += rhs;
00208 }
00209
00210 template< template<typename, const index_t, const index_t, typename> class Multivector,
00211 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00212 inline
00213 auto
00214 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00215 {
00216 auto result = lhs;
00217 return result -= scr;
00218 }
00219
00220 template< template<typename, const index_t, const index_t, typename> class Multivector,
00221 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00222 inline
00223 auto
00224 operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00225 { return -rhs + scr; }
00226
00227 template
00228 <
00229 template<typename, const index_t, const index_t, typename> class Multivector,
00230 template<typename, const index_t, const index_t, typename> class RHS,
00231 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232 >
00233 inline
00234 auto
00235 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00236 {
00237 auto result = lhs;
00238 return result -= rhs;
00239 }
00240
00241 template< template<typename, const index_t, const index_t, typename> class Multivector,
00242 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00243 inline
00244 auto
00245 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00246 {
00247 auto result = lhs;
00248 return result *= scr;
00249 }
00250
00251 template< template<typename, const index_t, const index_t, typename> class Multivector,
00252 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00253 inline
00254 auto
00255 operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>

```



```

00263 { // Note: this assumes that scalar commutes with multivector.
00264 // This excludes Clifford algebras over non-commuting rings.
00265 return rhs * scr;
00266 }
00267
00269 template
00270 <
00271 template<typename, const index_t, const index_t, typename> class Multivector,
00272 template<typename, const index_t, const index_t, typename> class RHS,
00273 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00274 >
00275 inline
00276 auto
00277 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00278 {
00279 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00280 return lhs * multivector_t(rhs);
00281 }
00282
00284 template
00285 <
00286 template<typename, const index_t, const index_t, typename> class Multivector,
00287 template<typename, const index_t, const index_t, typename> class RHS,
00288 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00289 >
00290 inline
00291 auto
00292 operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00293 {
00294 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00295 return lhs ^ multivector_t(rhs);
00296 }
00297
00299 template
00300 <
00301 template<typename, const index_t, const index_t, typename> class Multivector,
00302 template<typename, const index_t, const index_t, typename> class RHS,
00303 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304 >
00305 inline
00306 auto
00307 operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00308 {
00309 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00310 return lhs & multivector_t(rhs);
00311 }
00312
00314 template
00315 <
00316 template<typename, const index_t, const index_t, typename> class Multivector,
00317 template<typename, const index_t, const index_t, typename> class RHS,
00318 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00319 >
00320 inline
00321 auto
00322 operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00323 {
00324 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00325 return lhs % multivector_t(rhs);
00326 }
00327
00329 template
00330 <
00331 template<typename, const index_t, const index_t, typename> class Multivector,
00332 template<typename, const index_t, const index_t, typename> class RHS,
00333 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00334 >
00335 inline
00336 auto
00337 star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T
00338 {
00339 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00340 return star(lhs, multivector_t(rhs));
00341 }
00342
00344 template< template<typename, const index_t, const index_t, typename> class Multivector,
00345 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00346 inline
00347 auto
00348 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00349 {

```

```

00350 auto result = lhs;
00351 return result /= scr;
00352 }
00353
00354 template< template<typename, const index_t, const index_t, typename> class Multivector,
00355 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00356 inline
00357 auto
00358 operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00359 Multivector<Scalar_T,LO,HI,Tune_P>
00360 {
00361 Multivector<Scalar_T,LO,HI,Tune_P> result = scr;
00362 return result /= rhs;
00363 }
00364
00365 template
00366 <
00367 template<typename, const index_t, const index_t, typename> class Multivector,
00368 template<typename, const index_t, const index_t, typename> class RHS,
00369 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00370 >
00371 inline
00372 auto
00373 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00374 const Multivector<Scalar_T,LO,HI,Tune_P>
00375 {
00376 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00377 return lhs / multivector_t(rhs);
00378 }
00379
00380 template
00381 <
00382 template<typename, const index_t, const index_t, typename> class Multivector,
00383 template<typename, const index_t, const index_t, typename> class RHS,
00384 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00385 >
00386 inline
00387 auto
00388 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00389 const Multivector<Scalar_T,LO,HI,Tune_P>
00390 {
00391 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00392 return lhs | multivector_t(rhs);
00393 }
00394
00395 template< template<typename, const index_t, const index_t, typename> class Multivector,
00396 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00397 inline
00398 auto
00399 inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00400 { return val.inv(); }
00401
00402 template< template<typename, const index_t, const index_t, typename> class Multivector,
00403 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00404 auto
00405 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
00406 Multivector<Scalar_T,LO,HI,Tune_P>
00407 {
00408 {
00409 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00410 if (lhs == Scalar_T(0))
00411 {
00412 using traits_t = numeric_traits<Scalar_T>;
00413 return
00414 (rhs < 0)
00415 ? traits_t::NaN()
00416 : (rhs == 0)
00417 ? Scalar_T(1)
00418 : Scalar_T(0);
00419 }
00420 auto result = multivector_t(Scalar_T(1));
00421 auto power =
00422 (rhs < 0)
00423 ? lhs.inv()
00424 : lhs;
00425 for (auto
00426 k = std::abs(rhs);
00427 k != 0;
00428 k /= 2)
00429 {
00430 if (k % 2)
00431 result *= power;
00432 power *= power;
00433 }
00434 return result;
00435 }
00436 }
00437
00438 template

```

```

00439 <
00440 template<typename, const index_t, const index_t, typename> class Multivector,
00441 template<typename, const index_t, const index_t, typename> class RHS,
00442 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00443 >
00444 inline
00445 auto
00446 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00447 {
00448 using traits_t = numeric_traits<Scalar_T>;
00449
00450 if (lhs == Scalar_T(0))
00451 {
00452 const Scalar_T m = rhs.scalar();
00453 if (rhs == m)
00454 return
00455 (m < 0)
00456 ? traits_t::NaN()
00457 : (m == 0)
00458 ? Scalar_T(1)
00459 : Scalar_T(0);
00460 else
00461 return Scalar_T(0);
00462 }
00463 return exp(log(lhs) * rhs);
00464 }
00465
00466 template< template<typename, const index_t, const index_t, typename> class Multivector,
00467 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00468 auto
00469 outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00470 { return lhs.outer_pow(rhs); }
00471
00472 template< template<typename, const index_t, const index_t, typename> class Multivector,
00473 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00474 inline
00475 auto
00476 scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00477 { return val.scalar(); }
00478
00479 template< template<typename, const index_t, const index_t, typename> class Multivector,
00480 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00481 inline
00482 auto
00483 real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00484 { return val.scalar(); }
00485
00486 template
00487 <
00488 template<typename, const index_t, const index_t, typename> class Multivector,
00489 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00490 >
00491 inline
00492 auto
00493 imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00494 { return Scalar_T(0); }
00495
00496 template< template<typename, const index_t, const index_t, typename> class Multivector,
00497 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00498 inline
00499 auto
00500 pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00501 { return val - val.scalar(); }
00502
00503 template< template<typename, const index_t, const index_t, typename> class Multivector,
00504 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00505 inline
00506 auto
00507 even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00508 { return val.even(); }
00509
00510 template< template<typename, const index_t, const index_t, typename> class Multivector,
00511 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00512 inline
00513 auto
00514 odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00515 { return val.odd(); }
00516
00517 template< template<typename, const index_t, const index_t, typename> class Multivector,
00518 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00519 inline
00520 auto
00521 vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00522 { return val.vector_part(); }
00523
00524 template< template<typename, const index_t, const index_t, typename> class Multivector,
00525 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00526 inline
00527 auto
00528 vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00529 { return val.vector_part(); }
00530
00531

```

```

00533 template< template<typename, const index_t, const index_t, typename> class Multivector,
00534 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00535 inline
00536 auto
00537 involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00538 { return val.involute(); }
00539
00541 template< template<typename, const index_t, const index_t, typename> class Multivector,
00542 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00543 inline
00544 auto
00545 reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00546 { return val.reverse(); }
00547
00549 template< template<typename, const index_t, const index_t, typename> class Multivector,
00550 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00551 inline
00552 auto
00553 conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00554 { return val.conj(); }
00555
00557 template< template<typename, const index_t, const index_t, typename> class Multivector,
00558 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00559 inline
00560 auto
00561 quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00562 { return val.quad(); }
00563
00565 template< template<typename, const index_t, const index_t, typename> class Multivector,
00566 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00567 inline
00568 auto
00569 norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00570 { return val.norm(); }
00571
00573 template< template<typename, const index_t, const index_t, typename> class Multivector,
00574 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00575 inline
00576 auto
00577 abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00578 { return numeric_traits<Scalar_T>::sqrt(val.norm()); }
00579
00581 template< template<typename, const index_t, const index_t, typename> class Multivector,
00582 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00583 inline
00584 auto
00585 max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00586 { return val.max_abs(); }
00587
00589 template< template<typename, const index_t, const index_t, typename> class Multivector,
00590 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00591 auto
00592 complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00593 {
00594 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00595 using traits_t = numeric_traits<Scalar_T>;
00596
00597 auto frm = val.frame();
00598 using array_t = std::array<index_t, 4>;
00599 auto incp = array_t{0, 2, 1, 0};
00600 auto incq = array_t{1, 0, 0, 0};
00601 auto bott = pos_mod((frm.count_pos() - frm.count_neg()), 4);
00602 for (auto
00603 k = index_t(0);
00604 k != incp[bott];
00605 k++)
00606 for (auto
00607 idx = index_t(1);
00608 idx != HI+1;
00609 ++idx)
00610 if (!frm[idx])
00611 {
00612 frm.set(idx);
00613 break;
00614 }
00615 for (auto
00616 k = index_t(0);
00617 k != incq[bott];
00618 k++)
00619 for (auto
00620 idx = index_t(-1);
00621 idx != LO-1;
00622 --idx)
00623 if (!frm[idx])
00624 {
00625 frm.set(idx);

```

```

00626 break;
00627 }
00628 auto new_bott = pos_mod(frm.count_pos() - frm.count_neg(), 4);
00629
00630 if ((incp[new_bott] == 0) && (incq[new_bott] == 0))
00631 return multivector_t(frm, Scalar_T(1));
00632 else
00633 // Return IEEE NaN or -Inf
00634 return traits_t::NaN();
00635 }
00636
00637 template< template<typename, const index_t, const index_t, typename> class Multivector,
00638 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00639 inline
00640 auto
00641 elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00642 { return complexifier(val); }
00643
00644 template< template<typename, const index_t, const index_t, typename> class Multivector,
00645 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00646 inline
00647 static
00648 void
00649 check_complex(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00650 const Multivector<Scalar_T,LO,HI,Tune_P>& i, const bool prechecked = false)
00651 {
00652 if (!prechecked)
00653 {
00654 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00655 using error_t = typename multivector_t::error_t;
00656
00657 const auto i_frame = i.frame();
00658 // We need i to be a complexifier whose frame is large enough to represent val
00659 if (complexifier(i) != i ||
00660 (val.frame() | i_frame) != i_frame ||
00661 complexifier(val).frame().count() > i_frame.count())
00662 throw error_t("check_complex(val, i): i is not a valid complexifier for val");
00663 }
00664 }
00665
00666 template< template<typename, const index_t, const index_t, typename> class Multivector,
00667 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00668 inline
00669 auto
00670 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00671 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00672 { return sqrt(val, i, prechecked); }
00673
00674 template< template<typename, const index_t, const index_t, typename> class Multivector,
00675 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00676 inline
00677 auto
00678 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00679 { return sqrt(val, complexifier(val), true); }
00680
00681 template< template<typename, const index_t, const index_t, typename> class Multivector,
00682 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00683 auto
00684 clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00685 Multivector<Scalar_T,LO,HI,Tune_P>
00686 {
00687 // Scaling and squaring Pade' approximation of matrix exponential
00688 // Reference: [GL], Section 11.3, p572-576
00689 // Reference: [H]
00690
00691 using traits_t = numeric_traits<Scalar_T>;
00692
00693 const auto scalar_val = val.scalar();
00694 const auto scalar_exp = traits_t::exp(scalar_val);
00695 if (traits_t::isNaN_or_isInf(scalar_exp))
00696 return traits_t::NaN();
00697 if (val == scalar_val)
00698 return scalar_exp;
00699
00700 using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00701 auto A = val - scalar_val;
00702 const auto pure_scale2 = A.norm();
00703
00704 if (traits_t::isNaN_or_isInf(pure_scale2))
00705 return traits_t::NaN();
00706 if (pure_scale2 == Scalar_T(0))
00707 return scalar_exp;
00708
00709 const auto ilog2_scale =
00710 std::max(0, traits_t::to_int(ceil((log2(pure_scale2) +
00711 Scalar_T(A.frame().count()))/Scalar_T(2)))) - 3);
00712 const auto i_scale = traits_t::pow(Scalar_T(2), ilog2_scale);

```

```

00716 if (traits_t::isNaN_or_isInf(i_scale))
00717 return traits_t::NaN();
00718
00719 A /= i_scale;
00720 multivector_t pure_exp;
00721 {
00722 using limits_t = std::numeric_limits<Scalar_T>;
00723 const auto nbr_even_powers = 2*(limits_t::digits / 32) + 4;
00724 using nbr_t = decltype(nbr_even_powers);
00725
00726 // Create an array of coefficients
00727 const auto max_power = 2*nbr_even_powers + 1;
00728 static std::array<Scalar_T, max_power+1> c;
00729 if (c[0] != Scalar_T(1))
00730 {
00731 c[0] = Scalar_T(1);
00732 for (auto
00733 k = decltype(max_power)(0);
00734 k != max_power;
00735 ++k)
00736 c[k+1] = c[k]*(max_power-k) / ((2*max_power-k)*(k+1));
00737 }
00738
00739 // Create an array of even powers
00740 std::array<multivector_t, nbr_even_powers> AA;
00741 AA[0] = A * A;
00742 AA[1] = AA[0] * AA[0];
00743 for (auto
00744 k = nbr_t(2);
00745 k != nbr_even_powers;
00746 ++k)
00747 AA[k] = AA[k-2] * AA[1];
00748
00749 // Use compensated summation to calculate U and AV
00750 auto residual = multivector_t();
00751 auto U = multivector_t(c[0]);
00752 for (auto
00753 k = nbr_t(0);
00754 k != nbr_even_powers;
00755 ++k)
00756 {
00757 const auto& term = AA[k]*c[2*k + 2] - residual;
00758 const auto& sum = U + term;
00759 residual = (sum - U) - term;
00760 U = sum;
00761 }
00762 residual = multivector_t();
00763 auto AV = multivector_t(c[1]);
00764 for (auto
00765 k = nbr_t(0);
00766 k != nbr_even_powers;
00767 ++k)
00768 {
00769 const auto& term = AA[k]*c[2*k + 3] - residual;
00770 const auto& sum = AV + term;
00771 residual = (sum - AV) - term;
00772 AV = sum;
00773 }
00774 AV *= A;
00775 pure_exp = (U+AV) / (U-AV);
00776 }
00777 for (auto
00778 k = decltype(ilog2_scale)(0);
00779 k != ilog2_scale;
00780 ++k)
00781 pure_exp *= pure_exp;
00782 return pure_exp * scalar_exp;
00783 }
00784
00786 template< template<typename, const index_t, const index_t, typename> class Multivector,
00787 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788 inline
00789 auto
00790 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00791 { return log(val, i, prechecked); }
00792
00794 template< template<typename, const index_t, const index_t, typename> class Multivector,
00795 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 inline
00797 auto
00798 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00799 { return log(val, complexifier(val), true); }
00800
00802 template< template<typename, const index_t, const index_t, typename> class Multivector,
00803 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00804 inline

```

```

00805 auto
00806 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00807 {
00808 using traits_t = numeric_traits<Scalar_T>;
00809 if (val.isnan())
00810 return traits_t::NaN();
00811
00812 const auto& s = val.scalar();
00813 if (val == s)
00814 return traits_t::cosh(s);
00815 return (exp(val)+exp(-val)) / Scalar_T(2);
00816 }
00817
00818 // Reference: [AS], Section 4.6, p86-89
00819 template< template<typename, const index_t, const index_t, typename> class Multivector,
00820 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00821 inline
00822 auto
00823 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00824 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00825 {
00826 using traits_t = numeric_traits<Scalar_T>;
00827 check_complex(val, i, prechecked);
00828 if (val.isnan())
00829 return traits_t::NaN();
00830
00831 const auto radical = sqrt(val*val - Scalar_T(1), i, true);
00832 return (norm(val + radical) >= norm(val))
00833 ? log(val + radical, i, true)
00834 : -log(val - radical, i, true);
00835 }
00836
00837 // Reference: [AS], Section 4.6, p86-89
00838 template< template<typename, const index_t, const index_t, typename> class Multivector,
00839 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840 inline
00841 auto
00842 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00843 { return acosh(val, complexifier(val), true); }
00844
00845 template< template<typename, const index_t, const index_t, typename> class Multivector,
00846 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00847 auto
00848 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00849 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00850 {
00851 using traits_t = numeric_traits<Scalar_T>;
00852 if (val.isnan())
00853 return traits_t::NaN();
00854
00855 const auto& s = val.scalar();
00856 if (val == s)
00857 return traits_t::cos(s);
00858
00859 check_complex(val, i, prechecked);
00860
00861 static const auto& twopi = Scalar_T(2) * traits_t::pi();
00862 const auto& z = i *
00863 (val - s + traits_t::fmod(s, twopi));
00864 return (exp(z)+exp(-z)) / Scalar_T(2);
00865 }
00866
00867 template< template<typename, const index_t, const index_t, typename> class Multivector,
00868 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00869 inline
00870 auto
00871 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00872 { return cos(val, complexifier(val), true); }
00873
00874 // Reference: [AS], Section 4.4, p79-83
00875 template< template<typename, const index_t, const index_t, typename> class Multivector,
00876 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00877 inline
00878 auto
00879 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00880 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00881 {
00882 using traits_t = numeric_traits<Scalar_T>;
00883 if (val.isnan())
00884 return traits_t::NaN();
00885
00886 const auto& s = val.scalar();
00887 if (val == s && traits_t::abs(s) <= Scalar_T(1))
00888 return traits_t::acos(s);
00889
00890 check_complex(val, i, prechecked);
00891 return i * acosh(val, i, true);
00892 }

```

```

00894 }
00895
00896 // Reference: [AS], Section 4.4, p79-83
00897 template< template<typename, const index_t, const index_t, typename> class Multivector,
00898 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00899 inline
00900 auto
00901 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00902 { return acos(val, complexifier(val), true); }
00903
00904 template< template<typename, const index_t, const index_t, typename> class Multivector,
00905 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00906 inline
00907 auto
00908 sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00909 {
00910 {
00911 using traits_t = numeric_traits<Scalar_T>;
00912 if (val.isnan())
00913 return traits_t::NaN();
00914
00915 const auto& s = val.scalar();
00916 if (val == s)
00917 return traits_t::sinh(s);
00918
00919 return (exp(val)-exp(-val)) / Scalar_T(2);
00920 }
00921 }
00922
00923 // Reference: [AS], Section 4.6, p86-89
00924 template< template<typename, const index_t, const index_t, typename> class Multivector,
00925 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00926 inline
00927 auto
00928 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00929 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00930 {
00931 {
00932 using traits_t = numeric_traits<Scalar_T>;
00933 check_complex(val, i, prechecked);
00934 if (val.isnan())
00935 return traits_t::NaN();
00936
00937 const auto radical = sqrt(val*val + Scalar_T(1), i, true);
00938 return (norm(val + radical) >= norm(val))
00939 ? log(val + radical, i, true)
00940 : -log(-val + radical, i, true);
00941 }
00942 }
00943
00944 // Reference: [AS], Section 4.6, p86-89
00945 template< template<typename, const index_t, const index_t, typename> class Multivector,
00946 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00947 inline
00948 auto
00949 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00950 { return asinh(val, complexifier(val), true); }
00951
00952 template< template<typename, const index_t, const index_t, typename> class Multivector,
00953 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954 inline
00955 auto
00956 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00957 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00958 {
00959 {
00960 using traits_t = numeric_traits<Scalar_T>;
00961 if (val.isnan())
00962 return traits_t::NaN();
00963
00964 const auto& s = val.scalar();
00965 if (val == s)
00966 return traits_t::sin(s);
00967
00968 check_complex(val, i, prechecked);
00969
00970 static const auto& twopi = Scalar_T(2) * traits_t::pi();
00971 const auto& z = i *
00972 (val - s + traits_t::fmod(s, twopi));
00973 return i * (exp(-z)-exp(z)) / Scalar_T(2);
00974 }
00975 }
00976
00977 template< template<typename, const index_t, const index_t, typename> class Multivector,
00978 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00979 inline
00980 auto
00981 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00982 { return sin(val, complexifier(val), true); }
00983
00984 // Reference: [AS], Section 4.4, p79-83
00985 template< template<typename, const index_t, const index_t, typename> class Multivector,
00986 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00987 inline

```



```

00986 auto
00987 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00988 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00989 {
00989 using traits_t = numeric_traits<Scalar_T>;
00990 if (val.isnan())
00991 return traits_t::NaN();
00992
00993 const auto& s = val.scalar();
00994 if (val == s && traits_t::abs(s) <= Scalar_T(1))
00995 return traits_t::asin(s);
00996
00997 check_complex(val, i, prechecked);
00998 return -i * asinh(i * val, i, true);
00999 }
01000
01002 // Reference: [AS], Section 4.4, p79-83
01003 template< template<typename, const index_t, const index_t, typename> class Multivector,
01004 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01005 inline
01006 auto
01007 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01008 { return asin(val, complexifier(val), true); }
01009
01011 template< template<typename, const index_t, const index_t, typename> class Multivector,
01012 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01013 inline
01014 auto
01015 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01016 {
01017 using traits_t = numeric_traits<Scalar_T>;
01018 if (val.isnan())
01019 return traits_t::NaN();
01020
01021 const auto& s = val.scalar();
01022 if (val == s)
01023 return traits_t::tanh(s);
01024
01025 return sinh(val) / cosh(val);
01026 }
01027
01029 // Reference: [AS], Section 4.6, p86-89
01030 template< template<typename, const index_t, const index_t, typename> class Multivector,
01031 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01032 inline
01033 auto
01034 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01035 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01036 {
01037 using traits_t = numeric_traits<Scalar_T>;
01038 check_complex(val, i, prechecked);
01039 return val.isnan()
01040 ? traits_t::NaN()
01041 : (norm(val + Scalar_T(1)) > norm(val - Scalar_T(1)))
01042 ? (log(val + Scalar_T(1), i, true) - log(-val + Scalar_T(1), i, true)) / Scalar_T(2)
01043 : log((val + Scalar_T(1)) / (-val + Scalar_T(1)), i, true) / Scalar_T(2);
01044 }
01046 // Reference: [AS], Section 4.6, p86-89
01047 template< template<typename, const index_t, const index_t, typename> class Multivector,
01048 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01049 inline
01050 auto
01051 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01052 { return atanh(val, complexifier(val), true); }
01053
01055 template< template<typename, const index_t, const index_t, typename> class Multivector,
01056 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01057 inline
01058 auto
01059 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
01060 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01061 {
01062 using traits_t = numeric_traits<Scalar_T>;
01063 if (val.isnan())
01064 return traits_t::NaN();
01065
01066 const auto& s = val.scalar();
01067 if (val == s)
01068 return traits_t::tan(s);
01069
01070 check_complex(val, i, prechecked);
01071 return sin(val, i, true) / cos(val, i, true);
01072 }
01073
01074 template< template<typename, const index_t, const index_t, typename> class Multivector,
01075 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```

```

01076 inline
01077 auto
01078 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01079 { return tan(val, complexifier(val), true); }
01080
01082 // Reference: [AS], Section 4.4, p79-83
01083 template< template<typename, const index_t, const index_t, typename> class Multivector,
01084 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01085 inline
01086 auto
01087 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01088 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01089 {
01089 using traits_t = numeric_traits<Scalar_T>;
01090 if (val.isnan())
01091 return traits_t::NaN();
01092
01093 const auto& s = val.scalar();
01094 if (val == s)
01095 return traits_t::atan(s);
01096
01097 check_complex(val, i, prechecked);
01098 return -i * atanh(i * val, i, true);
01099 }
01100
01102 // Reference: [AS], Section 4.4, p79-83
01103 template< template<typename, const index_t, const index_t, typename> class Multivector,
01104 typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01105 inline
01106 auto
01107 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01108 { return atan(val, complexifier(val), true); }
01109
01110 }
01111 #endif // _GLUCAT_CLIFFORD_ALGEBRA_IMP_H

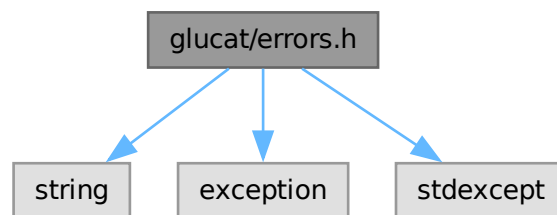
```

## 9.5 glucat/errors.h File Reference

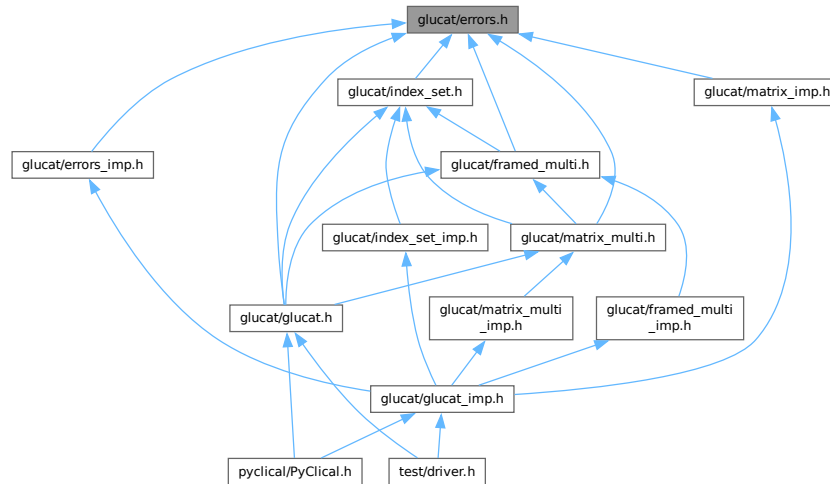
```

#include <string>
#include <exception>
#include <stdexcept>
Include dependency graph for errors.h:

```



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::glucat_error`  
*Abstract exception class.*
- class `glucat::error< Class_T >`  
*Specific exception class.*

## Namespaces

- namespace `glucat`

## 9.6 errors.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_H
00002 #define _GLUCAT_ERRORS_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 errors.h : Declare error classes and functions
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/

```

```

00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include <string>
00035 #include <exception>
00036 #include <stdexcept>
00037
00038 namespace glucat
00039 {
00041 class glucat_error : public std::logic_error
00042 {
00043 public:
00044 glucat_error(const std::string& context, const std::string& msg)
00045 : logic_error(msg), name(context)
00046 { }
00047 ~glucat_error() noexcept override = default;
00048 virtual auto heading() const noexcept -> const std::string =0;
00049 virtual auto classname() const noexcept -> const std::string =0;
00050 virtual void print_error_msg() const =0;
00051 std::string name;
00052 };
00053
00055 template< class Class_T >
00056 class error : public glucat_error
00057 {
00058 public:
00059 error(const std::string& msg);
00060 error(const std::string& context, const std::string& msg);
00061 auto heading() const noexcept -> const std::string override;
00062 auto classname() const noexcept -> const std::string override;
00063 void print_error_msg() const override;
00064 };
00065 }
00066 #endif // _GLUCAT_ERRORS_H

```

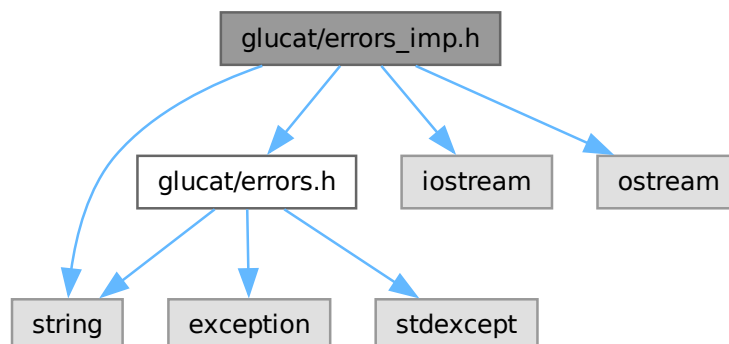
## 9.7 glucat/errors\_imp.h File Reference

```

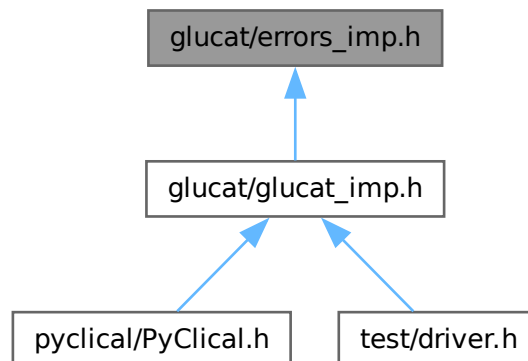
#include "glucat/errors.h"
#include <string>
#include <iostream>
#include <ostream>

```

Include dependency graph for errors\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## 9.8 errors\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_IMP_H
00002 #define _GLUCAT_ERRORS_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 errors_imp.h : Define error functions
00006 -----
00007 begin : Sun 2001-12-20
00008 copyright : (C) 2001-2007 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/errors.h"
00035
00036 #include <string>
00037 #include <iostream>
00038 #include <ostream>

```

```

00039
00040 namespace glucat
00041 {
00042 template< class Class_T >
00043 error<Class_T>::
00044 error(const std::string& msg)
00045 : glucat_error(Class_T::classname(), msg)
00046 { }
00047
00048 template< class Class_T >
00049 error<Class_T>::
00050 error(const std::string& context, const std::string& msg)
00051 : glucat_error(context, msg)
00052 { }
00053
00054 template< class Class_T >
00055 auto
00056 error<Class_T>::
00057 heading() const noexcept -> const std::string
00058 { return "Error in glucat::"; }
00059
00060 template< class Class_T >
00061 auto
00062 error<Class_T>::
00063 classname() const noexcept -> const std::string
00064 { return name; }
00065
00066 template< class Class_T >
00067 void
00068 error<Class_T>::
00069 print_error_msg() const
00070 { std::cerr << heading() << classname() << std::endl << what() << std::endl; }
00071 }
00072 #endif // _GLUCAT_ERRORS_IMP_H
00073

```

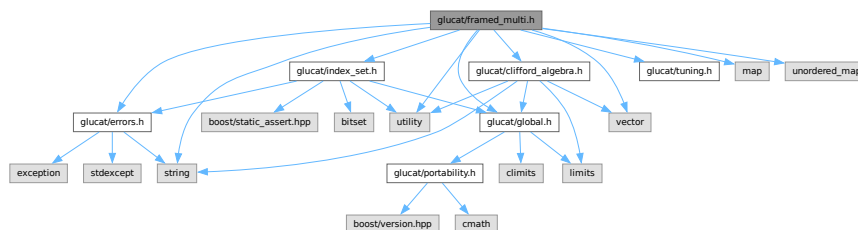
## 9.9 glucat/framed\_multi.h File Reference

```

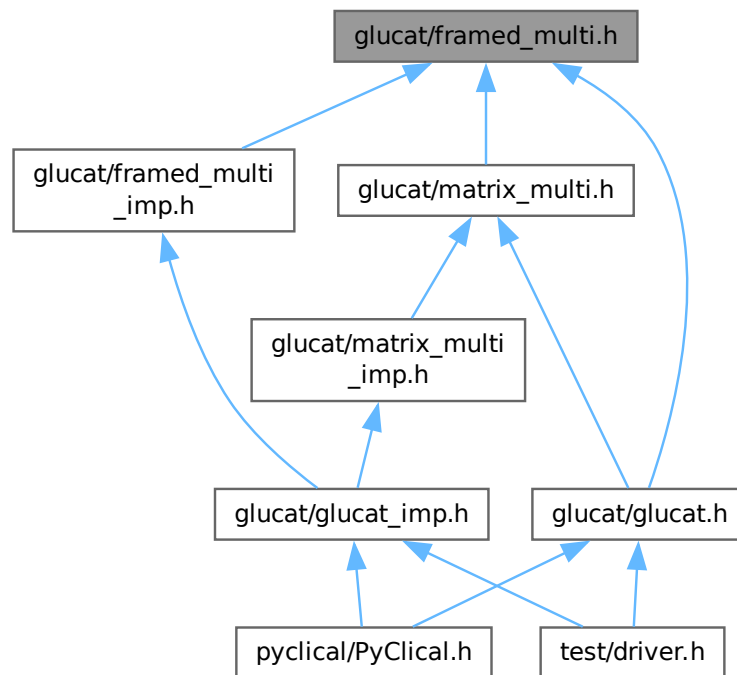
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include <string>
#include <utility>
#include <map>
#include <unordered_map>
#include <vector>

```

Include dependency graph for framed\_multi.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::index_set_hash< LO, HI >`
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`  
*A `framed_multi<Scalar_T,LO,HI,Tune_P>` is a framed approximation to a multivector.*
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t`
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term`  
*Variable term.*
- struct `std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >`  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*

## Namespaces

- namespace `glucat`
- namespace `std`

## Functions

- template<typename `Scalar_T`, const `index_t` `LO`, const `index_t` `HI`, typename `Tune_P`>  
 auto `glucat::operator*` (const `framed_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `framed_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi<`  
`Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream`  
`&`  
*Read multivector from input.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) ->`  
`std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term)`  
`-> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T,`  
`LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`static auto glucat::crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::↵`  
`pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*



## 9.10 framed\_multi.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_FRAMED_MULTI_H
00002 #define _GLUCAT_FRAMED_MULTI_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 framed_multi.h : Declare a class for the framed representation of a multivector
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039
00040 #if defined(_GLUCAT_USE_BOOST_POOL_ALLOC)
00041 // Use the Boost pool allocator
00042 #include <boost/pool/pool_fwd.hpp>
00043 #endif
00044
00045 #include <string>
00046 #include <utility>
00047 #include <map>
00048 #include <unordered_map>
00049 #include <vector>
00050
00051 namespace glucat
00052 {
00053 // Forward declarations for friends
00054
00055 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00056 class framed_multi; // forward
00057
00058 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059 class matrix_multi; // forward
00060
00061 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00062 auto
00063 operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00064 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00065
00066 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00067 auto
00068 operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00069 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00070
00071 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00072 auto
00073 operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00074 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00075
00076 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00077 auto
00078 operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00079 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00080
00081 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00082 auto

```

```

00084 star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00085
00086 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00087 auto
00088 operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00089 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00090
00091 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092 auto
00093 operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00094 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00095
00096 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097 auto
00098 operator» (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00100 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00101 auto
00102 operator« (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00103
00104 template< typename Scalar_T, const index_t LO, const index_t HI >
00105 auto
00106 operator« (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
00107 std::ostream&;
00108
00109 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00110 auto
00111 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00112
00113 template< const index_t LO, const index_t HI>
00114 class index_set_hash
00115 {
00116 public:
00117 using index_set_t = index_set<LO, HI>;
00118 inline auto operator()(index_set_t val) const -> size_t { return val.hash_fn(); }
00119 };
00120
00121 template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
00122 typename Tune_P = tuning<> >
00123 class framed_multi {
00124 public: clifford_algebra< Scalar_T, index_set<LO,HI>, framed_multi<Scalar_T,LO,HI,Tune_P> >,
00125 private: std::unordered_map< index_set<LO,HI>, Scalar_T, index_set_hash<LO,HI> >
00126 {
00127 public:
00128 using multivector_t = framed_multi;
00129 using framed_multi_t = multivector_t;
00130 using scalar_t = Scalar_T;
00131 using tune_p = Tune_P;
00132 using index_set_t = index_set<LO, HI>;
00133 using term_t = std::pair<const index_set_t, Scalar_T>;
00134 using vector_t = std::vector<Scalar_T>;
00135 using error_t = error<multivector_t>;
00136 using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >;
00137 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00138 Other_Tune_P >
00139 friend class matrix_multi;
00140 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00141 Other_Tune_P >
00142 friend class framed_multi;
00143 private:
00144 class var_term; // forward
00145 using var_term_t = class var_term;
00146 using matrix_t = typename matrix_multi_t::matrix_t;
00147 using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >;
00148 using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>;
00149
00150 class hash_size_t
00151 {
00152 public:
00153 hash_size_t(size_t hash_size)
00154 : n(hash_size)
00155 { };
00156 auto operator()() const -> size_t
00157 { return n; }
00158 private:
00159 size_t n;
00160 };
00161
00162 using framed_pair_t = std::pair<const multivector_t, const multivector_t>;
00163 using size_type = typename map_t::size_type;
00164 using iterator = typename map_t::iterator;
00165 using const_iterator = typename map_t::const_iterator;
00166 public:
00167 static auto classname() -> const std::string;

```

```

00173 ~framed_multi() override = default;
00175 framed_multi();
00176
00177 private:
00179 framed_multi(const hash_size_t& hash_size);
00180 public:
00182 template< typename Other_Scalar_T >
00183 framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00185 template< typename Other_Scalar_T >
00186 framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00187 const index_set_t frm, const bool prechecked = false);
00189 framed_multi(const framed_multi_t& val,
00190 const index_set_t frm, const bool prechecked = false);
00192 framed_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00194 framed_multi(const index_set_t ist, const Scalar_T& crd,
00195 const index_set_t frm, const bool prechecked = false);
00197 framed_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00199 framed_multi(const int scr, const index_set_t frm = index_set_t());
00201 framed_multi(const vector_t& vec,
00202 const index_set_t frm, const bool prechecked = false);
00204 framed_multi(const std::string& str);
00206 framed_multi(const std::string& str,
00207 const index_set_t frm, const bool prechecked = false);
00209 framed_multi(const char* str)
00210 { *this = framed_multi(std::string(str)); };
00212 framed_multi(const char* str,
00213 const index_set_t frm, const bool prechecked = false)
00214 { *this = framed_multi(std::string(str), frm, prechecked); };
00216 template< typename Other_Scalar_T >
00217 framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P >& val);
00219 template< typename Other_Scalar_T >
00220 auto fast_matrix_multi(const index_set_t frm) const -> const
matrix_multi<Other_Scalar_T,LO,HI,Tune_P >;
00222 auto fast_framed_multi() const -> const framed_multi_t;
00223
00224 _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00225
00227 auto nbr_terms() const -> unsigned long;
00228
00230 static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const multivector_t;
00231
00232 // Friend declarations
00233
00234 friend auto
00235 operator* <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00236 friend auto
00237 operator^ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00238 friend auto
00239 operator& <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00240 friend auto
00241 operator% <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00242 friend auto
00243 star <>(const multivector_t& lhs, const multivector_t& rhs) -> Scalar_T;
00244 friend auto
00245 operator/ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00246 friend auto
00247 operator| <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00248
00249 friend auto
00250 operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00251 friend auto
00252 operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00253 friend auto
00254 operator« <>(std::ostream& os, const term_t& term) -> std::ostream&;
00255
00256 friend auto
00257 exp <>(const multivector_t& val) -> const multivector_t;
00258
00260 auto operator+= (const term_t& term) -> multivector_t;
00261
00262 private:
00264 auto fold(const index_set_t frm) const -> multivector_t;
00266 auto unfold(const index_set_t frm) const -> multivector_t;
00268 auto centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&;
00270 auto centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&;
00272 auto centre_qp1_pml(index_t& p, index_t& q) -> multivector_t&;
00274 auto divide(const index_set_t ist) const -> const framed_pair_t;
00276 auto fast(const index_t level, const bool odd) const -> const matrix_t;
00277
00279 class var_term :
00280 public std::pair<index_set<LO,HI>, Scalar_T>
00281 {
00282 public:
00283 using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>;
00284
00286 static auto classname() -> const std::string
00287 { return "var_term"; };

```

```

00289 ~var_term() = default;
00291 var_term()
00292 : var_pair_t(index_set_t(), Scalar_T(1))
00293 { };
00295 var_term(const index_set_t ist, const Scalar_T& crd = Scalar_T(1))
00296 : var_pair_t(ist, crd)
00297 { };
00299 auto operator*=(const term_t& rhs) -> var_term_t&
00300 {
00301 this->second *= rhs.second * this->first.sign_of_mult(rhs.first);
00302 this->first ^= rhs.first;
00303 return *this;
00304 }
00305 };
00306 };
00307
00308 // Non-members
00309
00311 template< typename Scalar_T, const index_t LO, const index_t HI >
00312 inline
00313 static
00314 auto
00315 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00316 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T;
00317
00319 template< typename Scalar_T, const index_t LO, const index_t HI >
00320 auto
00321 operator*
00322 (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00323 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const index_set<LO,HI>,
00324 Scalar_T>;
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 auto
00328 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00329 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00331 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00332 auto
00333 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00334
00336 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00337 auto
00338 log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00339 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00339 }
00340
00341 namespace std
00342 {
00344 template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00345 struct numeric_limits< glucat::framed_multi<Scalar_T,LO,HI,Tune_P> > :
00346 public numeric_limits<Scalar_T>
00347 { };
00348 }
00349 #endif // _GLUCAT_FRAMED_MULTI_H

```

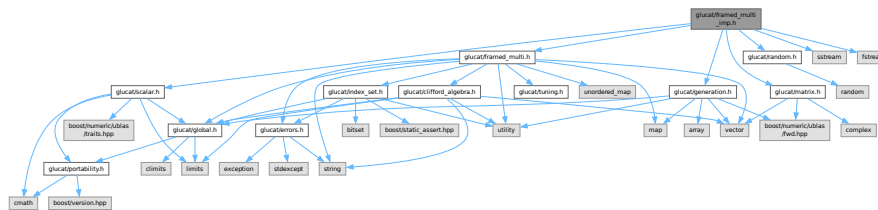
## 9.11 glucat/framed\_multi\_imp.h File Reference

```

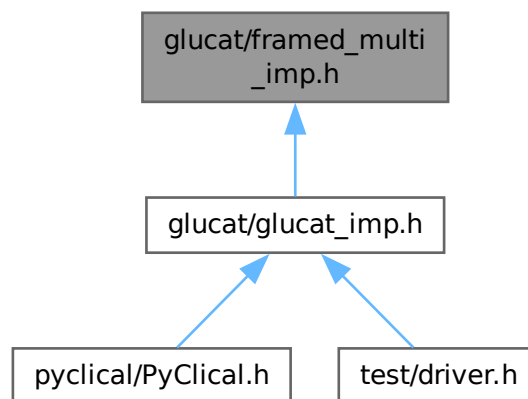
#include "glucat/framed_multi.h"
#include "glucat/scalar.h"
#include "glucat/random.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <sstream>
#include <fstream>

```

Include dependency graph for framed\_multi\_imp.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >](#)  
*Sorted range for use with output.*
- class [glucat::sorted\\_range< Sorted\\_Map\\_T, Sorted\\_Map\\_T >](#)

## Namespaces

- namespace [glucat](#)

## Macros

- [#define \\_GLUCAT\\_HASH\\_N\(x\)](#)
- [#define \\_GLUCAT\\_HASH\\_SIZE\\_T\(x\)](#)

## Functions

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) ->`  
`std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term)`  
`-> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream`  
`&`  
*Read multivector from input.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`static auto glucat::ord_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::↵`  
`pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>`  
`auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T,`  
`LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*

## 9.11.1 Macro Definition Documentation

### 9.11.1.1 [\\_GLUCAT\\_HASH\\_N](#)

```
#define _GLUCAT_HASH_N(
 x)
```

#### Value:

(x)

Definition at line 54 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)(), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()), and [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)([glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)()).

### 9.11.1.2 [\\_GLUCAT\\_HASH\\_SIZE\\_T](#)

```
#define _GLUCAT_HASH_SIZE_T(
 x)
```

#### Value:

([typename](#) multivector\_t::hash\_size\_t) (x)

Definition at line 55 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >::[framed\\_multi](#)(), [glucat::operator%](#)(), [glucat::operator&](#)(), [glucat::operator\\*](#)(), and [glucat::operator^](#)().

## 9.12 framed\_multi\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_FRAMED_MULTI_IMP_H
00002 #define _GLUCAT_FRAMED_MULTI_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 framed_multi_imp.h : Implement the coordinate map representation of a
00006 Clifford algebra element
00007 -----
00008 begin : Sun 2001-12-09
00009 copyright : (C) 2001-2021 by Paul C. Leopardi
00010 *****/
00011
00012 This library is free software: you can redistribute it and/or modify
00013 it under the terms of the GNU Lesser General Public License as published
00014 by the Free Software Foundation, either version 3 of the License, or
00015 (at your option) any later version.
00016
00017 This library is distributed in the hope that it will be useful,
00018 but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 GNU Lesser General Public License for more details.
00021
00022 You should have received a copy of the GNU Lesser General Public License
00023 along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025 *****/
00026 This library is based on a prototype written by Arvind Raja and was
00027 licensed under the LGPL with permission of the author. See Arvind Raja,
00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****/
00032 See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include "glucat/framed_multi.h"
00036
00037 #include "glucat/scalar.h"
00038 #include "glucat/random.h"
00039 #include "glucat/generation.h"
00040 #include "glucat/matrix.h"
00041
00042 #include <sstream>
00043 #include <fstream>
00044
00045 namespace glucat
00046 {
00047 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00048 auto
00049 framed_multi<Scalar_T,LO,HI,Tune_P>::
00050 classname() -> const std::string
00051 { return "framed_multi"; }
00052
00053 #define _GLUCAT_HASH_N(x) (x)
00054 #define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)
00055
00056 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00057 framed_multi<Scalar_T,LO,HI,Tune_P>::
00058 framed_multi()
00059 : map_t(_GLUCAT_HASH_N(0))
00060 { }
00061
00062 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063 framed_multi<Scalar_T,LO,HI,Tune_P>::
00064 framed_multi(const hash_size_t& hash_size)
00065 : map_t(_GLUCAT_HASH_N(hash_size()))
00066 { }
00067
00068 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00069 template< typename Other_Scalar_T >
00070 framed_multi<Scalar_T,LO,HI,Tune_P>::
00071 framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00072 : map_t(_GLUCAT_HASH_N(val.size()))
00073 {
00074 for (auto& val_term : val)
00075 this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00076 }
00077
00078 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00079 template< typename Other_Scalar_T >
00080 framed_multi<Scalar_T,LO,HI,Tune_P>::
00081 framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00082 const index_set_t frm, const bool prechecked)

```



```

00088 : map_t(_GLUCAT_HASH_N(val.size()))
00089 {
00090 if (!prechecked && (val.frame() | frm) != frm)
00091 throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00092 for (auto& val_term : val)
00093 this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00094 }
00095
00097 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098 framed_multi<Scalar_T,LO,HI,Tune_P>::
00099 framed_multi(const multivector_t& val,
00100 const index_set_t frm, const bool prechecked)
00101 : map_t(_GLUCAT_HASH_N(val.size()))
00102 {
00103 if (!prechecked && (val.frame() | frm) != frm)
00104 throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00105 for (auto& val_term : val)
00106 this->insert(val_term);
00107 }
00108
00110 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111 framed_multi<Scalar_T,LO,HI,Tune_P>::
00112 framed_multi(const index_set_t ist, const Scalar_T& crd)
00113 : map_t(_GLUCAT_HASH_N(1))
00114 {
00115 if (crd != Scalar_T(0))
00116 this->insert(term_t(ist, crd));
00117 }
00118
00120 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00121 framed_multi<Scalar_T,LO,HI,Tune_P>::
00122 framed_multi(const index_set_t ist, const Scalar_T& crd,
00123 const index_set_t frm, const bool prechecked)
00124 : map_t(_GLUCAT_HASH_N(1))
00125 {
00126 if (!prechecked && (ist | frm) != frm)
00127 throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00128 if (crd != Scalar_T(0))
00129 this->insert(term_t(ist, crd));
00130 }
00131
00133 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00134 framed_multi<Scalar_T,LO,HI,Tune_P>::
00135 framed_multi(const Scalar_T& scr, const index_set_t frm)
00136 : map_t(_GLUCAT_HASH_N(1))
00137 {
00138 if (scr != Scalar_T(0))
00139 this->insert(term_t(index_set_t(), scr));
00140 }
00141
00143 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00144 framed_multi<Scalar_T,LO,HI,Tune_P>::
00145 framed_multi(const int scr, const index_set_t frm)
00146 : map_t(_GLUCAT_HASH_N(1))
00147 {
00148 if (scr != Scalar_T(0))
00149 this->insert(term_t(index_set_t(), Scalar_T(scr)));
00150 }
00151
00153 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00154 framed_multi<Scalar_T,LO,HI,Tune_P>::
00155 framed_multi(const vector_t& vec,
00156 const index_set_t frm, const bool prechecked)
00157 : map_t(_GLUCAT_HASH_N(vec.size()))
00158 {
00159 if (!prechecked && index_t(vec.size()) != frm.count())
00160 throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00161 auto idx = frm.min();
00162 const auto frm_end = frm.max()+1;
00163 for (auto& crd : vec)
00164 {
00165 *this += term_t(index_set_t(idx), crd);
00166 for (
00167 ++idx;
00168 idx != frm_end && !frm[idx];
00169 ++idx)
00170 ;
00171 }
00172 }
00173
00175 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00176 framed_multi<Scalar_T,LO,HI,Tune_P>::
00177 framed_multi(const std::string& str)
00178 : map_t(_GLUCAT_HASH_N(0))
00179 {
00180 std::istringstream ss(str);
00181 ss >> *this;

```

```

00182 if (!ss)
00183 throw error_t("multivector_t(str): could not parse string");
00184 // Peek to see if the end of the string has been reached.
00185 ss.peek();
00186 if (!ss.eof())
00187 throw error_t("multivector_t(str): could not parse entire string");
00188 }
00189
00191 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00192 framed_multi<Scalar_T,LO,HI,Tune_P>::
00193 framed_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00194 : map_t(_GLUCAT_HASH_N(0))
00195 {
00196 if (prechecked)
00197 *this = multivector_t(str);
00198 else
00199 *this = multivector_t(multivector_t(str), frm, false);
00200 }
00201
00203 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00204 template< typename Other_Scalar_T >
00205 framed_multi<Scalar_T,LO,HI,Tune_P>::
00206 framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00207 : map_t(_GLUCAT_HASH_N(1))
00208 {
00209 if (val == Other_Scalar_T(0))
00210 return;
00211
00212 const auto dim = val.m_matrix.size();
00213 using traits_t = numeric_traits<Scalar_T>;
00214 if (dim == 1)
00215 {
00216 this->insert(term_t(index_set_t(), traits_t::to_scalar_t(val.m_matrix(0, 0))));
00217 return;
00218 }
00219 if (dim >= Tune_P::inv_fast_dim_threshold)
00220 try
00221 {
00222 *this = (val.template fast_framed_multi<Scalar_T>()).truncated();
00223 return;
00224 }
00225 catch (const glucat_error& e)
00226 { }
00227
00228 const auto val_norm = traits_t::to_scalar_t(val.norm());
00229 if (traits_t::isNaN_or_isInf(val_norm))
00230 {
00231 *this = traits_t::NaN();
00232 return;
00233 }
00234 const auto frm = val.frame();
00235 const auto algebra_dim = set_value_t(1) << frm.count();
00236 auto result = multivector_t(
00237 _GLUCAT_HASH_SIZE_T(std::min<size_t>(algebra_dim, matrix::nnz(val.m_matrix))));
00238 for (auto
00239 stv = set_value_t(0);
00240 stv != algebra_dim;
00241 stv++)
00242 {
00243 const auto ist = index_set_t(stv, frm, true);
00244 const auto crd =
00245 traits_t::to_scalar_t(matrix::inner<Other_Scalar_T>(val.basis_element(ist), val.m_matrix));
00246 if (crd != Scalar_T(0))
00247 result.insert(term_t(ist, crd));
00248 }
00249 *this = result.truncated();
00250 }
00251
00253 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00254 auto
00255 framed_multi<Scalar_T,LO,HI,Tune_P>::
00256 operator==(const multivector_t& rhs) const -> bool
00257 {
00258 if (this->size() != rhs.size())
00259 return false;
00260 const auto rhs_end = rhs.end();
00261 for (auto& this_term : *this)
00262 {
00263 const const_iterator& rhs_it = rhs.find(this_term.first);
00264 if (rhs_it == rhs_end || rhs_it->second != this_term.second)
00265 return false;
00266 }
00267 return true;
00268 }
00269
00271 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00272 inline

```

```

00273 auto
00274 framed_multi<Scalar_T,LO,HI,Tune_P>::
00275 operator== (const Scalar_T& scr) const -> bool
00276 {
00277 switch (this->size())
00278 {
00279 case 0:
00280 return scr == Scalar_T(0);
00281 case 1:
00282 {
00283 const auto& this_it = this->begin();
00284 return this_it->first == index_set_t() && this_it->second == scr;
00285 }
00286 default:
00287 return false;
00288 }
00289 }
00290
00291 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00292 inline
00293 auto
00294 framed_multi<Scalar_T,LO,HI,Tune_P>::
00295 operator+= (const Scalar_T& scr) -> multivector_t&
00296 {
00297 *this += term_t(index_set_t(), scr);
00298 return *this;
00299 }
00300
00301 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00302 inline
00303 auto
00304 framed_multi<Scalar_T,LO,HI,Tune_P>::
00305 operator+= (const multivector_t& rhs) -> multivector_t&
00306 { // simply add terms
00307 for (auto& rhs_term : rhs)
00308 *this += rhs_term;
00309 return *this;
00310 }
00311
00312 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00313 inline
00314 auto
00315 framed_multi<Scalar_T,LO,HI,Tune_P>::
00316 operator-= (const Scalar_T& scr) -> multivector_t&
00317 {
00318 *this += term_t(index_set_t(), -scr);
00319 return *this;
00320 }
00321
00322 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00323 inline
00324 auto
00325 framed_multi<Scalar_T,LO,HI,Tune_P>::
00326 operator-= (const multivector_t& rhs) -> multivector_t&
00327 {
00328 for (auto& rhs_term : rhs)
00329 *this += term_t(rhs_term.first, -(rhs_term.second));
00330 return *this;
00331 }
00332
00333 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00334 inline
00335 auto
00336 framed_multi<Scalar_T,LO,HI,Tune_P>::
00337 operator- () const -> const multivector_t
00338 { // multiply coordinates of all terms by -1
00339 auto result = *this;
00340 for (auto& result_term : result)
00341 result_term.second *= Scalar_T(-1);
00342 return result;
00343 }
00344
00345 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00346 auto
00347 framed_multi<Scalar_T,LO,HI,Tune_P>::
00348 operator*= (const Scalar_T& scr) -> multivector_t&
00349 { // multiply coordinates of all terms by scalar
00350 using traits_t = numeric_traits<Scalar_T>;
00351
00352 if (traits_t::isNaN_or_isInf(scr))
00353 return *this = traits_t::NaN();
00354 if (scr == Scalar_T(0))
00355 if (this->isnan())
00356 *this = traits_t::NaN();
00357 else
00358 this->clear();
00359 else
00360 *this *= scr;
00361 }

```

```

00366 for (auto& this_term : *this)
00367 this_term.second *= scr;
00368 return *this;
00369 }
00370
00371 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00372 auto
00373 operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00374 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00375 {
00376 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00377 using traits_t = numeric_traits<Scalar_T>;
00378
00379 if (lhs.isnan() || rhs.isnan())
00380 return traits_t::NaN();
00381
00382 const double lhs_size = lhs.size();
00383 const double rhs_size = rhs.size();
00384 const auto our_frame = lhs.frame() | rhs.frame();
00385 const auto frm_count = our_frame.count();
00386 const auto algebra_dim = set_value_t(1) << frm_count;
00387 const auto direct_mult = lhs_size * rhs_size <= double(algebra_dim);
00388 if (direct_mult)
00389 { // If we have a sparse multiply, store the result directly
00390 auto result = multivector_t(
00391 _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00392 for (auto& lhs_term : lhs)
00393 for (auto& rhs_term : rhs)
00394 result += lhs_term * rhs_term;
00395 return result;
00396 }
00397 else
00398 { // Past a certain threshold, the matrix algorithm is fastest
00399 using matrix_multi_t = typename multivector_t::matrix_multi_t;
00400 return matrix_multi_t(lhs, our_frame, true) *
00401 matrix_multi_t(rhs, our_frame, true);
00402 }
00403 }
00404
00405 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00406 inline
00407 auto
00408 framed_multi<Scalar_T,LO,HI,Tune_P>::
00409 operator*= (const multivector_t& rhs) -> multivector_t&
00410 { return *this = *this * rhs; }
00411
00412 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00413 auto
00414 operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00415 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00416 { // Arvind Raja's original reference:
00417 // "old clical, outerproduct(p,q:pterm):pterm in file compmod.pas"
00418
00419 if (lhs.empty() || rhs.empty())
00420 return Scalar_T(0);
00421
00422 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00423 using index_set_t = typename multivector_t::index_set_t;
00424 using term_t = typename multivector_t::term_t;
00425
00426 const auto empty_set = index_set_t();
00427
00428 const double lhs_size = lhs.size();
00429 const double rhs_size = rhs.size();
00430 const auto lhs_frame = lhs.frame();
00431 const auto rhs_frame = rhs.frame();
00432 const auto our_frame = lhs_frame | rhs_frame;
00433 const auto algebra_dim = set_value_t(1) << our_frame.count();
00434 auto result = multivector_t(
00435 _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00436 const auto lhs_end = lhs.end();
00437 const auto rhs_end = rhs.end();
00438
00439 if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00440 {
00441 for (auto
00442 result_stv = set_value_t(0);
00443 result_stv != algebra_dim;
00444 ++result_stv)
00445 {
00446 const auto result_ist = index_set_t(result_stv, our_frame, true);
00447 const auto lhs_result_frame = lhs_frame & result_ist;
00448 const auto lhs_result_dim = set_value_t(1) << lhs_result_frame.count();
00449 auto result_crd = Scalar_T(0);
00450 for (auto
00451 lhs_stv = set_value_t(0);
00452 lhs_stv != lhs_result_dim;

```

```

00454 ++lhs_stv)
00455 {
00456 const auto lhs_ist = index_set_t(lhs_stv, lhs_result_frame, true);
00457 const auto rhs_ist = result_ist ^ lhs_ist;
00458 if ((rhs_ist | rhs_frame) == rhs_frame)
00459 {
00460 const auto lhs_it = lhs.find(lhs_ist);
00461 if (lhs_it != lhs_end)
00462 {
00463 const auto rhs_it = rhs.find(rhs_ist);
00464 if (rhs_it != rhs_end)
00465 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00466 }
00467 }
00468 }
00469 if (result_crd != Scalar_T(0))
00470 result.insert(term_t(result_ist, result_crd));
00471 }
00472 return result;
00473 }
00474 else
00475 {
00476 for (auto& lhs_term : lhs)
00477 for (auto& rhs_term : rhs)
00478 if ((lhs_term.first & rhs_term.first) == empty_set)
00479 result += lhs_term * rhs_term;
00480 return result;
00481 }
00482 }
00483
00484 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00485 inline
00486 auto
00487 framed_multi<Scalar_T,LO,HI,Tune_P>::
00488 operator^= (const multivector_t& rhs) -> multivector_t&
00489 { return *this = *this ^ rhs; }
00490
00491 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00492 auto
00493 operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00494 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00495 { // Arvind Raja's original reference:
00496 // "old clical, innerproduct(p,q:pterm):pterm in file compmod.pas"
00497
00498 if (lhs.empty() || rhs.empty())
00499 return Scalar_T(0);
00500
00501 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00502 using index_set_t = typename multivector_t::index_set_t;
00503 using term_t = typename multivector_t::term_t;
00504
00505 const auto lhs_end = lhs.end();
00506 const auto rhs_end = rhs.end();
00507 const double lhs_size = lhs.size();
00508 const double rhs_size = rhs.size();
00509
00510 const auto lhs_frame = lhs.frame();
00511 const auto rhs_frame = rhs.frame();
00512
00513 const auto our_frame = lhs_frame | rhs_frame;
00514 const auto algebra_dim = set_value_t(1) << our_frame.count();
00515 auto result = multivector_t(
00516 _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00517 if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00518 {
00519 for (auto
00520 result_stv = set_value_t(0);
00521 result_stv != algebra_dim;
00522 ++result_stv)
00523 {
00524 const auto result_ist = index_set_t(result_stv, our_frame, true);
00525 const auto comp_frame = our_frame & ~result_ist;
00526 const auto comp_dim = set_value_t(1) << comp_frame.count();
00527 auto result_crd = Scalar_T(0);
00528 for (auto
00529 comp_stv = set_value_t(1);
00530 comp_stv != comp_dim;
00531 ++comp_stv)
00532 {
00533 const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00534 const auto our_ist = result_ist ^ comp_ist;
00535 if ((our_ist | lhs_frame) == lhs_frame)
00536 {
00537 const auto lhs_it = lhs.find(our_ist);
00538 if (lhs_it != lhs_end)
00539 {
00540 const auto rhs_it = rhs.find(comp_ist);

```

```

00542 if (rhs_it != rhs_end)
00543 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00544 }
00545 }
00546 if (result_stv != 0)
00547 {
00548 if ((our_ist | rhs_frame) == rhs_frame)
00549 {
00550 const auto rhs_it = rhs.find(our_ist);
00551 if (rhs_it != rhs_end)
00552 {
00553 const auto lhs_it = lhs.find(comp_ist);
00554 if (lhs_it != lhs_end)
00555 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00556 }
00557 }
00558 }
00559 }
00560 if (result_crd != Scalar_T(0))
00561 result.insert(term_t(result_ist, result_crd));
00562 }
00563 }
00564 else
00565 {
00566 const auto empty_set = index_set_t();
00567 for (auto& lhs_term : lhs)
00568 {
00569 const auto lhs_ist = lhs_term.first;
00570 if (lhs_ist != empty_set)
00571 for (auto& rhs_term : rhs)
00572 {
00573 const auto rhs_ist = rhs_term.first;
00574 if (rhs_ist != empty_set)
00575 {
00576 const auto our_ist = lhs_ist | rhs_ist;
00577 if ((lhs_ist == our_ist) || (rhs_ist == our_ist))
00578 result += lhs_term * rhs_term;
00579 }
00580 }
00581 }
00582 }
00583 return result;
00584 }
00585
00586 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00587 inline
00588 auto
00589 framed_multi<Scalar_T,LO,HI,Tune_P>::
00590 operator&= (const multivector_t& rhs) -> multivector_t&
00591 { return *this = *this & rhs; }
00592
00593 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00594 auto
00595 operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00596 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00597 {
00598 // Reference: Leo Dorst, "Honing geometric algebra for its use in the computer sciences",
00599 // in Geometric Computing with Clifford Algebras, ed. G. Sommer,
00600 // Springer 2001, Chapter 6, pp. 127-152.
00601 // http://staff.science.uva.nl/~leo/clifford/index.html
00602
00603 if (lhs.empty() || rhs.empty())
00604 return Scalar_T(0);
00605
00606 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00607 using index_set_t = typename multivector_t::index_set_t;
00608 using term_t = typename multivector_t::term_t;
00609
00610 const auto lhs_end = lhs.end();
00611 const auto rhs_end = rhs.end();
00612 const double lhs_size = lhs.size();
00613 const double rhs_size = rhs.size();
00614 const auto lhs_frame = lhs.frame();
00615 const auto rhs_frame = rhs.frame();
00616
00617 const auto our_frame = lhs_frame | rhs_frame;
00618 const auto algebra_dim = set_value_t(1) << our_frame.count();
00619 auto result = multivector_t(
00620 _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00621
00622 if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00623 {
00624 for (auto
00625 result_stv = set_value_t(0);
00626 result_stv != algebra_dim;
00627 ++result_stv)
00628 {
00629

```

```

00630 const auto result_ist = index_set_t(result_stv, our_frame, true);
00631 const auto comp_frame = lhs_frame & ~result_ist;
00632 const auto comp_dim = set_value_t(1) << comp_frame.count();
00633 auto result_crd = Scalar_T(0);
00634 for (auto
00635 comp_stv = set_value_t(0);
00636 comp_stv != comp_dim;
00637 ++comp_stv)
00638 {
00639 const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00640 const auto rhs_ist = result_ist ^ comp_ist;
00641 if ((rhs_ist | rhs_frame) == rhs_frame)
00642 {
00643 const auto rhs_it = rhs.find(rhs_ist);
00644 if (rhs_it != rhs_end)
00645 {
00646 const auto lhs_it = lhs.find(comp_ist);
00647 if (lhs_it != lhs_end)
00648 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00649 }
00650 }
00651 }
00652 if (result_crd != Scalar_T(0))
00653 result.insert(term_t(result_ist, result_crd));
00654 }
00655 }
00656 else
00657 {
00658 for (auto& rhs_term : rhs)
00659 {
00660 const auto rhs_ist = rhs_term.first;
00661 for (auto& lhs_term : lhs)
00662 {
00663 const index_set_t lhs_ist = lhs_term.first;
00664 if ((lhs_ist | rhs_ist) == rhs_ist)
00665 result += lhs_term * rhs_term;
00666 }
00667 }
00668 }
00669 return result;
00670 }
00671
00672 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00673 inline
00674 auto
00675 framed_multi<Scalar_T,LO,HI,Tune_P>::
00676 operator%=(const multivector_t& rhs) -> multivector_t&
00677 { return *this = *this % rhs; }
00678
00679 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00680 auto
00681 star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00682 -> Scalar_T
00683 {
00684 {
00685 auto result = Scalar_T(0);
00686 const auto small_star_large = lhs.size() < rhs.size();
00687 const auto* smallp =
00688 small_star_large
00689 ? &lhs
00690 : &rhs;
00691 const auto* largep =
00692 small_star_large
00693 ? &rhs
00694 : &lhs;
00695
00696 for (auto& small_term : *smallp)
00697 {
00698 const auto small_ist = small_term.first;
00699 const auto large_crd = (*largep)[small_ist];
00700 if (large_crd != Scalar_T(0))
00701 result += small_ist.sign_of_square() * small_term.second * large_crd;
00702 }
00703 return result;
00704 }
00705
00706 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00707 auto
00708 framed_multi<Scalar_T,LO,HI,Tune_P>::
00709 operator/=(const Scalar_T& scr) -> multivector_t&
00710 { // Divide coordinates of all terms by scr
00711 using traits_t = numeric_traits<Scalar_T>;
00712
00713 if (traits_t::isNaN(scr))
00714 return *this = traits_t::NaN();
00715 if (traits_t::isInf(scr))
00716 if (this->isnan())
00717 *this = traits_t::NaN();
00718 }

```

```

00719 else
00720 this->clear();
00721 else
00722 for (auto& this_term : *this)
00723 this_term.second /= scr;
00724 return *this;
00725 }
00726
00727 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00728 inline
00729 auto
00730 operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00731 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00732 {
00733 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00734 using traits_t = numeric_traits<Scalar_T>;
00735 using matrix_multi_t = typename multivector_t::matrix_multi_t;
00736
00737 if (rhs == Scalar_T(0))
00738 return traits_t::NaN();
00739
00740 const auto our_frame = lhs.frame() | rhs.frame();
00741 return matrix_multi_t(lhs, our_frame, true) / matrix_multi_t(rhs, our_frame, true);
00742 }
00743
00744 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00745 inline
00746 auto
00747 framed_multi<Scalar_T,LO,HI,Tune_P>::
00748 operator/= (const multivector_t& rhs) -> multivector_t&
00749 { return *this = *this / rhs; }
00750
00751 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00752 inline
00753 auto
00754 operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00755 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00756 {
00757 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00758 using matrix_multi_t = typename multivector_t::matrix_multi_t;
00759
00760 return matrix_multi_t(rhs) * matrix_multi_t(lhs) / matrix_multi_t(rhs.involute());
00761 }
00762
00763 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00764 inline
00765 auto
00766 framed_multi<Scalar_T,LO,HI,Tune_P>::
00767 operator|= (const multivector_t& rhs) -> multivector_t&
00768 { return *this = *this | rhs; }
00769
00770 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00771 inline
00772 auto
00773 framed_multi<Scalar_T,LO,HI,Tune_P>::
00774 inv() const -> const multivector_t
00775 {
00776 auto result = matrix_multi_t(Scalar_T(1), this->frame());
00777 return result /= matrix_multi_t(*this);
00778 }
00779
00780 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00781 auto
00782 framed_multi<Scalar_T,LO,HI,Tune_P>::
00783 pow(int m) const -> const multivector_t
00784 { return glucat::pow(*this, m); }
00785
00786 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00787 auto
00788 framed_multi<Scalar_T,LO,HI,Tune_P>::
00789 outer_pow(int m) const -> const multivector_t
00790 {
00791 if (m < 0)
00792 throw error_t("outer_pow(int): negative exponent");
00793 auto result = multivector_t(Scalar_T(1));
00794 auto a = *this;
00795 for (;
00796 m != 0;
00797 m >= 1, a = a ^ a)
00798 if (m & 1)
00799 result ^= a;
00800 return result;
00801 }
00802
00803 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00804 inline
00805 auto

```



```

00812 framed_multi<Scalar_T,LO,HI,Tune_P>::
00813 frame() const -> const index_set_t
00814 {
00815 auto result = index_set_t();
00816 for (auto& this_term : *this)
00817 result |= this_term.first;
00818 return result;
00819 }
00820
00822 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00823 inline
00824 auto
00825 framed_multi<Scalar_T,LO,HI,Tune_P>::
00826 grade() const -> index_t
00827 {
00828 auto result = index_t(0);
00829 for (auto& this_term : *this)
00830 result = std::max(result, this_term.first.count());
00831 return result;
00832 }
00833
00835 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00836 inline
00837 auto
00838 framed_multi<Scalar_T,LO,HI,Tune_P>::
00839 operator[] (const index_set_t ist) const -> Scalar_T
00840 {
00841 const auto& this_it = this->find(ist);
00842 if (this_it == this->end())
00843 return Scalar_T(0);
00844 else
00845 return this_it->second;
00846 }
00847
00849 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00850 auto
00851 framed_multi<Scalar_T,LO,HI,Tune_P>::
00852 operator() (index_t grade) const -> const multivector_t
00853 {
00854 if ((grade < 0) || (grade > HI-LO))
00855 return Scalar_T(0);
00856 else
00857 {
00858 auto result = multivector_t();
00859 for (auto& this_term : *this)
00860 if (this_term.first.count() == grade)
00861 result += this_term;
00862 return result;
00863 }
00864 }
00865
00867 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00868 inline
00869 auto
00870 framed_multi<Scalar_T,LO,HI,Tune_P>::
00871 scalar() const -> Scalar_T
00872 { return (*this)[index_set_t()]; }
00873
00875 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00876 inline
00877 auto
00878 framed_multi<Scalar_T,LO,HI,Tune_P>::
00879 pure() const -> const multivector_t
00880 { return *this - this->scalar(); }
00881
00883 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00884 auto
00885 framed_multi<Scalar_T,LO,HI,Tune_P>::
00886 even() const -> const multivector_t
00887 { // even part of x, sum of the pure(count) with even count
00888 auto result = multivector_t();
00889 for (auto& this_term : *this)
00890 if ((this_term.first.count() % 2) == 0)
00891 result.insert(this_term);
00892 return result;
00893 }
00894
00896 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00897 auto
00898 framed_multi<Scalar_T,LO,HI,Tune_P>::
00899 odd() const -> const multivector_t
00900 { // even part of x, sum of the pure(count) with even count
00901 auto result = multivector_t();
00902 for (auto& this_term : *this)
00903 if ((this_term.first.count() % 2) == 1)
00904 result.insert(this_term);
00905 return result;

```

```

00906 }
00907
00909 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00910 auto
00911 framed_multi<Scalar_T,LO,HI,Tune_P>::
00912 vector_part() const -> const vector_t
00913 { return this->vector_part(this->frame(), true); }
00914
00916 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00917 auto
00918 framed_multi<Scalar_T,LO,HI,Tune_P>::
00919 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00920 {
00921 if (!prechecked && (this->frame() | frm) != frm)
00922 throw error_t("vector_part(frm): value is outside of requested frame");
00923 auto result = vector_t();
00924 result.reserve(frm.count());
00925 const auto frm_end = frm.max()+1;
00926 for (auto
00927 idx = frm.min();
00928 idx != frm_end;
00929 ++idx)
00930 // Frame may contain indices which do not correspond to a grade 1 term but
00931 // frame cannot omit any index corresponding to a grade 1 term
00932 if (frm[idx])
00933 result.push_back((*this)[index_set_t(idx)]);
00934 return result;
00935 }
00936
00938 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00939 auto
00940 framed_multi<Scalar_T,LO,HI,Tune_P>::
00941 involute() const -> const multivector_t
00942 {
00943 auto result = *this;
00944 for (auto& result_term : result)
00945 { // for a k-vector u, involute(u) == (-1)^k * u
00946 if ((result_term.first.count() % 2) == 1)
00947 result_term.second *= Scalar_T(-1);
00948 }
00949 return result;
00950 }
00951
00953 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954 auto
00955 framed_multi<Scalar_T,LO,HI,Tune_P>::
00956 reverse() const -> const multivector_t
00957 {
00958 auto result = *this;
00959 for (auto& result_term : result)
00960 // For a k-vector u, reverse(u) = { -u, k == 2,3 (mod 4)
00961 // { u, k == 0,1 (mod 4)
00962 switch (result_term.first.count() % 4)
00963 {
00964 case 2:
00965 case 3:
00966 result_term.second *= Scalar_T(-1);
00967 break;
00968 default:
00969 break;
00970 }
00971 return result;
00972 }
00973
00975 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976 auto
00977 framed_multi<Scalar_T,LO,HI,Tune_P>::
00978 conj() const -> const multivector_t
00979 {
00980 auto result = *this;
00981 for (auto& result_term : result)
00982 // For a k-vector u, conj(u) = { -u, k == 1,2 (mod 4)
00983 // { u, k == 0,3 (mod 4)
00984 switch (result_term.first.count() % 4)
00985 {
00986 case 1:
00987 case 2:
00988 result_term.second *= Scalar_T(-1);
00989 break;
00990 default:
00991 break;
00992 }
00993 return result;
00994 }
00995
00997 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00998 auto

```

```

00999 framed_multi<Scalar_T, LO, HI, Tune_P>::
01000 quad() const -> Scalar_T
01001 {
01002 // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
01003 // ref: old clical: quadfunction(p:pterm):pterm in file compmod.pas
01004 auto result = Scalar_T(0);
01005 for (auto& this_term : *this)
01006 {
01007 const auto sign =
01008 (this_term.first.count_neg() % 2)
01009 ? -Scalar_T(1)
01010 : Scalar_T(1);
01011 result += sign * (this_term.second) * (this_term.second);
01012 }
01013 return result;
01014 }
01015
01016 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01017 auto
01018 framed_multi<Scalar_T, LO, HI, Tune_P>::
01019 norm() const -> Scalar_T
01020 {
01021 using traits_t = numeric_traits<Scalar_T>;
01022
01023 auto result = Scalar_T(0);
01024 for (auto& this_term : *this)
01025 {
01026 const auto abs_crd = traits_t::abs(this_term.second);
01027 result += abs_crd * abs_crd;
01028 }
01029 return result;
01030 }
01031
01032 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01033 auto
01034 framed_multi<Scalar_T, LO, HI, Tune_P>::
01035 max_abs() const -> Scalar_T
01036 {
01037 using traits_t = numeric_traits<Scalar_T>;
01038
01039 auto result = Scalar_T(0);
01040 for (auto& this_term : *this)
01041 {
01042 const auto abs_crd = traits_t::abs(this_term.second);
01043 if (abs_crd > result)
01044 result = abs_crd;
01045 }
01046 return result;
01047 }
01048
01049 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01050 auto
01051 framed_multi<Scalar_T, LO, HI, Tune_P>::
01052 random(const index_set_t frm, Scalar_T fill) -> const multivector_t
01053 {
01054 using multivector_t = framed_multi<Scalar_T, LO, HI, Tune_P>;
01055 using index_set_t = typename multivector_t::index_set_t;
01056 using term_t = typename multivector_t::term_t;
01057
01058 using random_generator_t = random_generator<Scalar_T>;
01059 auto& generator = random_generator_t::generator();
01060
01061 fill =
01062 (fill < Scalar_T(0))
01063 ? Scalar_T(0)
01064 : (fill > Scalar_T(1))
01065 ? Scalar_T(1)
01066 : fill;
01067
01068 const auto algebra_dim = set_value_t(1) << frm.count();
01069 using traits_t = numeric_traits<Scalar_T>;
01070 const auto mean_abs = traits_t::sqrt(Scalar_T(double(algebra_dim)));
01071 auto result = multivector_t();
01072 for (auto
01073 stv = set_value_t(0);
01074 stv != algebra_dim;
01075 ++stv)
01076 {
01077 if (generator.uniform() < fill)
01078 {
01079 const auto& result_crd = generator.normal() / mean_abs;
01080 result.insert(term_t(index_set_t(stv, frm, true), result_crd));
01081 }
01082 }
01083 return result;
01084 }
01085
01086 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01087 inline
01088 void

```

```

01090 framed_multi<Scalar_T,LO,HI,Tune_P>::
01091 write(const std::string& msg) const
01092 { std::cout << msg << std::endl << " " << (*this) << std::endl; }
01093
01094 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01095 inline
01096 void
01097 framed_multi<Scalar_T,LO,HI,Tune_P>::
01098 write(std::ofstream& ofile, const std::string& msg) const
01099 {
01100 {
01101 if (!ofile)
01102 throw error_t("write(ofile,msg): cannot write to output file");
01103 ofile << msg << std::endl << " " << (*this) << std::endl;
01104 }
01105
01106 template< typename Map_T,typename Sorted_Map_T >
01107 class sorted_range
01108 {
01109 public:
01110 using map_t = Map_T;
01111 using sorted_map_t = Sorted_Map_T;
01112 using sorted_iterator = typename Sorted_Map_T::const_iterator;
01113
01114 sorted_range (Sorted_Map_T &sorted_val, const Map_T& val)
01115 {
01116 for (auto& val_term : val)
01117 sorted_val.insert(val_term);
01118 sorted_begin = sorted_val.begin();
01119 sorted_end = sorted_val.end();
01120 }
01121 sorted_iterator sorted_begin;
01122 sorted_iterator sorted_end;
01123 };
01124
01125 template< typename Sorted_Map_T >
01126 class sorted_range< Sorted_Map_T, Sorted_Map_T >
01127 {
01128 public:
01129 using map_t = Sorted_Map_T;
01130 using sorted_map_t = Sorted_Map_T;
01131 using sorted_iterator = typename Sorted_Map_T::const_iterator;
01132
01133 sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T& val)
01134 : sorted_begin(val.begin()),
01135 sorted_end(val.end())
01136 { }
01137 sorted_iterator sorted_begin;
01138 sorted_iterator sorted_end;
01139 };
01140
01141 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01142 auto
01143 operator<< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
01144 {
01145 using limits_t = std::numeric_limits<Scalar_T>;
01146 if (val.empty())
01147 os << 0;
01148 else if (val.isnan())
01149 os << limits_t::quiet_NaN();
01150 else if (val.isinf())
01151 {
01152 const Scalar_T& inf = limits_t::infinity();
01153 os << (scalar(val) < 0.0 ? -inf : inf);
01154 }
01155 else
01156 {
01157 using traits_t = numeric_traits<Scalar_T>;
01158 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01159 Scalar_T truncation;
01160 switch (os.flags() & std::ios::floatfield)
01161 {
01162 case std::ios_base::scientific:
01163 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()) + 1);
01164 break;
01165 case std::ios_base::fixed:
01166 truncation = Scalar_T(1) / (traits_t::pow(Scalar_T(10), int(os.precision())) *
01167 val.max_abs());
01168 break;
01169 case std::ios_base::fixed | std::ios_base::scientific:
01170 truncation = multivector_t::default_truncation;
01171 break;
01172 default:
01173 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()));
01174 break;
01175 }
01176 auto truncated_val = val.truncated(truncation);
01177 if (truncated_val.empty())
01178

```

```

01179 os << 0;
01180 else
01181 {
01182 using map_t = typename multivector_t::map_t;
01183 using sorted_map_t = typename multivector_t::sorted_map_t;
01184 auto sorted_val = sorted_map_t();
01185 const auto sorted_val_range = sorted_range< map_t, sorted_map_t >(sorted_val, truncated_val);
01186 auto sorted_it = sorted_val_range.sorted_begin();
01187 os << *sorted_it;
01188 for (++sorted_it;
01189 sorted_it != sorted_val_range.sorted_end;
01190 ++sorted_it)
01191 {
01192 const Scalar_T& scr = sorted_it->second;
01193 if (scr >= 0.0)
01194 os << '+';
01195 os << *sorted_it;
01196 }
01197 }
01198 }
01199 return os;
01200 }
01201
01202 template< typename Scalar_T, const index_t LO, const index_t HI >
01203 auto
01204 operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
01205 std::ostream&
01206 {
01207 const auto second_as_double = numeric_traits<Scalar_T>::to_double(term.second);
01208 const auto use_double =
01209 (os.precision() <= std::numeric_limits<double>::digits10) ||
01210 (term.second == Scalar_T(second_as_double));
01211 if (term.first.count() == 0)
01212 if (use_double)
01213 os << second_as_double;
01214 else
01215 os << term.second;
01216 else if (term.second == Scalar_T(-1))
01217 {
01218 os << '-';
01219 os << term.first;
01220 }
01221 else if (term.second != Scalar_T(1))
01222 {
01223 if (use_double)
01224 {
01225 auto tol = std::pow(10.0, -os.precision());
01226 if (std::fabs(second_as_double + 1.0) < tol)
01227 os << '-';
01228 else if (std::fabs(second_as_double - 1.0) >= tol)
01229 os << second_as_double;
01230 }
01231 else
01232 os << term.second;
01233 os << term.first;
01234 }
01235 else
01236 os << term.first;
01237 return os;
01238 }
01239
01240 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01241 auto
01242 operator<< (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P> & val) -> std::istream&
01243 { // Input looks like 1.0-2.0{1,2}+3.2{3,4}.
01244 using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01245 // Parsing variables.
01246 auto local_val = multivector_t();
01247 auto c = 0;
01248 // Parsing control variables.
01249 auto negative = false;
01250 auto expect_term = true;
01251 // The multivector may begin with '+' or '-'. Check for this.
01252 c = s.peek();
01253 if (s.good() && (c == int('+') || c == int('-')))
01254 { // A '-' here negates the following term.
01255 negative = (c == int('-'));
01256 // Consume the '+' or '-'.
01257 s.get();
01258 }
01259 while (s.good())
01260 { // Parse a term.
01261 // A term consists of an optional scalar, followed by an optional index set.
01262 // At least one of the two must be present.
01263 // Default coordinate is Scalar_T(1).
01264 auto coordinate = Scalar_T(1);
01265 // Default index set is empty.

```

```

01267 auto ist = index_set<LO,HI>();
01268 // First, check for an opening brace.
01269 c = s.peek();
01270 if (s.good())
01271 { // If the character is not an opening brace,
01272 // a coordinate value is expected here.
01273 if (c != int('{'))
01274 { // Try to read a coordinate value.
01275 double coordinate_as_double;
01276 s » coordinate_as_double;
01277 // Reading the coordinate may have resulted in an end of file condition.
01278 // This is not a failure.
01279 if (s)
01280 coordinate = Scalar_T(coordinate_as_double);
01281 }
01282 }
01283 else
01284 { // End of file here ends parsing while a term may still be expected.
01285 break;
01286 }
01287 // Coordinate is now Scalar_T(1) or a Scalar_T value.
01288 // Parse an optional index set.
01289 if (s.good())
01290 {
01291 c = s.peek();
01292 if (s.good() && c == int('{'))
01293 { // Try to read index set.
01294 s » ist;
01295 }
01296 }
01297 // Reading the term may have resulted in an end of file condition.
01298 // This is not a failure.
01299 if (s)
01300 {
01301 // Immediately after parsing a term, another term is not expected.
01302 expect_term = false;
01303 if (coordinate != Scalar_T(0))
01304 {
01305 // Add the term to the local multivector.
01306 coordinate =
01307 negative
01308 ? -coordinate
01309 : coordinate;
01310 using term_t = typename multivector_t::term_t;
01311 local_val += term_t(ist, coordinate);
01312 }
01313 }
01314 // Check if anything follows the current term.
01315 if (s.good())
01316 {
01317 c = s.peek();
01318 if (s.good())
01319 { // Only '+' and '-' are valid here.
01320 if (c == int('+') || c == int('-'))
01321 { // A '-' here negates the following term.
01322 negative = (c == int('-'));
01323 // Consume the '+' or '-'.
01324 s.get();
01325 // Immediately after '+' or '-',
01326 // expect another term.
01327 expect_term = true;
01328 }
01329 else
01330 { // Any other character here is a not failure,
01331 // but still ends the parsing of the multivector.
01332 break;
01333 }
01334 }
01335 }
01336 }
01337 // If a term is still expected, this is a failure.
01338 if (expect_term)
01339 s.clear(std::istream::failbit);
01340 // End of file is not a failure.
01341 if (s)
01342 { // The multivector has been successfully parsed.
01343 val = local_val;
01344 }
01345 return s;
01346 }
01347
01348 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01349 auto
01350 framed_multi<Scalar_T,LO,HI,Tune_P>::
01351 nbr_terms () const -> unsigned long
01352 { return this->size(); }
01353
01354

```

```

01356 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01357 inline
01358 auto
01359 framed_multi<Scalar_T,LO,HI,Tune_P>::
01360 operator+= (const term_t& term) -> multivector_t&
01361 { // Do not insert terms with 0 coordinate
01362 if (term.second != Scalar_T(0))
01363 {
01364 const auto& this_it = this->find(term.first);
01365 if (this_it == this->end())
01366 this->insert(term);
01367 else if (this_it->second + term.second == Scalar_T(0))
01368 // Erase term if resulting coordinate is 0
01369 this->erase(this_it);
01370 else
01371 this_it->second += term.second;
01372 }
01373 return *this;
01374 }
01375
01377 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01378 auto
01379 framed_multi<Scalar_T,LO,HI,Tune_P>::
01380 isinf() const -> bool
01381 {
01382 using traits_t = numeric_traits<Scalar_T>;
01383
01384 if (std::numeric_limits<Scalar_T>::has_infinity)
01385 for (auto& this_term : *this)
01386 if (traits_t::isInf(this_term.second))
01387 return true;
01388 return false;
01389 }
01390
01392 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01393 auto
01394 framed_multi<Scalar_T,LO,HI,Tune_P>::
01395 isnan() const -> bool
01396 {
01397 using traits_t = numeric_traits<Scalar_T>;
01398
01399 if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
01400 for (auto& this_term : *this)
01401 if (traits_t::isNaN(this_term.second))
01402 return true;
01403 return false;
01404 }
01405
01407 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01408 auto
01409 framed_multi<Scalar_T,LO,HI,Tune_P>::
01410 truncated(const Scalar_T& limit) const -> const multivector_t
01411 {
01412 using traits_t = numeric_traits<Scalar_T>;
01413
01414 if (this->isnan() || this->isinf())
01415 return *this;
01416 const auto truncation = traits_t::abs(limit);
01417 const auto top = max_abs();
01418 auto result = multivector_t();
01419 if (top != Scalar_T(0))
01420 for (auto& this_term : *this)
01421 if (traits_t::abs(this_term.second) > top * truncation)
01422 result.insert(this_term);
01423 return result;
01424 }
01425
01427 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01428 auto
01429 framed_multi<Scalar_T,LO,HI,Tune_P>::
01430 fold(const index_set_t frm) const -> multivector_t
01431 {
01432 if (frm.is_contiguous())
01433 return *this;
01434 else
01435 {
01436 auto result = multivector_t();
01437 for (auto& this_term : *this)
01438 result.insert(term_t(this_term.first.fold(frm), this_term.second));
01439 return result;
01440 }
01441 }
01442
01444 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01445 auto
01446 framed_multi<Scalar_T,LO,HI,Tune_P>::
01447 unfold(const index_set_t frm) const -> multivector_t

```

```

01448 {
01449 if (frm.is_contiguous())
01450 return *this;
01451 else
01452 {
01453 auto result = multivector_t();
01454 for (auto& this_term : *this)
01455 result.insert(term_t(this_term.first.unfold(frm), this_term.second));
01456 return result;
01457 }
01458 }
01459
01461 // Reference: [L] 16.4 Periodicity of 8, p216
01462 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01463 auto
01464 framed_multi<Scalar_T,LO,HI,Tune_P>::
01465 centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&
01466 {
01467 // We add 4 to q by subtracting 4 from p
01468 if (q+4 > -LO)
01469 throw error_t("centre_pm4_qp4(p,q): LO is too high to represent this value");
01470 if (this->frame().max() > p-4)
01471 {
01472 using index_pair_t = typename index_set_t::index_pair_t;
01473 const auto pm3210 = index_set_t(index_pair_t(p-3,p), true);
01474 const auto qm4321 = index_set_t(index_pair_t(-q-4,-q-1), true);
01475 const auto& tqm4321 = term_t(qm4321, Scalar_T(1));
01476 auto result = multivector_t();
01477 for (auto& this_term : *this)
01478 {
01479 const auto ist = this_term.first;
01480 if (ist.max() > p-4)
01481 {
01482 auto var_term = var_term_t();
01483 for (auto
01484 n = index_t(0);
01485 n != index_t(4);
01486 ++n)
01487 if (ist[n+p-3])
01488 var_term *= term_t(index_set_t(n-q-4), Scalar_T(1)) * tqm4321;
01489 // Mask out {p-3}..{p}
01490 result.insert(term_t(ist & ~pm3210, this_term.second) *
01491 term_t(var_term.first, var_term.second));
01492 }
01493 else
01494 result.insert(this_term);
01495 }
01496 *this = result;
01497 }
01498 p -=4; q += 4;
01499 return *this;
01500 }
01501
01503 // Reference: [L] 16.4 Periodicity of 8, p216
01504 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01505 auto
01506 framed_multi<Scalar_T,LO,HI,Tune_P>::
01507 centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&
01508 {
01509 // We add 4 to p by subtracting 4 from q
01510 if (p+4 > HI)
01511 throw error_t("centre_pp4_qm4(p,q): HI is too low to represent this value");
01512 if (this->frame().min() < -q+4)
01513 {
01514 using index_pair_t = typename index_set_t::index_pair_t;
01515 const auto qp0123 = index_set_t(index_pair_t(-q,-q+3), true);
01516 const auto pp1234 = index_set_t(index_pair_t(p+1,p+4), true);
01517 const auto& tpp1234 = term_t(pp1234, Scalar_T(1));
01518 auto result = multivector_t();
01519 for (auto& this_term : *this)
01520 {
01521 index_set_t ist = this_term.first;
01522 if (ist.min() < -q+4)
01523 {
01524 auto var_term = var_term_t();
01525 for (auto
01526 n = index_t(0);
01527 n != index_t(4);
01528 ++n)
01529 if (ist[n-q])
01530 var_term *= term_t(index_set_t(n+p+1), Scalar_T(1)) * tpp1234;
01531 // Mask out {-q}..{-q+3}
01532 result.insert(term_t(var_term.first, var_term.second) *
01533 term_t(ist & ~qp0123, this_term.second));
01534 }
01535 else
01536 result.insert(this_term);

```



```

01537 }
01538 *this = result;
01539 }
01540 p +=4; q -= 4;
01541 return *this;
01542 }
01543
01544 // Reference: [P] Proposition 15.20, p 131
01545 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01546 auto
01547 framed_multi<Scalar_T,LO,HI,Tune_P>::
01548 centre_qpl_pml(index_t& p, index_t& q) -> multivector_t&
01549 {
01550 {
01551 if (q+1 > HI)
01552 throw error_t("centre_qpl_pml(p,q): HI is too low to represent this value");
01553 if (p-1 > -LO)
01554 throw error_t("centre_qpl_pml(p,q): LO is too high to represent this value");
01555 const auto qpl = index_set_t(q+1);
01556 const auto& tqpl = term_t(qpl, Scalar_T(1));
01557 auto result = multivector_t();
01558 for (auto& this_term : *this)
01559 {
01560 const auto ist = this_term.first;
01561 auto var_term = var_term_t(index_set_t(), this_term.second);
01562 for (auto
01563 n = -q;
01564 n != p;
01565 ++n)
01566 if (n != 0 && ist[n])
01567 var_term *= term_t(index_set_t(-n) | qpl, Scalar_T(1));
01568 if (p != 0 && ist[p])
01569 var_term *= tqpl;
01570 result.insert(term_t(var_term.first, var_term.second));
01571 }
01572 index_t orig_p = p;
01573 p = q+1;
01574 q = orig_p-1;
01575 return *this = result;
01576 }
01577
01578 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01579 auto
01580 framed_multi<Scalar_T,LO,HI,Tune_P>::
01581 divide(const index_set_t ist) const -> const framed_pair_t
01582 {
01583 {
01584 auto quo = multivector_t();
01585 auto rem = multivector_t();
01586 for (auto& this_term : *this)
01587 if ((this_term.first | ist) == this_term.first)
01588 quo.insert(term_t(this_term.first ^ ist, this_term.second));
01589 else
01590 rem.insert(this_term);
01591 return framed_pair_t(quo, rem);
01592 }
01593
01594 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01595 auto
01596 framed_multi<Scalar_T,LO,HI,Tune_P>::
01597 fast(const index_t level, const bool odd) const -> const matrix_t
01598 {
01599 {
01600 // Assume val is already folded and centred
01601 if (this->empty())
01602 {
01603 using matrix_index_t = typename matrix_multi_t::matrix_index_t;
01604 const auto dim = matrix_index_t(1) << level;
01605 auto result = matrix_t(dim, dim);
01606 result.clear();
01607 return result;
01608 }
01609 if (level == 0)
01610 return matrix::unit<matrix_t>(1) * this->scalar();
01611
01612 using basis_matrix_t = typename matrix_multi_t::basis_matrix_t;
01613 using basis_scalar_t = typename basis_matrix_t::value_type;
01614
01615 const auto& I = matrix::unit<basis_matrix_t>(2);
01616 auto J = basis_matrix_t(2,2,2);
01617 J.clear();
01618 J(0,1) = basis_scalar_t(-1);
01619 J(1,0) = basis_scalar_t(1);
01620 auto K = J;
01621 K(0,1) = basis_scalar_t(1);
01622 auto JK = I;
01623 JK(0,0) = basis_scalar_t(-1);
01624
01625 const auto ist_mn = index_set_t(-level);
01626 const auto ist_pn = index_set_t(level);

```

```

01627 if (level == 1)
01628 {
01629 if (odd)
01630 return matrix_t(J) * (*this)[ist_mn] + matrix_t(K) * (*this)[ist_pn];
01631 else
01632 return matrix_t(I) * this->scalar() + matrix_t(JK) * (*this)[ist_mn ^ ist_pn];
01633 }
01634 else
01635 {
01636 const auto& pair_mn = this->divide(ist_mn);
01637 const auto& quo_mn = pair_mn.first;
01638 const auto& rem_mn = pair_mn.second;
01639 const auto& pair_quo_mnpn = quo_mn.divide(ist_pn);
01640 const auto& val_mnpn = pair_quo_mnpn.first;
01641 const auto& val_mn = pair_quo_mnpn.second;
01642 const auto& pair_rem_mnpn = rem_mn.divide(ist_pn);
01643 const auto& val_pn = pair_rem_mnpn.first;
01644 const auto& val_l = pair_rem_mnpn.second;
01645 using matrix::kron;
01646 if (odd)
01647 return - kron(JK, val_l.fast (level-1, 1))
01648 + kron(I, val_mnpn.fast (level-1, 1))
01649 + kron(J, val_mn.fast (level-1, 0))
01650 + kron(K, val_pn.fast (level-1, 0));
01651 else
01652 return kron(I, val_l.fast (level-1, 0))
01653 + kron(JK, val_mnpn.fast (level-1, 0))
01654 + kron(K, val_mn.fast (level-1, 1))
01655 - kron(J, val_pn.fast (level-1, 1));
01656 }
01657 }
01658
01660 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01661 template< typename Other_Scalar_T >
01662 auto
01663 framed_multi<Scalar_T,LO,HI,Tune_P>::
01664 fast_matrix_multi(const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>
01665 {
01666 // Fold val
01667 auto val = this->fold(frm);
01668 auto p = frm.count_pos();
01669 auto q = frm.count_neg();
01670 const auto bott_offset = gen::offset_to_super[pos_mod(p - q, 8)];
01671 p += std::max(bott_offset, index_t(0));
01672 q -= std::min(bott_offset, index_t(0));
01673 if (p > HI)
01674 throw error_t("fast_matrix_multi(frm): HI is too low to represent this value");
01675 if (q > -LO)
01676 throw error_t("fast_matrix_multi(frm): LO is too high to represent this value");
01677 // Centre val
01678 while (p - q > 4)
01679 val.centre_pm4_qp4(p, q);
01680 while (p - q < -3)
01681 val.centre_pp4_qm4(p, q);
01682 if (p - q > 1)
01683 val.centre_qp1_pm1(p, q);
01684 const index_t level = (p + q)/2;
01685
01686 // Do the fast transform
01687 const auto& ev_val = val.even();
01688 const auto& od_val = val.odd();
01689 return matrix_multi<Other_Scalar_T,LO,HI,Tune_P>(ev_val.fast(level, 0) + od_val.fast(level, 1),
01690 frm);
01691 }
01692
01693 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01694 inline
01695 auto
01696 framed_multi<Scalar_T,LO,HI,Tune_P>::
01697 fast_framed_multi() const -> const multivector_t
01698 { return *this; }
01699
01700 template< typename Scalar_T, const index_t LO, const index_t HI >
01701 inline
01702 static
01703 auto
01704 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01705 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T
01706 { return lhs.first.sign_of_mult(rhs.first) * lhs.second * rhs.second; }
01707
01708 template< typename Scalar_T, const index_t LO, const index_t HI >
01709 inline
01710 auto
01711 operator* (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01712 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const
index_set<LO,HI>, Scalar_T>
01713 {
01714 {

```

```

01715 using term_t = std::pair<const index_set<LO,HI>, Scalar_T>;
01716 return term_t(lhs.first ^ rhs.first, crd_of_mult(lhs, rhs));
01717 }
01718
01720 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01721 auto
01722 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01723 {
01724 using traits_t = numeric_traits<Scalar_T>;
01725 if (val.isnan())
01726 return traits_t::NaN();
01727
01728 check_complex(val, i, prechecked);
01729
01730 const auto realval = val.scalar();
01731 if (val == realval)
01732 {
01733 if (realval < Scalar_T(0))
01734 return i * traits_t::sqrt(-realval);
01735 else
01736 return traits_t::sqrt(realval);
01737 }
01738 using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01739 return sqrt(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01740 }
01741
01743 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01744 auto
01745 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01746 {
01747 using traits_t = numeric_traits<Scalar_T>;
01748 if (val.isnan())
01749 return traits_t::NaN();
01750
01751 const auto s = scalar(val);
01752 if (val == s)
01753 return traits_t::exp(s);
01754
01755 const double size = val.size();
01756 const auto frm_count = val.frame().count();
01757 const auto algebra_dim = set_value_t(1) << frm_count;
01758
01759 if ((size * size <= double(algebra_dim)) || (frm_count < Tune_P::mult_matrix_threshold))
01760 {
01761 switch (Tune_P::function_precision)
01762 {
01763 case precision_demoted:
01764 {
01765 using demoted_scalar_t = typename traits_t::demoted::type;
01766 using demoted_multivector_t = framed_multi<demoted_scalar_t,LO,HI,Tune_P>;
01767
01768 const auto& demoted_val = demoted_multivector_t(val);
01769 return clifford_exp(demoted_val);
01770 }
01771 break;
01772 case precision_promoted:
01773 {
01774 using promoted_scalar_t = typename traits_t::promoted::type;
01775 using promoted_multivector_t = framed_multi<promoted_scalar_t,LO,HI,Tune_P>;
01776
01777 const auto& promoted_val = promoted_multivector_t(val);
01778 return clifford_exp(promoted_val);
01779 }
01780 break;
01781 default:
01782 return clifford_exp(val);
01783 }
01784 }
01785 else
01786 {
01787 using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01788 return exp(matrix_multi_t(val));
01789 }
01790 }
01791
01793 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01794 auto
01795 log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01796 {
01797 using traits_t = numeric_traits<Scalar_T>;
01798 if (val == Scalar_T(0) || val.isnan())
01799 return traits_t::NaN();
01800
01801 check_complex(val, i, prechecked);
01802

```

```

01803 const auto realval = val.scalar();
01804 if (val == realval)
01805 {
01806 if (realval < Scalar_T(0))
01807 return i * traits_t::pi() + traits_t::log(-realval);
01808 else
01809 return traits_t::log(realval);
01810 }
01811 using matrix_multi_t = typename framed_multi<Scalar_T,I,O,HI,Tune_P>::matrix_multi_t;
01812 return log(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01813 }
01814 }
01815 #endif // _GLUCAT_FRAMED_MULTI_IMP_H

```

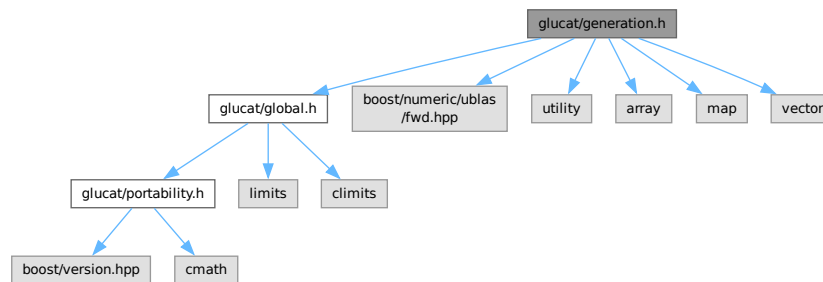
## 9.13 glucat/generation.h File Reference

```

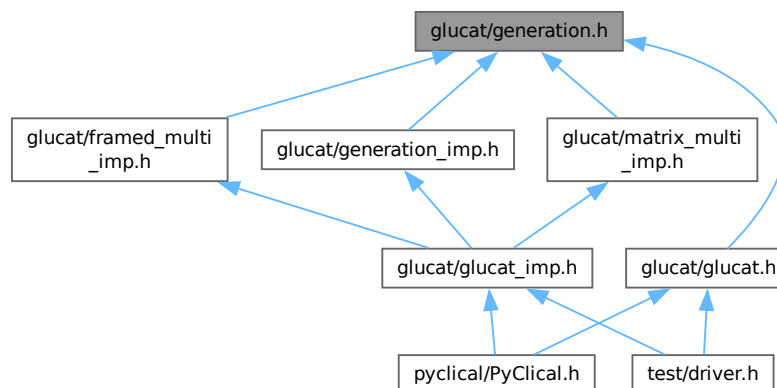
#include "glucat/global.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <utility>
#include <array>
#include <map>
#include <vector>

```

Include dependency graph for generation.h:



This graph shows which files directly or indirectly include this file:



**Classes**

- class `glucat::gen::generator_table< Matrix_T >`  
*Table of generators for specific signatures.*

**Namespaces**

- namespace `glucat`
- namespace `glucat::gen`

**Typedefs**

- using `glucat::gen::signature_t = std::pair<index_t, index_t>`  
*A signature is a pair of indices,  $p, q$ , with  $p == \text{frame.max}()$ ,  $q == -\text{frame.min}()$ .*

**Variables**

- static const `std::array< index_t, 8 > glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}`  
*Offsets between the current signature and that of the real superalgebra.*

## 9.14 generation.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_H
00002 #define _GLUCAT_GENERATION_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 generation.h : Declare functions for generation of the matrix representation
00006 -----
00007 begin : Wed Jan 23 2002
00008 copyright : (C) 2002-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <boost/numeric/ublas/fwd.hpp>
00037
00038 #include <utility>
00039 #include <array>
00040 #include <map>
00041 #include <vector>
00042

```

```

00043 namespace glucat { namespace gen
00044 {
00045 namespace ublas = boost::numeric::ublas;
00046
00048 using signature_t = std::pair<index_t, index_t>;
00049
00051 template< class Matrix_T >
00052 class generator_table :
00053 private std::map< signature_t, std::vector<Matrix_T> >
00054 {
00055 public:
00057 auto operator() (const index_t p, const index_t q) -> const Matrix_T*;
00059 static auto generator() -> generator_table<Matrix_T>&;
00060 private:
00062 auto gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&;
00064 void gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig);
00066 void gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig);
00068 void gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig);
00070 void gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig);
00071
00075 friend class friend_for_private_destructor;
00076 // Enforce singleton
00077 // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00078 generator_table() = default;
00079 ~generator_table() = default;
00080 public:
00081 generator_table(const generator_table&) = delete;
00082 auto operator= (const generator_table&) -> generator_table& = delete;
00083 };
00084
00086 static const std::array<index_t, 8> offset_to_super = {0, -1, 0, -1, -2, 3, 2, 1};
00087 } }
00089 #endif // _GLUCAT_GENERATION_H

```

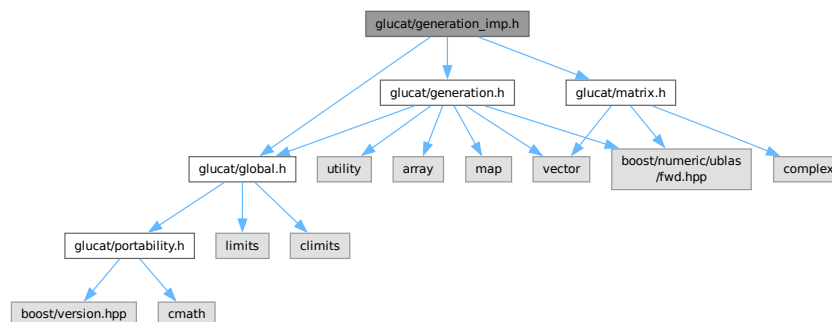
## 9.15 glucat/generation\_imp.h File Reference

```

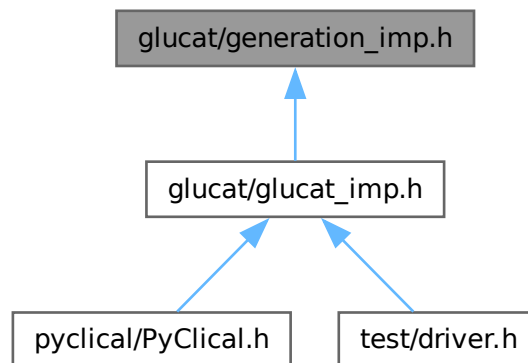
#include "glucat/global.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"

```

Include dependency graph for generation\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## 9.16 generation\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_IMP_H
00002 #define _GLUCAT_GENERATION_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 generation_imp.h : Implement functions for generation of the matrix representation
00006 *****/
00007 begin : Wed Jan 23 2002
00008 copyright : (C) 2002-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/generation.h"
00036 #include "glucat/matrix.h"

```

```

00037
00038 namespace glucat { namespace gen
00039 {
00040 // References for algorithms:
00041 // [M]: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.
00042 // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00043 // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00044
00046 // Reference: [M] Item 47
00047 template< class Matrix_T >
00048 auto
00049 generator_table<Matrix_T>::
00050 generator() -> generator_table<Matrix_T>&
00051 { static generator_table<Matrix_T> g; return g; }
00052
00054 // Reference: [P] Table 15.27, p 133
00055 template< class Matrix_T >
00056 inline
00057 auto
00058 generator_table<Matrix_T>::
00059 operator() (const index_t p, const index_t q) -> const Matrix_T*
00060 {
00061 const auto bott = pos_mod(p-q, 8);
00062 switch(bott)
00063 {
00064 case 0:
00065 case 2:
00066 // Construct generators
00067 return &(gen_vector(p, q)[q]);
00068 default:
00069 // Select generators from the vector for a larger frame
00070 const auto super_p = p + std::max(offset_to_super[bott], index_t(0));
00071 const auto super_q = q - std::min(offset_to_super[bott], index_t(0));
00072 return &(gen_vector(super_p, super_q)[super_q]);
00073 }
00074 }
00075
00077 template< class Matrix_T >
00078 auto
00079 generator_table<Matrix_T>::
00080 gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&
00081 {
00082 using result_t = std::vector<Matrix_T>;
00083 const auto card = p + q;
00084 const auto bias = p - q;
00085 const auto bott = pos_mod(bias, 8);
00086 const auto sig = signature_t(p, q);
00087 if (this->find(sig) == this->end())
00088 switch(bott)
00089 {
00090 case 0:
00091 if (bias < 0)
00092 // Construct generators for p,q given generators for p+4,q-4
00093 gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00094 else if (bias > 0)
00095 // Construct generators for p,q given generators for p-4,q+4
00096 gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00097 else if (card == 0)
00098 { // Base case. Save a generator vector containing one matrix, size 1.
00099 auto result = result_t(1, matrix::unit<Matrix_T>(1));
00100 this->insert(make_pair(sig, result));
00101 }
00102 else
00103 // Construct generators for p,q given generators for p-1,q-1
00104 gen_from_pml_qml(gen_vector(p-1, q-1), sig);
00105 break;
00106 case 2:
00107 if (bias < 2)
00108 // Construct generators for p,q given generators for p+4,q-4
00109 gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00110 else if (bias > 2)
00111 // Construct generators for p,q given generators for p-4,q+4
00112 gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00113 else
00114 // Construct generators for p,q given generators for q+1,p-1
00115 gen_from_qp1_pml(gen_vector(q+1, p-1), sig);
00116 break;
00117 default:
00118 break;
00119 }
00120 return (*this)[sig];
00121 }
00122
00124 // Reference: [P] Proposition 15.17, p 131
00125 template< class Matrix_T >
00126 void
00127 generator_table<Matrix_T>::

```



```

00128 gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig)
00129 {
00130 const auto new_size = old.size() + 2;
00131 using size_t = decltype(new_size);
00132 using result_t = std::vector<Matrix_T>;
00133 auto result = result_t(new_size);
00134
00135 const auto old_dim = old[0].size1();
00136 const auto& eye = matrix::unit<Matrix_T>(old_dim);
00137
00138 auto neg = Matrix_T(2,2,2);
00139 neg(0,1) = -1;
00140 neg(1,0) = 1;
00141
00142 auto pos = neg;
00143 pos(0,1) = 1;
00144
00145 auto dup = Matrix_T(2,2,2);
00146 dup(0,0) = 1;
00147 dup(1,1) = -1;
00148
00149 result[0] = matrix::mono_kron(neg, eye);
00150 for (auto
00151 k = size_t(1);
00152 k != new_size-1;
00153 ++k)
00154 result[k] = matrix::mono_kron(dup, old[k-1]);
00155 result[new_size-1] = matrix::mono_kron(pos, eye);
00156
00157 // Save the resulting generator array.
00158 this->insert(make_pair(sig, result));
00159 }
00160
00162 // Reference: [L] 16.4 Periodicity of 8, p216
00163 template< class Matrix_T >
00164 void
00165 generator_table<Matrix_T>::
00166 gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig)
00167 {
00168 const auto old_size = old.size();
00169 using size_t = decltype(old_size);
00170 using result_t = std::vector<Matrix_T>;
00171 auto result = result_t(old_size);
00172
00173 auto h = old[0];
00174 for (auto
00175 k = size_t(1);
00176 k != size_t(4);
00177 ++k)
00178 h = matrix::mono_prod(old[k], h);
00179
00180 for (auto
00181 k = size_t(0);
00182 k != old_size-4;
00183 ++k)
00184 result[k] = old[k+4];
00185 for (auto
00186 k = old_size-4;
00187 k != old_size;
00188 ++k)
00189 result[k] = matrix::mono_prod(old[k+4-old_size], h);
00190 // Save the resulting generator array.
00191 this->insert(make_pair(sig, result));
00192 }
00193
00195 // Reference: [L] 16.4 Periodicity of 8, p216
00196 template< class Matrix_T >
00197 void
00198 generator_table<Matrix_T>::
00199 gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig)
00200 {
00201 const auto old_size = old.size();
00202 using size_t = decltype(old_size);
00203 using result_t = std::vector<Matrix_T>;
00204 auto result = result_t(old_size);
00205
00206 auto h = old[old_size-1];
00207 for (auto
00208 k = size_t(1);
00209 k != size_t(4);
00210 ++k)
00211 h = matrix::mono_prod(old[old_size-1-k], h);
00212
00213 for (auto
00214 k = size_t(0);
00215 k != size_t(4);
00216 ++k)

```

```

00217 result[k] = matrix::mono_prod(old[k+old_size-4], h);
00218 for (auto
00219 k = size_t(4);
00220 k != old_size;
00221 ++k)
00222 result[k] = old[k-4];
00223 // Save the resulting generator array.
00224 this->insert(make_pair(sig, result));
00225 }
00226
00228 // Reference: [P] Proposition 15.20, p 131
00229 template< class Matrix_T >
00230 void
00231 generator_table<Matrix_T>::
00232 gen_from_qpl_pml(const std::vector<Matrix_T>& old, const signature_t sig)
00233 {
00234 const auto old_size = old.size();
00235 using size_t = decltype(old_size);
00236 using result_t = std::vector<Matrix_T>;
00237 auto result = result_t(old_size);
00238
00239 const auto& h = old[old_size-1];
00240 for (auto
00241 k = size_t(0);
00242 k != old_size-1;
00243 ++k)
00244 result[k] = matrix::mono_prod(old[old_size-2-k], h);
00245 result[old_size-1] = h;
00246
00247 // Save the resulting generator array.
00248 this->insert(make_pair(sig, result));
00249 }
00250
00251 } }
00252 #endif // _GLUCAT_GENERATION_IMP_H

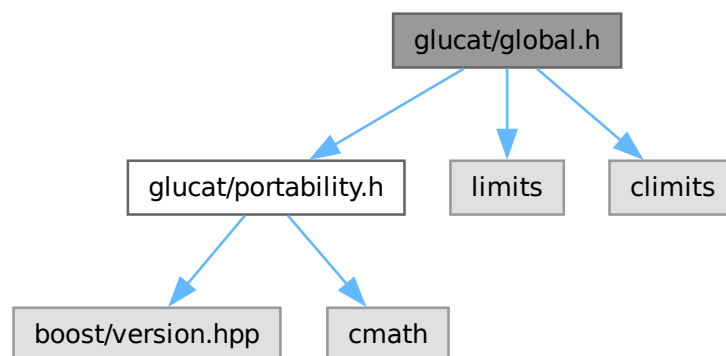
```

## 9.17 glucat/global.h File Reference

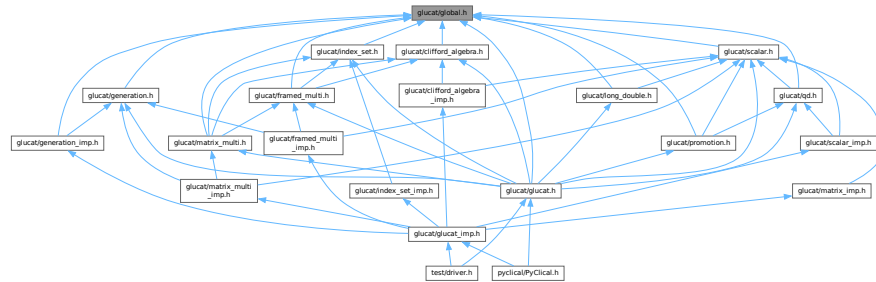
```

#include "glucat/portability.h"
#include <limits>
#include <climits>
Include dependency graph for global.h:

```



This graph shows which files directly or indirectly include this file:



## Classes

- struct `glucat::CTAssertion< true >`
- class `glucat::compare_types< LHS_T, RHS_T >`  
*Type comparison.*
- class `glucat::compare_types< T, T >`
- class `glucat::bool_to_type< truth_value >`  
*Bool to type.*

## Namespaces

- namespace **glucat**

## Macros

- #define `_GLUCAT_CTAssert`(expr, msg)

## Typedefs

- using `glucat::index_t` = int  
*Size of `index_t` should be enough to represent LO, HI.*
- using `glucat::set_value_t` = unsigned long  
*Size of `set_value_t` should be enough to contain `index_set<LO,HI>`.*

## Functions

- `glucat::_GLUCAT_CTAssert` (std::numeric\_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const `index_t` BITS\_PER\_CHAR  
*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*
- `glucat::_GLUCAT_CTAssert` (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const `index_t` DEFAULT\_LO  
*Default lowest index in an index set.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::pos_mod` (LHS\_T lhs, RHS\_T rhs) -> LHS\_T  
*Modulo function which works reliably for lhs < 0.*

## Variables

- const double `glucat::MS_PER_S` = 1000.0  
*Timing constant: deprecated here - moved to [test/timing.h](#).*
- const `index_t` `glucat::BITS_PER_SET_VALUE` = `std::numeric_limits<set_value_t>::digits`  
*Number of bits in `set_value_t`.*
- const `index_t` `glucat::DEFAULT_HI` = `index_t(BITS_PER_SET_VALUE / 2)`  
*Default highest index in an index set.*

## 9.17.1 Macro Definition Documentation

### 9.17.1.1 `_GLUCAT_CTAssert`

```
#define _GLUCAT_CTAssert(
 expr,
 msg)
```

#### Value:

```
namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
```

Definition at line 48 of file `global.h`.

Referenced by `_GLUCAT_CTAssert()`.

## 9.18 `global.h`

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLOBAL_H
00002 #define _GLUCAT_GLOBAL_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 global.h : Global declarations
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/portability.h"
00035
00036 #include <limits>
00037 #include <climits>
```

```

00038
00039 namespace glucat
00040 {
00041 // References:
00042 // [AA]: A. Alexandrescu, "Modern C++ Design", Addison-Wesley, 2001.
00043
00044 // Reference: [AA], p. 25
00045 template<bool> struct CTAAssertion;
00046 template<> struct CTAAssertion<true> { };
00047 #define _GLUCAT_CTAssert(expr, msg) \
00048 namespace { struct msg { glucat::CTAAssertion<(expr)> ERROR_##msg; }; }
00049
00050
00051 // Reference: [AA], pp. 34--37
00052 template < typename LHS_T, typename RHS_T >
00053 class compare_types
00054 {
00055 public:
00056 enum { are_same = false };
00057 };
00058 template < typename T >
00059 class compare_types<T, T>
00060 {
00061 public:
00062 enum { are_same = true };
00063 };
00064
00065 // Reference: [AA], 2.4, p. 29
00066 template< bool truth_value >
00067 class bool_to_type
00068 {
00069 private:
00070 enum { value = truth_value };
00071 };
00072
00073 // Global types which determine sizes
00074 using index_t = int;
00075 using set_value_t = unsigned long;
00076
00077 // Global constants
00078 const double MS_PER_S = 1000.0;
00079
00080 // Constants which determine sizes
00081 // Bits per unsigned long
00082 #if (ULONG_MAX == (4294967295UL))
00083 #define _GLUCAT_BITS_PER_ULONG 32
00084 #elif (ULONG_MAX == (18446744073709551615UL))
00085 #define _GLUCAT_BITS_PER_ULONG 64
00086 #elif defined(__WORDSIZE)
00087 #define _GLUCAT_BITS_PER_ULONG __WORDSIZE
00088 #endif
00089
00090 _GLUCAT_CTAssert(std::numeric_limits<unsigned char>::radix == 2, CannotDetermineBitsPerChar)
00091
00092 const index_t BITS_PER_CHAR = std::numeric_limits<unsigned char>::digits;
00093
00094 const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits;
00095
00096 _GLUCAT_CTAssert(_GLUCAT_BITS_PER_ULONG == BITS_PER_SET_VALUE, BitsPerULongDoesNotMatchSetValueT)
00097
00098 // Constants which are determined by size
00099 const index_t DEFAULT_LO = -index_t(BITS_PER_SET_VALUE / 2);
00100 const index_t DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2);
00101
00102 template< typename LHS_T, typename RHS_T >
00103 inline
00104 auto
00105 pos_mod(LHS_T lhs, RHS_T rhs) -> LHS_T
00106 { return lhs > 0? lhs % rhs : (-lhs) % rhs == 0 ? 0 : rhs - (-lhs) % rhs; }
00107
00108 }
00109 #endif // _GLUCAT_GLOBAL_H

```

## 9.19 glucat/glucat.h File Reference

```

#include "glucat/portability.h"
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"

```



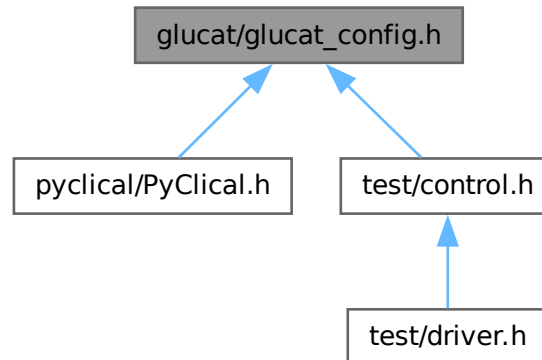
```

00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 Arvind Raja's original header comments and references follow.
00032 *****
00033 // clifford algebra package, Arvind.Raja@hut.fi
00034 // ref: Press et.al. "Numerical Recipes in C", 2nd ed., C.U.P., 1992.
00035 // ref: LEDA, v 3.0, Stefan N\aher, Max-Planck-Institut f\"ur Informatik
00036 // ref: Stroustrup B., "The C++ Programming Language", 2nd ed.,
00037 // Addison-Wesley, 1991.
00038 // ref: R. Sedgewick, "Algorithms in C++", Addison-Wesley, 1992.
00039 // ref: S. Meyers, "Effective C++ ", Addison-Wesley, 1992.
00040 *****/
00041
00042 #include "glucat/portability.h"
00043
00044 #include "glucat/global.h"
00045
00046 #include "glucat/errors.h"
00047
00048 #include "glucat/index_set.h"
00049
00050 #include "glucat/scalar.h"
00051
00052 #include "glucat/long_double.h"
00053
00054 #include "glucat/qd.h"
00055
00056 #include "glucat/promotion.h"
00057
00058 #include "glucat/random.h"
00059
00060 #include "glucat/clifford_algebra.h"
00061
00062 #include "glucat/tuning.h"
00063
00064 #include "glucat/framed_multi.h"
00065
00066 #include "glucat/generation.h"
00067
00068 #include "glucat/matrix.h"
00069
00070 #include "glucat/matrix_multi.h"
00071
00072 #endif // _GLUCAT_GLUCAT_H

```

## 9.21 glucat/glucat\_config.h File Reference

This graph shows which files directly or indirectly include this file:



### Macros

- #define [GLUCAT\\_HAVE\\_CXX11](#) 1
- #define [GLUCAT\\_HAVE\\_INTTYPES\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_STDINT\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_STDIO\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_STDLIB\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_STRINGS\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_STRING\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_SYS\\_STAT\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_SYS\\_TYPES\\_H](#) 1
- #define [GLUCAT\\_HAVE\\_UNISTD\\_H](#) 1
- #define [GLUCAT\\_PACKAGE](#) "glucat"
- #define [GLUCAT\\_PACKAGE\\_BUGREPORT](#) ""
- #define [GLUCAT\\_PACKAGE\\_NAME](#) "glucat"
- #define [GLUCAT\\_PACKAGE\\_STRING](#) "glucat 0.13.0"
- #define [GLUCAT\\_PACKAGE\\_TARNAME](#) "glucat"
- #define [GLUCAT\\_PACKAGE\\_URL](#) ""
- #define [GLUCAT\\_PACKAGE\\_VERSION](#) "0.13.0"
- #define [GLUCAT\\_STDC\\_HEADERS](#) 1
- #define [GLUCAT\\_VERSION](#) "0.13.0"

### 9.21.1 Macro Definition Documentation

#### 9.21.1.1 GLUCAT\_HAVE\_CXX11

```
#define GLUCAT_HAVE_CXX11 1
```

Definition at line 20 of file [glucat\\_config.h](#).



#### 9.21.1.2 GLUCAT\_HAVE\_INTTYPES\_H

```
#define GLUCAT_HAVE_INTTYPES_H 1
```

Definition at line 28 of file [glucat\\_config.h](#).

#### 9.21.1.3 GLUCAT\_HAVE\_STDINT\_H

```
#define GLUCAT_HAVE_STDINT_H 1
```

Definition at line 39 of file [glucat\\_config.h](#).

#### 9.21.1.4 GLUCAT\_HAVE\_STDIO\_H

```
#define GLUCAT_HAVE_STDIO_H 1
```

Definition at line 44 of file [glucat\\_config.h](#).

#### 9.21.1.5 GLUCAT\_HAVE\_STDLIB\_H

```
#define GLUCAT_HAVE_STDLIB_H 1
```

Definition at line 49 of file [glucat\\_config.h](#).

#### 9.21.1.6 GLUCAT\_HAVE\_STRING\_H

```
#define GLUCAT_HAVE_STRING_H 1
```

Definition at line 59 of file [glucat\\_config.h](#).

#### 9.21.1.7 GLUCAT\_HAVE\_STRINGS\_H

```
#define GLUCAT_HAVE_STRINGS_H 1
```

Definition at line 54 of file [glucat\\_config.h](#).

#### 9.21.1.8 GLUCAT\_HAVE\_SYS\_STAT\_H

```
#define GLUCAT_HAVE_SYS_STAT_H 1
```

Definition at line 64 of file [glucat\\_config.h](#).

#### 9.21.1.9 GLUCAT\_HAVE\_SYS\_TYPES\_H

```
#define GLUCAT_HAVE_SYS_TYPES_H 1
```

Definition at line 69 of file [glucat\\_config.h](#).

#### 9.21.1.10 GLUCAT\_HAVE\_UNISTD\_H

```
#define GLUCAT_HAVE_UNISTD_H 1
```

Definition at line 74 of file [glucat\\_config.h](#).

#### 9.21.1.11 GLUCAT\_PACKAGE

```
#define GLUCAT_PACKAGE "glucat"
```

Definition at line 79 of file [glucat\\_config.h](#).

#### 9.21.1.12 GLUCAT\_PACKAGE\_BUGREPORT

```
#define GLUCAT_PACKAGE_BUGREPORT ""
```

Definition at line 84 of file [glucat\\_config.h](#).

#### 9.21.1.13 GLUCAT\_PACKAGE\_NAME

```
#define GLUCAT_PACKAGE_NAME "glucat"
```

Definition at line 89 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).

#### 9.21.1.14 GLUCAT\_PACKAGE\_STRING

```
#define GLUCAT_PACKAGE_STRING "glucat 0.13.0"
```

Definition at line 94 of file [glucat\\_config.h](#).

#### 9.21.1.15 GLUCAT\_PACKAGE\_TARNAME

```
#define GLUCAT_PACKAGE_TARNAME "glucat"
```

Definition at line 99 of file [glucat\\_config.h](#).

#### 9.21.1.16 GLUCAT\_PACKAGE\_URL

```
#define GLUCAT_PACKAGE_URL ""
```

Definition at line 104 of file [glucat\\_config.h](#).

**9.21.1.17 GLUCAT\_PACKAGE\_VERSION**

```
#define GLUCAT_PACKAGE_VERSION "0.13.0"
```

Definition at line 109 of file [glucat\\_config.h](#).

**9.21.1.18 GLUCAT\_STDC\_HEADERS**

```
#define GLUCAT_STDC_HEADERS 1
```

Definition at line 116 of file [glucat\\_config.h](#).

**9.21.1.19 GLUCAT\_VERSION**

```
#define GLUCAT_VERSION "0.13.0"
```

Definition at line 121 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).

**9.22 glucat\_config.h**

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLUCAT_CONFIG_H
00002 #define _GLUCAT_GLUCAT_CONFIG_H 1
00003
00004 /* glucat/glucat_config.h. Generated automatically at end of configure. */
00005 /* config.h. Generated from config.h.in by configure. */
00006 /* config.h.in. Generated from configure.ac by autoheader. */
00007
00008 /* Define to dummy 'main' function (if any) required to link to the Fortran
00009 libraries. */
00010 /* #undef F77_DUMMY_MAIN */
00011
00012 /* Define if F77 and FC dummy 'main' functions are identical. */
00013 /* #undef FC_DUMMY_MAIN_EQ_F77 */
00014
00015 /* Define if you have a BLAS library. */
00016 /* #undef HAVE_BLAS */
00017
00018 /* define if the compiler supports basic C++11 syntax */
00019 #ifndef GLUCAT_HAVE_CXX11
00020 #define GLUCAT_HAVE_CXX11 1
00021 #endif
00022
00023 /* define if the compiler supports basic C++14 syntax */
00024 /* #undef HAVE_CXX14 */
00025
00026 /* Define to 1 if you have the <inttypes.h> header file. */
00027 #ifndef GLUCAT_HAVE_INTTYPES_H
00028 #define GLUCAT_HAVE_INTTYPES_H 1
00029 #endif
00030
00031 /* Define if you have LAPACK library. */
00032 /* #undef HAVE_LAPACK */
00033
00034 /* Define to 1 if you have the 'lmf' library (-lmf). */
00035 /* #undef HAVE_LIBIMF */
00036
00037 /* Define to 1 if you have the <stdint.h> header file. */
00038 #ifndef GLUCAT_HAVE_STDINT_H
00039 #define GLUCAT_HAVE_STDINT_H 1
00040 #endif
00041
00042 /* Define to 1 if you have the <stdio.h> header file. */
00043 #ifndef GLUCAT_HAVE_STDIO_H
```

```

00044 #define GLUCAT_HAVE_STDIO_H 1
00045 #endif
00046
00047 /* Define to 1 if you have the <stdlib.h> header file. */
00048 #ifndef GLUCAT_HAVE_STDLIB_H
00049 #define GLUCAT_HAVE_STDLIB_H 1
00050 #endif
00051
00052 /* Define to 1 if you have the <strings.h> header file. */
00053 #ifndef GLUCAT_HAVE_STRINGS_H
00054 #define GLUCAT_HAVE_STRINGS_H 1
00055 #endif
00056
00057 /* Define to 1 if you have the <string.h> header file. */
00058 #ifndef GLUCAT_HAVE_STRING_H
00059 #define GLUCAT_HAVE_STRING_H 1
00060 #endif
00061
00062 /* Define to 1 if you have the <sys/stat.h> header file. */
00063 #ifndef GLUCAT_HAVE_SYS_STAT_H
00064 #define GLUCAT_HAVE_SYS_STAT_H 1
00065 #endif
00066
00067 /* Define to 1 if you have the <sys/types.h> header file. */
00068 #ifndef GLUCAT_HAVE_SYS_TYPES_H
00069 #define GLUCAT_HAVE_SYS_TYPES_H 1
00070 #endif
00071
00072 /* Define to 1 if you have the <unistd.h> header file. */
00073 #ifndef GLUCAT_HAVE_UNISTD_H
00074 #define GLUCAT_HAVE_UNISTD_H 1
00075 #endif
00076
00077 /* Name of package */
00078 #ifndef GLUCAT_PACKAGE
00079 #define GLUCAT_PACKAGE "glucat"
00080 #endif
00081
00082 /* Define to the address where bug reports for this package should be sent. */
00083 #ifndef GLUCAT_PACKAGE_BUGREPORT
00084 #define GLUCAT_PACKAGE_BUGREPORT ""
00085 #endif
00086
00087 /* Define to the full name of this package. */
00088 #ifndef GLUCAT_PACKAGE_NAME
00089 #define GLUCAT_PACKAGE_NAME "glucat"
00090 #endif
00091
00092 /* Define to the full name and version of this package. */
00093 #ifndef GLUCAT_PACKAGE_STRING
00094 #define GLUCAT_PACKAGE_STRING "glucat 0.13.0"
00095 #endif
00096
00097 /* Define to the one symbol short name of this package. */
00098 #ifndef GLUCAT_PACKAGE_TARNAME
00099 #define GLUCAT_PACKAGE_TARNAME "glucat"
00100 #endif
00101
00102 /* Define to the home page for this package. */
00103 #ifndef GLUCAT_PACKAGE_URL
00104 #define GLUCAT_PACKAGE_URL ""
00105 #endif
00106
00107 /* Define to the version of this package. */
00108 #ifndef GLUCAT_PACKAGE_VERSION
00109 #define GLUCAT_PACKAGE_VERSION "0.13.0"
00110 #endif
00111
00112 /* Define to 1 if all of the C89 standard headers exist (not just the ones
00113 required in a freestanding environment). This macro is provided for
00114 backward compatibility; new code need not use it. */
00115 #ifndef GLUCAT_STDC_HEADERS
00116 #define GLUCAT_STDC_HEADERS 1
00117 #endif
00118
00119 /* Version number of package */
00120 #ifndef GLUCAT_VERSION
00121 #define GLUCAT_VERSION "0.13.0"
00122 #endif
00123
00124 /* once: _GLUCAT_GLUCAT_CONFIG_H */
00125 #endif

```

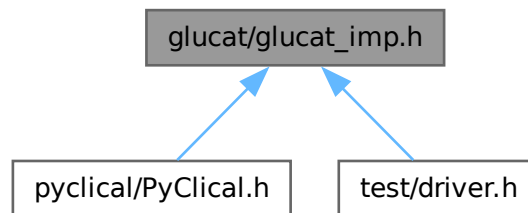
## 9.23 glucat/glucat\_imp.h File Reference

```
#include "glucat/errors_imp.h"
#include "glucat/index_set_imp.h"
#include "glucat/scalar_imp.h"
#include "glucat/clifford_algebra_imp.h"
#include "glucat/random.h"
#include "glucat/framed_multi_imp.h"
#include "glucat/matrix_imp.h"
#include "glucat/generation_imp.h"
#include "glucat/matrix_multi_imp.h"
```

Include dependency graph for glucat\_imp.h:



This graph shows which files directly or indirectly include this file:



## 9.24 glucat\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLUCAT_IMP_H
00002 #define _GLUCAT_GLUCAT_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 glucat_imp.h : Organize GluCat template definitions which cannot be compiled separately
00006 -----
00007 begin : Sun 2001-12-25
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
```

```

00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 For Arvind Raja's original header comments, see glucat.h
00032 *****/
00033
00034 // Template definitions which cannot be compiled separately
00035
00036 #include "glucat/errors_imp.h"
00037
00038 #include "glucat/index_set_imp.h"
00039
00040 #include "glucat/scalar_imp.h"
00041
00042 #include "glucat/clifford_algebra_imp.h"
00043
00044 #include "glucat/random.h"
00045
00046 #include "glucat/framed_multi_imp.h"
00047
00048 #include "glucat/matrix_imp.h"
00049
00050 #include "glucat/generation_imp.h"
00051
00052 #include "glucat/matrix_multi_imp.h"
00053
00054 #endif // _GLUCAT_GLUCAT_IMP_H

```

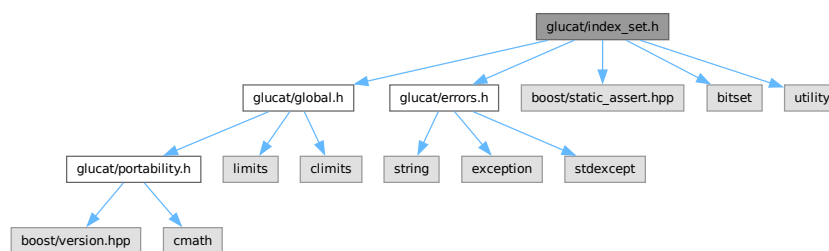
## 9.25 glucat/index\_set.h File Reference

```

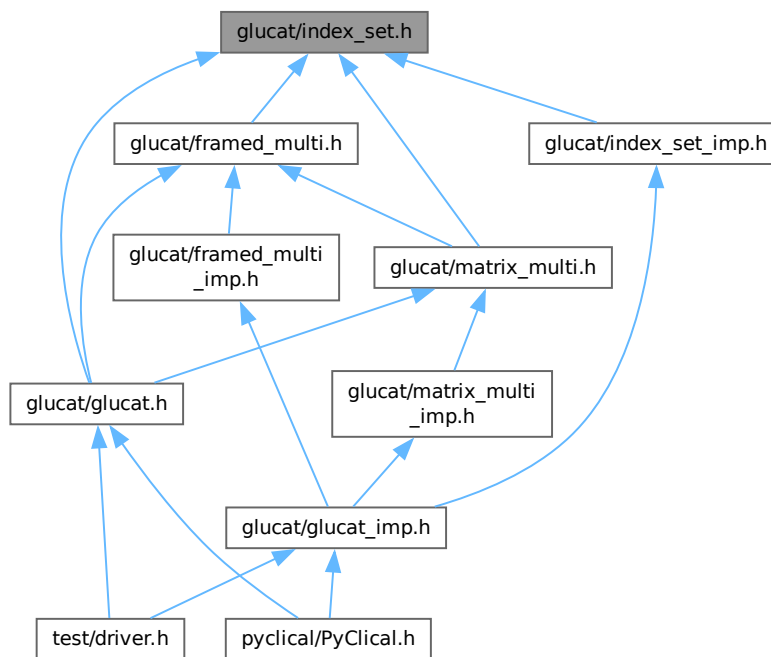
#include "glucat/global.h"
#include "glucat/errors.h"
#include <boost/static_assert.hpp>
#include <bitset>
#include <utility>

```

Include dependency graph for index\_set.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::index\\_set< LO, HI >](#)  
*Index set class based on `std::bitset<>` in Gnu standard C++ library.*
- class [glucat::index\\_set< LO, HI >::reference](#)  
*Index set member reference.*

## Namespaces

- namespace [glucat](#)

## Functions

- template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [glucat::operator^](#) (const [index\\_set](#)< LO, HI > &lhs, const [index\\_set](#)< LO, HI > &rhs) -> const [index\\_set](#)< LO, HI >  
*Symmetric set difference: exclusive or.*
- template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [glucat::operator&](#) (const [index\\_set](#)< LO, HI > &lhs, const [index\\_set](#)< LO, HI > &rhs) -> const [index\\_set](#)< LO, HI >  
*Set intersection: and.*
- template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [glucat::operator|](#) (const [index\\_set](#)< LO, HI > &lhs, const [index\\_set](#)< LO, HI > &rhs) -> const [index\\_set](#)< LO, HI >

Set union: or.

- `template<const index_t LO, const index_t HI>`  
`auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `glucat::GLUCAT_CTAssert (sizeof(set_value_t) >=sizeof(std::bitset< DEFAULT_HI-DEFAULT_LO >),`  
`Default_index_set_too_big_for_value) template< const index_t LO`  
*Size of set\_value\_t should be enough to contain bitset<DEFAULT\_HI-DEFAULT\_LO>.*
- `const index_t HI auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &`
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &`  
*Read in index set.*
- `auto glucat::sign_of_square (index_t j) -> int`  
*Square of generator {j}.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t`  
*Minimum negative index, or 0 if none.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t`  
*Maximum positive index, or 0 if none.*

## 9.26 index\_set.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_INDEX_SET_H
00002 #define _GLUCAT_INDEX_SET_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 index_set.h : Declare a class for a set of non-zero integer indices
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036
00037 #include <boost/static_assert.hpp>
00038
00039 #include <bitset>
00040 #include <utility>
00041
00042 namespace glucat
00043 {
00044 template<const index_t LO, const index_t HI>

```



```

00045 class index_set; // forward
00046
00047 template<const index_t LO, const index_t HI>
00048 auto
00049 operator^ (const index_set<LO,HI>& lhs,
00050 const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00051
00052 template<const index_t LO, const index_t HI>
00053 auto
00054 operator& (const index_set<LO,HI>& lhs,
00055 const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00056
00057 template<const index_t LO, const index_t HI>
00058 auto
00059 operator| (const index_set<LO,HI>& lhs,
00060 const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00061
00062 // -1 if a<b, +1 if a>b, 0 if a==b
00063 template<const index_t LO, const index_t HI>
00064 auto
00065 compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int;
00066
00067 template<const index_t LO, const index_t HI>
00068 class index_set :
00069 private std::bitset<HI-LO>
00070 {
00071 private:
00072 BOOST_STATIC_ASSERT((LO <= 0) && (0 <= HI) && (LO < HI) && \
00073 (-LO < _GLUCAT_BITS_PER_ULONG) && \
00074 (HI < _GLUCAT_BITS_PER_ULONG) && \
00075 (HI-LO <= _GLUCAT_BITS_PER_ULONG));
00076 using bitset_t = std::bitset<HI - LO>;
00077 using error_t = error<index_set>;
00078 public:
00079 using index_set_t = index_set;
00080 using index_pair_t = std::pair<index_t, index_t>;
00081
00082 static const index_t v_lo = LO;
00083 static const index_t v_hi = HI;
00084
00085 static auto classname() -> const std::string;
00086 index_set() = default;
00087 index_set(const bitset_t bst);
00088 index_set(const index_t idx);
00089 index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked = false);
00090 index_set(const index_pair_t& range, const bool prechecked = false);
00091 index_set(const std::string& str);
00092
00093 auto operator== (const index_set_t rhs) const -> bool;
00094 auto operator!= (const index_set_t rhs) const -> bool;
00095 auto operator~ () const -> index_set_t;
00096 auto operator^= (const index_set_t rhs) -> index_set_t&;
00097 auto operator&= (const index_set_t rhs) -> index_set_t&;
00098 auto operator|= (const index_set_t rhs) -> index_set_t&;
00099 auto operator[] (const index_t idx) const -> bool;
00100 auto test(const index_t idx) const -> bool;
00101 auto set() -> index_set_t&;
00102 auto set(const index_t idx) -> index_set_t&;
00103 auto set(const index_t idx, const int val) -> index_set_t&;
00104 auto reset() -> index_set_t&;
00105 auto reset(const index_t idx) -> index_set_t&;
00106 auto flip() -> index_set_t&;
00107 auto flip(const index_t idx) -> index_set_t&;
00108 auto count() const -> index_t;
00109 auto count_neg() const -> index_t;
00110 auto count_pos() const -> index_t;
00111 auto min() const -> index_t;
00112 auto max() const -> index_t;
00113
00114 // Functions which support Clifford algebra operations
00115 auto operator< (const index_set_t rhs) const -> bool;
00116 auto is_contiguous () const -> bool;
00117 auto fold () const -> const index_set_t;
00118 auto fold (const index_set_t frm, const bool prechecked = false) const -> const
00119 index_set_t;
00120 auto unfold (const index_set_t frm, const bool prechecked = false) const -> const
00121 index_set_t;
00122 auto value_of_fold (const index_set_t frm) const -> set_value_t;
00123 auto sign_of_mult (const index_set_t ist) const -> int;
00124 auto sign_of_square () const -> int;
00125
00126 auto hash_fn () const -> size_t;
00127
00128 // Friends
00129 friend auto operator^<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00130 friend auto operator&<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00131 friend auto operator|<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;

```

```

00170 friend auto compare<> (const index_set_t& lhs, const index_set_t& rhs) -> int;
00171
00172 // Member reference:
00173 class reference;
00174 friend class reference;
00175
00176 class reference {
00177 friend class index_set;
00178
00179 public:
00180 reference() = delete;
00181 reference (index_set_t& ist, index_t idx);
00182 ~reference () = default;
00183 auto operator== (const reference& c_j) const -> bool;
00184 auto operator= (const bool x) -> reference&;
00185 auto operator= (const reference& c_j) -> reference&;
00186 auto operator~ () const -> bool;
00187 operator bool () const;
00188 auto flip() -> reference&;
00189
00190 private:
00191 index_set_t* m_pst;
00192 index_t m_idx;
00193 };
00194 auto operator[] (index_t idx) -> reference;
00195 private:
00196 auto lex_less_than (const index_set_t rhs) const -> bool;
00197 };
00198
00199 _GLUCAT_CTAssert(sizeof(set_value_t) >= sizeof(std::bitset<DEFAULT_HI-DEFAULT_LO>),
00200 Default_index_set_too_big_for_value)
00201
00202 // non-members
00203
00204 template<const index_t LO, const index_t HI>
00205 auto
00206 operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&;
00207
00208 template<const index_t LO, const index_t HI>
00209 auto
00210 operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&;
00211
00212 // Functions which support Clifford algebra operations
00213 auto sign_of_square(index_t j) -> int;
00214
00215 template<const index_t LO, const index_t HI>
00216 auto
00217 min_neg(const index_set<LO,HI>& ist) -> index_t;
00218
00219 template<const index_t LO, const index_t HI>
00220 auto
00221 max_pos(const index_set<LO,HI>& ist) -> index_t;
00222 }
00223 #endif // _GLUCAT_INDEX_SET_H

```

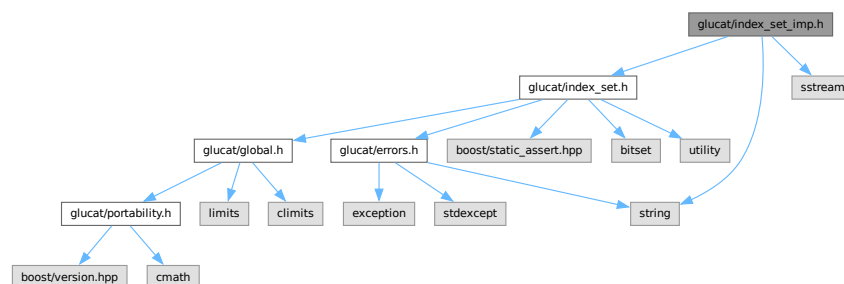
## 9.27 glucat/index\_set\_imp.h File Reference

```
#include "glucat/index_set.h"
```

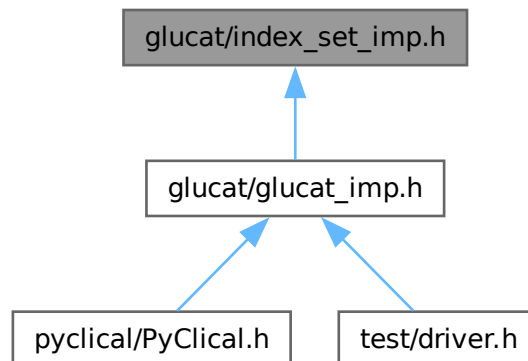
```
#include <string>
```

```
#include <sstream>
```

Include dependency graph for index\_set\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## Functions

- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator^ (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >`  
*Symmetric set difference: exclusive or.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator& (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >`  
*Set intersection: and.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >`  
*Set union: or.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &`  
*Write out index set.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &`  
*Read in index set.*
- `static auto glucat::inverse_reversed_gray (unsigned long x) -> unsigned long`  
*Inverse reversed Gray code.*
- `static auto glucat::inverse_gray (unsigned long x) -> unsigned long`  
*Inverse Gray code.*
- `auto glucat::sign_of_square (index_t j) -> int`

*Square of generator [j].*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t`  
*Minimum negative index, or 0 if none.*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t`  
*Maximum positive index, or 0 if none.*

## 9.28 index\_set\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_INDEX_SET_IMP_H
00002 #define _GLUCAT_INDEX_SET_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 index_set_imp.h : Implement a class for a set of non-zero integer indices
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/index_set.h"
00035
00036 #include <string>
00037 #include <sstream>
00038
00039 namespace glucat
00040 {
00041 // References for algorithms:
00042 // [JA]: Joerg Arndt, "Algorithms for programmers", http://www.jjj.de/fxt/fxtbook.pdf
00043 // Chapter 1, Bit wizardry, http://www.jjj.de/bitwizdary/bitwizdarypage.html
00044 // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00045
00046 template<const index_t LO, const index_t HI>
00047 inline
00048 auto
00049 index_set<LO,HI>::
00050 classname() -> const std::string
00051 { return "index_set"; }
00052
00053 template<const index_t LO, const index_t HI>
00054 index_set<LO,HI>::
00055 index_set(const index_t idx)
00056 { this->set(idx); }
00057
00058 template<const index_t LO, const index_t HI>
00059 index_set<LO,HI>::
00060 index_set(const bitset_t bst):
00061 bitset_t(bst)
00062 { }
00063
00064 template<const index_t LO, const index_t HI>
00065 index_set<LO,HI>::
00066 index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked)

```

```

00070 {
00071 if (!prechecked && folded_val >= (set_value_t(1) « frm.count()))
00072 throw error_t("index_set(val,frm): cannot create: value gives an index set outside of frame");
00073 const index_set_t folded_frame = frm.fold();
00074 const index_t min_index = folded_frame.min();
00075 const index_t skip = min_index > 0 ? 1 : 0;
00076 const index_set_t folded_set = index_set_t(bitset_t(folded_val) « (min_index - skip - LO));
00077 *this = folded_set.unfold(frm);
00078 }
00079
00080 template<const index_t LO, const index_t HI>
00081 index_set<LO,HI>::
00082 index_set(const index_pair_t& range, const bool prechecked)
00083 {
00084 if (!prechecked && (range.first < LO || range.second > HI))
00085 throw error_t("index_set(range): cannot create: range is too large");
00086 const index_t begin_bit = (range.first < 0)
00087 ? range.first-LO
00088 : range.first-LO-1;
00089 const index_t end_bit = (range.second < 0)
00090 ? range.second-LO+1
00091 : range.second-LO;
00092 unsigned long mask = ((end_bit == _GLUCAT_BITS_PER_ULONGLONG)
00093 ? -1UL
00094 : (1UL « end_bit)-1UL)
00095 & ~(1UL « begin_bit)-1UL);
00096 *this = bitset_t(mask);
00097 }
00098
00099 template<const index_t LO, const index_t HI>
00100 index_set<LO,HI>::
00101 index_set(const std::string& str)
00102 {
00103 std::istringstream ss(str);
00104 ss » *this;
00105 if (!ss)
00106 throw error_t("index_set_t(str): could not parse string");
00107 // Peek to see if the end of the string has been reached.
00108 ss.peek();
00109 if (!ss.eof())
00110 throw error_t("index_set_t(str): could not parse entire string");
00111 }
00112
00113 template<const index_t LO, const index_t HI>
00114 inline
00115 auto
00116 index_set<LO,HI>::
00117 operator== (const index_set_t rhs) const -> bool
00118 {
00119 const auto* pthis = static_cast<const bitset_t*>(this);
00120 return *pthis == static_cast<bitset_t>(rhs);
00121 }
00122
00123 template<const index_t LO, const index_t HI>
00124 inline
00125 auto
00126 index_set<LO,HI>::
00127 operator!= (const index_set_t rhs) const -> bool
00128 {
00129 const auto* pthis = static_cast<const bitset_t*>(this);
00130 return *pthis != static_cast<bitset_t>(rhs);
00131 }
00132
00133 template<const index_t LO, const index_t HI>
00134 inline
00135 auto
00136 index_set<LO,HI>::
00137 operator~ () const -> index_set_t
00138 { return bitset_t::operator~(); }
00139
00140 template<const index_t LO, const index_t HI>
00141 inline
00142 auto
00143 index_set<LO,HI>::
00144 operator^= (const index_set_t rhs) -> index_set_t&
00145 {
00146 bitset_t* pthis = this;
00147 *pthis ^= static_cast<bitset_t>(rhs);
00148 return *this;
00149 }
00150
00151 template<const index_t LO, const index_t HI>
00152 inline
00153 auto
00154 index_set<LO,HI>::
00155 operator^ (const index_set<LO,HI>& lhs,
00156 const index_set<LO,HI>& rhs) -> const
00157 index_set<LO,HI>

```

```

00164 {
00165 using index_set_t = index_set<LO, HI>;
00166 using bitset_t = typename index_set_t::bitset_t;
00167 return static_cast<bitset_t>(lhs) ^ static_cast<bitset_t>(rhs);
00168 }
00169
00171 template<const index_t LO, const index_t HI>
00172 inline
00173 auto
00174 index_set<LO, HI>::
00175 operator&= (const index_set_t rhs) -> index_set_t&
00176 {
00177 bitset_t* pthis = this;
00178 *pthis &= static_cast<bitset_t>(rhs);
00179 return *this;
00180 }
00181
00183 template<const index_t LO, const index_t HI>
00184 inline
00185 auto
00186 operator& (const index_set<LO, HI>& lhs,
00187 const index_set<LO, HI>& rhs) -> const
00188 index_set<LO, HI>
00189 {
00190 using index_set_t = index_set<LO, HI>;
00191 using bitset_t = typename index_set_t::bitset_t;
00192 return static_cast<bitset_t>(lhs) & static_cast<bitset_t>(rhs);
00193 }
00194
00196 template<const index_t LO, const index_t HI>
00197 inline
00198 auto
00199 index_set<LO, HI>::
00200 operator|= (const index_set_t rhs) -> index_set_t&
00201 {
00202 bitset_t* pthis = this;
00203 *pthis |= static_cast<bitset_t>(rhs);
00204 return *this;
00205 }
00206
00208 template<const index_t LO, const index_t HI>
00209 inline
00210 auto
00211 operator| (const index_set<LO, HI>& lhs,
00212 const index_set<LO, HI>& rhs) -> const
00213 index_set<LO, HI>
00214 {
00215 using index_set_t = index_set<LO, HI>;
00216 using bitset_t = typename index_set_t::bitset_t;
00217 return static_cast<bitset_t>(lhs) | static_cast<bitset_t>(rhs);
00218 }
00219
00221 template<const index_t LO, const index_t HI>
00222 inline
00223 auto
00224 index_set<LO, HI>::
00225 operator[] (const index_t idx) -> reference
00226 { return reference(*this, idx); }
00227
00229 template<const index_t LO, const index_t HI>
00230 inline
00231 auto
00232 index_set<LO, HI>::
00233 operator[] (const index_t idx) const -> bool
00234 { return this->test(idx); }
00235
00237 template<const index_t LO, const index_t HI>
00238 inline
00239 auto
00240 index_set<LO, HI>::
00241 test(const index_t idx) const -> bool
00242 {
00243 // Reference: [JA], 1.2.1
00244 return (idx < 0)
00245 ? bool(bitset_t::to_ulong() & (1UL << (idx - LO)))
00246 : (idx > 0)
00247 ? bool(bitset_t::to_ulong() & (1UL << (idx - LO - 1)))
00248 : false;
00249 }
00250
00252 template<const index_t LO, const index_t HI>
00253 inline
00254 auto
00255 index_set<LO, HI>::
00256 set() -> index_set_t&
00257 {
00258 bitset_t::set();

```

```

00259 return *this;
00260 }
00261
00263 template<const index_t LO, const index_t HI>
00264 inline
00265 auto
00266 index_set<LO,HI>::
00267 set(index_t idx) -> index_set_t&
00268 {
00269 if (idx > 0)
00270 bitset_t::set(idx-LO-1);
00271 else if (idx < 0)
00272 bitset_t::set(idx-LO);
00273 return *this;
00274 }
00275
00277 template<const index_t LO, const index_t HI>
00278 inline
00279 auto
00280 index_set<LO,HI>::
00281 set(const index_t idx, const int val) -> index_set_t&
00282 {
00283 if (idx > 0)
00284 bitset_t::set(idx-LO-1, val);
00285 else if (idx < 0)
00286 bitset_t::set(idx-LO, val);
00287 return *this;
00288 }
00289
00291 template<const index_t LO, const index_t HI>
00292 inline
00293 auto
00294 index_set<LO,HI>::
00295 reset() -> index_set_t&
00296 {
00297 bitset_t::reset();
00298 return *this;
00299 }
00300
00302 template<const index_t LO, const index_t HI>
00303 inline
00304 auto
00305 index_set<LO,HI>::
00306 reset(const index_t idx) -> index_set_t&
00307 {
00308 if (idx > 0)
00309 bitset_t::reset(idx-LO-1);
00310 else if (idx < 0)
00311 bitset_t::reset(idx-LO);
00312 return *this;
00313 }
00314
00316 template<const index_t LO, const index_t HI>
00317 inline
00318 auto
00319 index_set<LO,HI>::
00320 flip() -> index_set<LO,HI>&
00321 {
00322 bitset_t::flip();
00323 return *this;
00324 }
00325
00327 template<const index_t LO, const index_t HI>
00328 inline
00329 auto
00330 index_set<LO,HI>::
00331 flip(const index_t idx) -> index_set_t&
00332 {
00333 if (idx > 0)
00334 bitset_t::flip(idx-LO-1);
00335 else if (idx < 0)
00336 bitset_t::flip(idx-LO);
00337 return *this;
00338 }
00339
00341 template<const index_t LO, const index_t HI>
00342 inline
00343 auto
00344 index_set<LO,HI>::
00345 count() const -> index_t
00346 {
00347 unsigned long val = bitset_t::to_ulong();
00348 // Reference: [JA], 1.3
00349 if (val == 0)
00350 return 0;
00351 else
00352 {

```

```

00353 index_t result = 1;
00354 while (val &= val-1)
00355 ++result;
00356 return result;
00357 }
00358 }
00359
00361 template<const index_t LO, const index_t HI>
00362 inline
00363 auto
00364 index_set<LO,HI>::
00365 count_neg() const -> index_t
00366 {
00367 static const index_set_t lo_mask = bitset_t((1UL < -LO) - 1UL);
00368 const index_set_t neg_part = *this & lo_mask;
00369 return neg_part.count();
00370 }
00371
00373 template<const index_t LO, const index_t HI>
00374 inline
00375 auto
00376 index_set<LO,HI>::
00377 count_pos() const -> index_t
00378 {
00379 const auto* pthis = static_cast<const bitset_t*>(this);
00380 const index_set_t pos_part = *pthis > -LO;
00381 return pos_part.count();
00382 }
00383
00384 #if (_GLUCAT_BITS_PER_ULONG == 64)
00386 template<const index_t LO, const index_t HI>
00387 inline
00388 auto
00389 index_set<LO,HI>::
00390 min() const -> index_t
00391 {
00392 // Reference: [JA], 1.3
00393 unsigned long val = bitset_t::to_ulong();
00394 if (val == 0)
00395 return 0;
00396 else
00397 {
00398 val -= val & (val-1); // isolate lowest bit
00399
00400 index_t idx = 0;
00401 const index_t nbits = HI - LO;
00402
00403 if (nbits > 8)
00404 {
00405 if (val & 0xffffffff00000000ul)
00406 idx += 32;
00407 if (val & 0xffff0000ffff0000ul)
00408 idx += 16;
00409 if (val & 0xff00ff00ff00ff00ul)
00410 idx += 8;
00411 }
00412 if (val & 0xf0f0f0f0f0f0f0f0ul)
00413 idx += 4;
00414 if (val & 0xcccccccccccccccul)
00415 idx += 2;
00416 if (val & 0aaaaaaaaaaaaaaul)
00417 idx += 1;
00418
00419 return idx + ((idx < -LO) ? LO : LO+1);
00420 }
00421 }
00422 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00424 template<const index_t LO, const index_t HI>
00425 inline
00426 index_t
00427 index_set<LO,HI>::
00428 min() const
00429 {
00430 // Reference: [JA], 1.3
00431 unsigned long val = bitset_t::to_ulong();
00432 if (val == 0)
00433 return 0;
00434 else
00435 {
00436 val -= val & (val-1); // isolate lowest bit
00437
00438 index_t idx = 0;
00439 const index_t nbits = HI - LO;
00440 if (nbits > 8)
00441 {
00442 if (val & 0xffff0000ul)
00443 idx += 16;

```



```

00444 if (val & 0xff00ff00ul)
00445 idx += 8;
00446 }
00447 if (val & 0xf0f0f0f0ul)
00448 idx += 4;
00449 if (val & 0xcccccccul)
00450 idx += 2;
00451 if (val & 0xaaaaaaaaul)
00452 idx += 1;
00453
00454 return idx + ((idx < -LO) ? LO : LO+1);
00455 }
00456 }
00457 #else
00458 template<const index_t LO, const index_t HI>
00459 auto
00460 index_set<LO,HI>::
00461 min() const -> index_t
00462 {
00463 for (auto
00464 idx = LO;
00465 idx != 0;
00466 ++idx)
00467 if (this->test(idx))
00468 return idx;
00469 for (auto
00470 idx = index_t(1);
00471 idx <= HI;
00472 ++idx)
00473 if (this->test(idx))
00474 return idx;
00475 return 0;
00476 }
00477 }
00478 #endif
00479
00480 #if (_GLUCAT_BITS_PER_ULONG == 64)
00481 template<const index_t LO, const index_t HI>
00482 inline
00483 auto
00484 index_set<LO,HI>::
00485 max() const -> index_t
00486 {
00487 // Reference: [JA], 1.6
00488 auto val = bitset_t::to_ulong();
00489 if (val == 0)
00490 return 0;
00491 else
00492 {
00493 auto idx = index_t(0);
00494 const auto nbits = HI - LO;
00495 if (nbits > 8)
00496 {
00497 if (val & 0xffffffff00000000ul)
00498 { val >>= 32; idx += 32; }
00499 if (val & 0x00000000ffff0000ul)
00500 { val >>= 16; idx += 16; }
00501 if (val & 0x000000000000ff00ul)
00502 { val >>= 8; idx += 8; }
00503 }
00504 if (val & 0x00000000000000f0ul)
00505 { val >>= 4; idx += 4; }
00506 if (val & 0x000000000000000cul)
00507 { val >>= 2; idx += 2; }
00508 if (val & 0x0000000000000002ul)
00509 { idx += 1; }
00510 return idx + ((idx < -LO) ? LO : LO+1);
00511 }
00512 }
00513 }
00514 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00515 template<const index_t LO, const index_t HI>
00516 inline
00517 auto
00518 index_set<LO,HI>::
00519 max() const -> index_t
00520 {
00521 // Reference: [JA], 1.6
00522 auto val = bitset_t::to_ulong();
00523 if (val == 0)
00524 return 0;
00525 else
00526 {
00527 auto idx = index_t(0);
00528 const auto nbits = HI - LO;
00529 if (nbits > 8)
00530 {
00531 if (val & 0xffff0000ul)
00532 { val >>= 16; idx += 16; }

```

```

00534 if (val & 0x0000fff00ul)
00535 { val >>= 8; idx += 8; }
00536 }
00537 if (val & 0x000000f0ul)
00538 { val >>= 4; idx += 4; }
00539 if (val & 0x0000000cul)
00540 { val >>= 2; idx += 2; }
00541 if (val & 0x00000002ul)
00542 { idx += 1; }
00543 return idx + ((idx < -LO) ? LO : LO+1);
00544 }
00545 }
00546 #else
00547 template<const index_t LO, const index_t HI>
00548 auto
00549 index_set<LO,HI>::
00550 max() const -> index_t
00551 {
00552 for (auto
00553 idx = HI;
00554 idx != 0;
00555 --idx)
00556 if (this->test(idx))
00557 return idx;
00558 for (auto
00559 idx = index_t(-1);
00560 idx >= LO;
00561 --idx)
00562 if (this->test(idx))
00563 return idx;
00564 return 0;
00565 }
00566 #endif
00567 // eg. {3,4,5} is less than {3,7,8}
00571 template<const index_t LO, const index_t HI>
00572 inline
00573 auto
00574 compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int
00575 {
00576 return (a == b)
00577 ? 0
00578 : a.lex_less_than(b)
00579 ? -1
00580 : 1;
00581 }
00582 // eg. {3,4,5} is less than {3,7,8}
00584 template<const index_t LO, const index_t HI>
00585 inline
00586 auto
00587 index_set<LO,HI>::
00588 lex_less_than(const index_set_t rhs) const -> bool
00589 { return bitset_t::to_ulong() < rhs.bitset_t::to_ulong(); }
00591 // Order by count, then order lexicographically within the equivalence class of count.
00593 template<const index_t LO, const index_t HI>
00594 inline
00595 auto
00596 index_set<LO,HI>::
00597 operator< (const index_set_t rhs) const -> bool
00598 {
00599 const auto this_grade = this->count();
00600 const auto rhs_grade = rhs.count();
00601 return (this_grade < rhs_grade)
00602 ? true
00603 : (this_grade > rhs_grade)
00604 ? false
00605 : this->lex_less_than(rhs);
00606 }
00607
00608 template<const index_t LO, const index_t HI>
00611 auto
00612 operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&
00613 {
00614 index_t i;
00615 os << ' ';
00616 for (i = LO;
00617 (i <= HI) && !(ist[i]);
00618 ++i)
00619 { }
00620 if (i <= HI)
00621 os << i;
00622 for (++i;
00623 i <= HI;
00624 ++i)
00625 if (ist[i])

```

```

00626 os << ',' << i;
00627 os << '>';
00628 return os;
00629 }
00630
00631 template<const index_t LO, const index_t HI>
00632 auto
00633 operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&
00634 {
00635 // Parsing variables.
00636 auto i = index_t(0);
00637 using index_set_t = index_set<LO,HI>;
00638 auto local_ist = index_set_t();
00639 // Parsing control variables.
00640 auto parse_index_list = true;
00641 auto expect_closing_brace = false;
00642 auto expect_index = false;
00643 // Parse an optional opening brace.
00644 auto c = s.peek();
00645 // If there is a failure or end of file, this ends parsing.
00646 if (!s.good())
00647 parse_index_list = false;
00648 else
00649 { // Check for an opening brace.
00650 expect_closing_brace = (c == int('{'));
00651 if (expect_closing_brace)
00652 { // Consume the opening brace.
00653 s.get();
00654 // The next character may be a closing brace,
00655 // indicating the empty index set.
00656 c = s.peek();
00657 if (s.good() && (c == int('}')))
00658 { // A closing brace has been parsed and is no longer expected.
00659 expect_closing_brace = false;
00660 // Consume the closing brace.
00661 s.get();
00662 // This ends parsing.
00663 parse_index_list = false;
00664 }
00665 }
00666 }
00667 if (s.good() && parse_index_list)
00668 { // Parse an optional index list.
00669 // The index list starts with a first index.
00670 for (s >> i;
00671 !s.fail();
00672 s >> i)
00673 { // An index has been parsed. Check to see if it is in range.
00674 if ((i < LO) || (i > HI))
00675 { // An index out of range is a failure.
00676 s.clear(std::istream::failbit);
00677 break;
00678 }
00679 // Add the index to the index set local_ist.
00680 local_ist.set(i);
00681 // Immediately after parsing an index, an index is no longer expected.
00682 expect_index = false;
00683 // Reading the index may have resulted in an end of file condition.
00684 // If so, this ends the index list.
00685 if (s.eof())
00686 break;
00687 // The index list continues with a comma, and
00688 // may be ended by a closing brace, if it was begun with an opening brace.
00689 // Parse a possible comma or closing brace.
00690 c = s.peek();
00691 if (!s.good())
00692 break;
00693 // First, test for a closing brace, if expected.
00694 if (expect_closing_brace && (c == int('}')))
00695 { // Consume the closing brace.
00696 s.get();
00697 // Immediately after parsing the closing brace, it is no longer expected.
00698 expect_closing_brace = false;
00699 // A closing brace ends the index list.
00700 break;
00701 }
00702 // Now test for a comma.
00703 if (c == int(','))
00704 { // Consume the comma.
00705 s.get();
00706 // A index is expected after the comma.
00707 expect_index = true;
00708 }
00709 else
00710 { // Any other character here is a failure.
00711 s.clear(std::istream::failbit);
00712 break;
00713 }

```

```

00714 }
00715 }
00716 }
00717 // If an index or a closing brace is still expected, this is a failure.
00718 if (expect_index || expect_closing_brace)
00719 s.clear(std::istream::failbit);
00720 // End of file is not a failure.
00721 if (s)
00722 { // The index set has been successfully parsed.
00723 ist = local_ist;
00724 }
00725 return s;
00726 }
00727
00728 template<const index_t LO, const index_t HI>
00729 inline
00730 auto
00731 index_set<LO,HI>::
00732 is_contiguous () const -> bool
00733 {
00734 {
00735 const auto min_index = this->min();
00736 const auto max_index = this->max();
00737 return (min_index < 0 && max_index > 0)
00738 ? max_index - min_index == this->count()
00739 : (min_index == 1 || max_index == -1) &&
00740 (max_index - min_index == this->count() - 1);
00741 }
00742
00743 template<const index_t LO, const index_t HI>
00744 inline
00745 auto
00746 index_set<LO,HI>::
00747 fold() const -> const
00748 index_set<LO,HI>
00749 { return this->fold(*this, true); }
00750
00751
00752 template<const index_t LO, const index_t HI>
00753 auto
00754 index_set<LO,HI>::
00755 fold(const index_set_t frm, const bool prechecked) const -> const
00756 index_set<LO,HI>
00757 {
00758 {
00759 if (!prechecked && ((*this | frm) != frm))
00760 throw error_t("fold(frm): cannot fold from outside of frame");
00761 const auto frm_min = frm.min();
00762 const auto frm_max = frm.max();
00763 auto result = index_set_t();
00764 auto fold_idx = index_t(-1);
00765 for (auto
00766 unfold_idx = fold_idx;
00767 unfold_idx >= frm_min;
00768 --unfold_idx)
00769 {
00770 if (frm.test(unfold_idx))
00771 // result.set(fold_idx--, this->test(unfold_idx));
00772 {
00773 if (this->test(unfold_idx))
00774 result.set(fold_idx);
00775 --fold_idx;
00776 }
00777 }
00778 fold_idx = index_t(1);
00779 for (auto
00780 unfold_idx = fold_idx;
00781 unfold_idx <= frm_max;
00782 ++unfold_idx)
00783 {
00784 if (frm.test(unfold_idx))
00785 // result.set(fold_idx++, this->test(unfold_idx));
00786 {
00787 if (this->test(unfold_idx))
00788 result.set(fold_idx);
00789 ++fold_idx;
00790 }
00791 }
00792 return result;
00793 }
00794
00795 template<const index_t LO, const index_t HI>
00796 auto
00797 index_set<LO,HI>::
00798 unfold(const index_set_t frm, const bool prechecked) const -> const index_set_t
00799 {
00800 {
00801 const char* msg =
00802 "unfold(frm): cannot unfold into a smaller frame";
00803 const auto frm_min = frm.min();
00804 const auto frm_max = frm.max();
00805 auto result = index_set_t();
00806 auto fold_idx = index_t(-1);
00807 for (auto
00808 unfold_idx = fold_idx;

```

```

00805 unfold_idx >= frm_min;
00806 --unfold_idx)
00807 if (frm.test(unfold_idx))
00808 if (this->test(fold_idx--))
00809 result.set(unfold_idx);
00810 if (!prechecked && ((fold_idx+1) > this->min()))
00811 throw error_t(msg);
00812 fold_idx = index_t(1);
00813 for (auto
00814 unfold_idx = fold_idx;
00815 unfold_idx <= frm_max;
00816 ++unfold_idx)
00817 if (frm.test(unfold_idx))
00818 if (this->test(fold_idx++))
00819 result.set(unfold_idx);
00820 if (!prechecked && ((fold_idx-1) < this->max()))
00821 throw error_t(msg);
00822 return result;
00823 }
00824
00826 template<const index_t LO, const index_t HI>
00827 inline
00828 auto
00829 index_set<LO,HI>::
00830 value_of_fold(const index_set_t frm) const -> set_value_t
00831 {
00832 const auto min_index = frm.fold().min();
00833 if (min_index == 0)
00834 return 0;
00835 else
00836 {
00837 const auto folded_set = this->fold(frm);
00838 const auto skip = min_index > 0 ? index_t(1) : index_t(0);
00839 return folded_set.bitset_t::to_ulong() » (min_index-LO-skip);
00840 }
00841 }
00842
00844 inline
00845 static
00846 auto inverse_reversed_gray(unsigned long x) -> unsigned long
00847 {
00848 // Reference: [JA]
00849 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00850 x ^= x « 32; // for 64-bit words
00851 #endif
00852 x ^= x « 16; // reversed_gray ** 16
00853 x ^= x « 8; // reversed_gray ** 8
00854 x ^= x « 4; // reversed_gray ** 4
00855 x ^= x « 2; // reversed_gray ** 2
00856 x ^= x « 1; // reversed_gray ** 1
00857 return x;
00858 }
00859
00861 inline
00862 static
00863 auto inverse_gray(unsigned long x) -> unsigned long
00864 {
00865 // Reference: [JA]
00866 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00867 x ^= x » 32; // for 64-bit words
00868 #endif
00869 x ^= x » 16; // gray ** 16
00870 x ^= x » 8; // gray ** 8
00871 x ^= x » 4; // gray ** 4
00872 x ^= x » 2; // gray ** 2
00873 x ^= x » 1; // gray ** 1
00874 return x;
00875 }
00876
00878 template<const index_t LO, const index_t HI>
00879 auto
00880 index_set<LO,HI>::
00881 sign_of_mult(const index_set_t rhs) const -> int
00882 {
00883 // Implemented using Walsh functions and Gray codes.
00884 // Reference: [L] Chapter 21, 21.3
00885 // Reference: [JA]
00886 const auto uthis = this->bitset_t::to_ulong();
00887 const auto urhs = rhs.bitset_t::to_ulong();
00888 const auto nbits = HI - LO;
00889 auto negative = 0UL;
00890 if (nbits > 8)
00891 {
00892 // Set h to be the inverse reversed Gray code of rhs.
00893 // This sets each bit of h to be the cumulative ^ of
00894 // the same and lower bits of rhs.
00895 const auto h = inverse_reversed_gray(urhs);

```

```

00896 // Set k to be the inverse Gray code of *this & h.
00897 // This sets the low bit of k to be parity(*this & h).
00898 const auto k = inverse_gray(uthis & h);
00899 // Set q to be the inverse Gray code of the positive part of *this & rhs.
00900 const auto q = inverse_gray((uthis & urhs) >> -LO);
00901 negative = k ^ q;
00902 }
00903 else
00904 {
00905 auto h = 0UL;
00906 for (auto
00907 j = index_t(0);
00908 j < -LO;
00909 ++j)
00910 {
00911 h ^= urhs >> j;
00912 negative ^= h & (uthis >> j);
00913 }
00914 for (auto
00915 j = index_t(-LO);
00916 j < nbits;
00917 ++j)
00918 {
00919 negative ^= h & (uthis >> j);
00920 h ^= urhs >> j;
00921 }
00922 }
00923 return 1 - int((negative & 1) << 1);
00924 }
00925
00926 template<const index_t LO, const index_t HI>
00927 inline
00928 auto
00929 index_set<LO,HI>::
00930 sign_of_square() const -> int
00931 {
00932 auto result = 1 - int((this->count_neg() % 2) << 1);
00933 switch (this->count() % 4)
00934 {
00935 case 2:
00936 case 3:
00937 result *= -1;
00938 break;
00939 default:
00940 break;
00941 }
00942 return result;
00943 }
00944
00945 template<const index_t LO, const index_t HI>
00946 inline
00947 auto
00948 index_set<LO,HI>::
00949 hash_fn() const -> size_t
00950 {
00951 static const auto lo_mask = (1UL << -LO) - 1UL;
00952 const auto uthis = bitset_t::to_ulong();
00953 const auto neg_part = uthis & lo_mask;
00954 const auto pos_part = uthis >> -LO;
00955 return size_t(neg_part ^ pos_part);
00956 }
00957
00958 inline
00959 auto
00960 sign_of_square(index_t j) -> int
00961 { return (j < 0) ? -1 : 1; }
00962
00963 template<const index_t LO, const index_t HI>
00964 inline
00965 auto
00966 min_neg(const index_set<LO,HI>& ist) -> index_t
00967 { return std::min(ist.min(), 0); }
00968
00969 template<const index_t LO, const index_t HI>
00970 inline
00971 auto
00972 max_pos(const index_set<LO,HI>& ist) -> index_t
00973 { return std::max(ist.max(), 0); }
00974
00975 // index_set reference
00976 template<const index_t LO, const index_t HI>
00977 inline
00978 auto
00979 index_set<LO,HI>::reference::
00980 reference(index_set_t& ist, index_t idx) :
00981 m_pst(&ist),
00982 m_idx(idx)

```

```

00989 { }
00990
00992 template<const index_t LO, const index_t HI>
00993 inline
00994 auto
00995 index_set<LO,HI>::reference::
00996 operator== (const reference& c_j) const -> bool
00997 { return m_pst == c_j.m_pst && m_idx == c_j.m_idx; }
00998
01000 template<const index_t LO, const index_t HI>
01001 inline
01002 auto
01003 index_set<LO,HI>::reference::
01004 operator= (bool x) -> reference&
01005 {
01006 if (x)
01007 m_pst->set(m_idx);
01008 else
01009 m_pst->reset(m_idx);
01010 return *this;
01011 }
01012
01014 template<const index_t LO, const index_t HI>
01015 inline
01016 auto
01017 index_set<LO,HI>::reference::
01018 operator= (const reference& c_j) -> reference&
01019 {
01020 if (&c_j != this && c_j != *this)
01021 {
01022 if ((c_j.m_pst)[c_j.m_idx])
01023 m_pst->set(m_idx);
01024 else
01025 m_pst->reset(m_idx);
01026 }
01027 return *this;
01028 }
01029
01031 template<const index_t LO, const index_t HI>
01032 inline
01033 auto
01034 index_set<LO,HI>::reference::
01035 operator~ () const -> bool
01036 { return !(m_pst->test(m_idx)); }
01037
01039 template<const index_t LO, const index_t HI>
01040 inline
01041 index_set<LO,HI>::reference::
01042 operator bool () const
01043 { return m_pst->test(m_idx); }
01044
01046 template<const index_t LO, const index_t HI>
01047 inline
01048 auto
01049 index_set<LO,HI>::reference::
01050 flip() -> reference&
01051 {
01052 m_pst->flip(m_idx);
01053 return *this;
01054 }
01055 }
01056 #endif // _GLUCAT_INDEX_SET_IMP_H

```

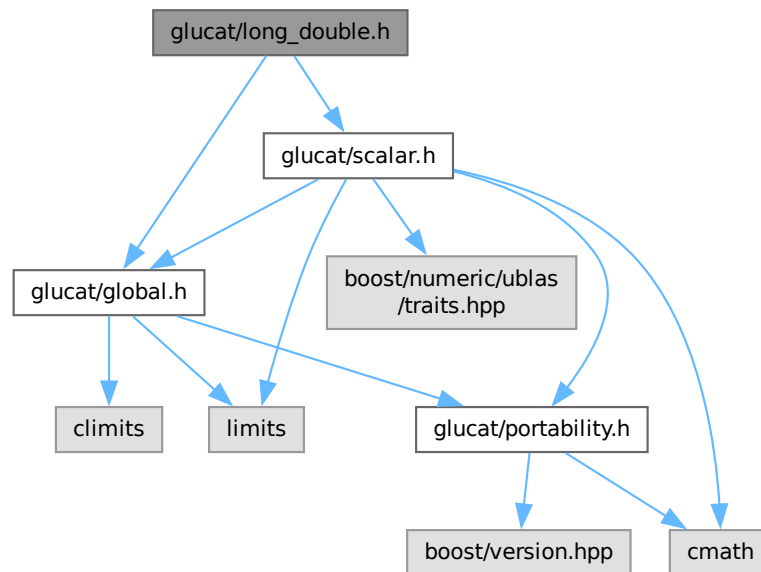
## 9.29 glucat/long\_double.h File Reference

```

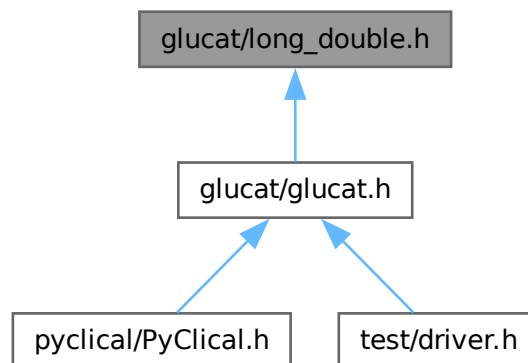
#include "glucat/global.h"
#include "glucat/scalar.h"

```

Include dependency graph for `long_double.h`:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## Variables

- static const long double `glucat::l_pi` = 3.1415926535897932384626433832795029L
- static const long double `glucat::l_ln2` = 0.6931471805599453094172321214581766L



## 9.30 long\_double.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_LONG_DOUBLE_H
00002 #define _GLUCAT_LONG_DOUBLE_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 long_double.h : Define std functions for long double
00006 -----
00007 begin : 2001-12-18
00008 copyright : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 namespace glucat
00038 {
00039 #if defined(__USE_GNU)
00040 static const long double l_pi = M_PI1;
00041 static const long double l_ln2 = M_LN21;
00042 #else
00043 static const long double l_pi = 3.1415926535897932384626433832795029L;
00044 static const long double l_ln2 = 0.6931471805599453094172321214581766L;
00045 #endif
00046
00047 template<>
00048 inline
00049 auto
00050 numeric_traits<long double>::
00051 pi() -> long double
00052 { return l_pi; }
00053
00054 template<>
00055 inline
00056 auto
00057 numeric_traits<long double>::
00058 ln_2() -> long double
00059 { return l_ln2; }
00060 }
00061 #endif // _GLUCAT_LONG_DOUBLE_H

```

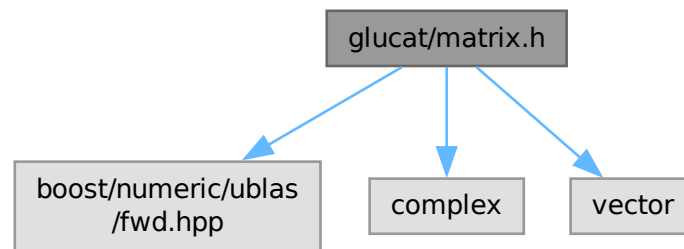
## 9.31 glucat/matrix.h File Reference

```

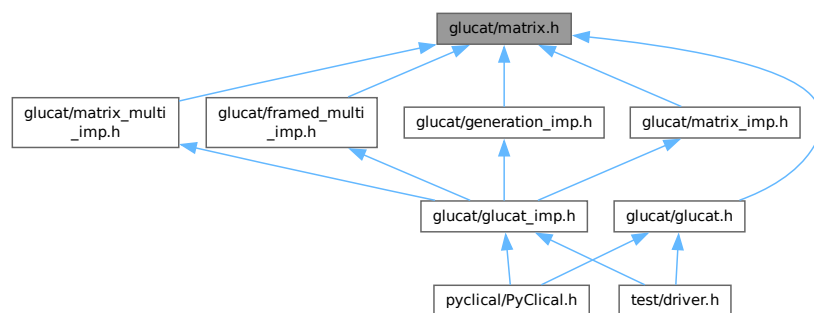
#include <boost/numeric/ublas/fwd.hpp>
#include <complex>
#include <vector>

```

Include dependency graph for matrix.h:



This graph shows which files directly or indirectly include this file:



## Classes

- struct [glucat::matrix::eig\\_genus< Matrix\\_T >](#)  
*Structure containing classification of eigenvalues.*

## Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

## Typedefs

- using [glucat::matrix::eig\\_case\\_t](#)  
*Classification of eigenvalues of a matrix.*

## Functions

- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
- template<typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression<  
 RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
- template<typename Scalar\_T, typename LHS\_T, typename RHS\_T>  
 auto [glucat::matrix::inner](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$ .*
- template<typename Matrix\_T>  
 auto [glucat::matrix::norm\\_frob2](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::trace](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::eigenvalues](#) (const Matrix\_T &val) -> std::vector< std::complex< double > >  
*Eigenvalues of a matrix.*
- template<typename Matrix\_T>  
 auto [glucat::matrix::classify\\_eigenvalues](#) (const Matrix\_T &val) -> [eig\\_genus](#)< Matrix\_T >  
*Classify the eigenvalues of a matrix.*

## 9.32 matrix.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_H
00002 #define _GLUCAT_MATRIX_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 matrix.h : Declare common matrix functions
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 : uBLAS interface contributed by Joerg Walter
00010 *****/
00011
00012 This library is free software: you can redistribute it and/or modify
00013 it under the terms of the GNU Lesser General Public License as published
00014 by the Free Software Foundation, either version 3 of the License, or
00015 (at your option) any later version.
00016
00017 This library is distributed in the hope that it will be useful,
00018 but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 GNU Lesser General Public License for more details.
00021
00022 You should have received a copy of the GNU Lesser General Public License
00023 along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025 *****/
00026 This library is based on a prototype written by Arvind Raja and was
00027 licensed under the LGPL with permission of the author. See Arvind Raja,
00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****/
00032 See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include <boost/numeric/ublas/fwd.hpp>
00036
00037 #include <complex>
00038 #include <vector>
00039
00040 namespace glucat
00041 {
00042 namespace ublas = boost::numeric::ublas;
00043
00044 namespace matrix
00045 {
00046 template< typename LHS_T, typename RHS_T >
00047 auto
00048 kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00049 RHS_T;
00050
00051 template< typename LHS_T, typename RHS_T >
00052 auto
00053 mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00054 RHS_T;
00055
00056 template< typename LHS_T, typename RHS_T >
00057 auto
00058 nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono = true) -> const
00059 RHS_T;
00060
00061 template< typename LHS_T, typename RHS_T >
00062 auto
00063 signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00064 RHS_T;
00065
00066 template< typename Matrix_T >
00067 auto
00068 nnz(const Matrix_T& m) -> typename Matrix_T::size_type;
00069
00070 template< typename Matrix_T >
00071 auto
00072 isinf(const Matrix_T& m) -> bool;
00073
00074 template< typename Matrix_T >
00075 auto
00076 isnan(const Matrix_T& m) -> bool;
00077
00078 template< typename Matrix_T >
00079 auto
00080 unit(const typename Matrix_T::size_type n) -> const
00081 Matrix_T;
00082
00083 }
00084
00085 }

```

```

00092 template< typename LHS_T, typename RHS_T >
00093 auto
00094 mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00095 const ublas::matrix_expression<RHS_T>& rhs) -> const
00096 typename RHS_T::expression_type;
00097
00099 template< typename LHS_T, typename RHS_T >
00100 auto
00101 sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00102 const ublas::matrix_expression<RHS_T>& rhs) -> const
00103 typename RHS_T::expression_type;
00104
00106 template< typename LHS_T, typename RHS_T >
00107 auto
00108 prod(const ublas::matrix_expression<LHS_T>& lhs,
00109 const ublas::matrix_expression<RHS_T>& rhs) -> const
00110 typename RHS_T::expression_type;
00111
00113 template< typename Scalar_T, typename LHS_T, typename RHS_T >
00114 auto
00115 inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T;
00116
00118 template< typename Matrix_T >
00119 auto
00120 norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type;
00121
00123 template< typename Matrix_T >
00124 auto
00125 trace(const Matrix_T& val) -> typename Matrix_T::value_type;
00126
00128 template< typename Matrix_T >
00129 auto
00130 eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >;
00131
00133 using eig_case_t = enum {
00134 safe_eigs,
00135 neg_real_eigs,
00136 both_eigs};
00137
00139 template< typename Matrix_T >
00140 struct eig_genus
00141 {
00142 using Scalar_T = typename Matrix_T::value_type;
00144 bool m_is_singular = false;
00146 eig_case_t m_eig_case = safe_eigs;
00148 Scalar_T m_safe_arg = Scalar_T(0);
00149 };
00150
00152 template< typename Matrix_T >
00153 auto
00154 classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>;
00155 }
00156 }
00157
00158 #endif // _GLUCAT_MATRIX_H

```

## 9.33 glucat/matrix\_imp.h File Reference

```

#include "glucat/errors.h"
#include "glucat/scalar.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/vector.hpp>
#include <boost/numeric/ublas/vector_proxy.hpp>
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <blaze/Math.h>
#include <blaze/math/DynamicMatrix.h>
#include <blaze/math/DynamicVector.h>
#include <set>

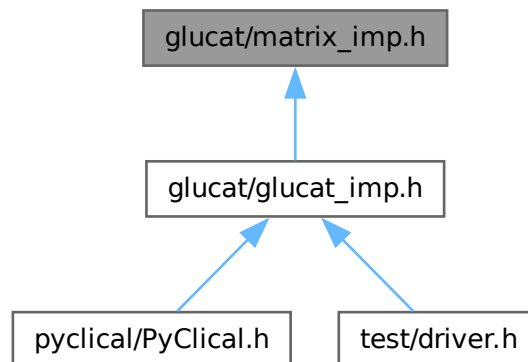
```

```
#include <vector>
```

Include dependency graph for matrix\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

## Functions

- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Kronecker tensor product of matrices - as per Matlab kron.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::mono\_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Sparse Kronecker tensor product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T>`  
`void glucat::matrix::nork\_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T`  
*Left inverse of Kronecker product.*
- `template<typename LHS_T, typename RHS_T>`  
`auto glucat::matrix::signed\_perm\_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`

- Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T>  
auto `glucat::matrix::nnz` (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::isinf` (const Matrix\_T &m) -> bool  
*Infinite.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::isnan` (const Matrix\_T &m) -> bool  
*Not a Number.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::unit` (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
  - template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::mono_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
  - template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::sparse_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
  - template<typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
  - template<typename Scalar\_T, typename LHS\_T, typename RHS\_T>  
auto `glucat::matrix::inner` (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{rows}()$ .*
  - template<typename Matrix\_T>  
auto `glucat::matrix::norm_frob2` (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::trace` (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*
  - template<typename Matrix\_T>  
static auto `glucat::matrix::to_blaze` (const Matrix\_T &val) -> blaze::DynamicMatrix< double, blaze::rowMajor >  
*Convert matrix to Blaze format.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::eigenvalues` (const Matrix\_T &val) -> std::vector< std::complex< double > >  
*Eigenvalues of a matrix.*
  - template<typename Matrix\_T>  
auto `glucat::matrix::classify_eigenvalues` (const Matrix\_T &val) -> eig\_genus< Matrix\_T >  
*Classify the eigenvalues of a matrix.*

## 9.34 matrix\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_MATRIX_IMP_H
00002 #define _GLUCAT_MATRIX_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 matrix_imp.h : Implement common matrix functions
00006 *****/
```

```

00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 : uBLAS interface contributed by Joerg Walter
00010 *****
00011
00012 This library is free software: you can redistribute it and/or modify
00013 it under the terms of the GNU Lesser General Public License as published
00014 by the Free Software Foundation, either version 3 of the License, or
00015 (at your option) any later version.
00016
00017 This library is distributed in the hope that it will be useful,
00018 but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 GNU Lesser General Public License for more details.
00021
00022 You should have received a copy of the GNU Lesser General Public License
00023 along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025 *****
00026 This library is based on a prototype written by Arvind Raja and was
00027 licensed under the LGPL with permission of the author. See Arvind Raja,
00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****
00032 See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include "glucat/errors.h"
00036 #include "glucat/scalar.h"
00037 #include "glucat/matrix.h"
00038
00039 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00040 # pragma GCC diagnostic push
00041 # pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00042 # endif
00043 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00044 # include <boost/serialization/array_wrapper.hpp>
00045 # endif
00046 #include <boost/numeric/ublas/vector.hpp>
00047 #include <boost/numeric/ublas/vector_proxy.hpp>
00048 #include <boost/numeric/ublas/matrix.hpp>
00049 #include <boost/numeric/ublas/matrix_expression.hpp>
00050 #include <boost/numeric/ublas/matrix_proxy.hpp>
00051 #include <boost/numeric/ublas/matrix_sparse.hpp>
00052 #include <boost/numeric/ublas/operation.hpp>
00053 #include <boost/numeric/ublas/operation_sparse.hpp>
00054
00055 #if defined(_GLUCAT_USE_BLAZE)
00056 #include <blaze/Math.h>
00057 #include <blaze/math/DynamicMatrix.h>
00058 #include <blaze/math/DynamicVector.h>
00059 #endif
00060
00061 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00062 # pragma GCC diagnostic pop
00063 # endif
00064
00065 #include <set>
00066 #include <vector>
00067
00068 namespace glucat { namespace matrix
00069 {
00070 // References for algorithms:
00071 // [v]: C. F. van Loan and N. Pitsianis, "Approximation with Kronecker products",
00072 // in Linear Algebra for Large Scale and Real-Time Applications, Marc S. Moonen,
00073 // Gene H. Golub, and Bart L. R. Moor (eds.), 1993, pp. 293--314.
00074
00075 template< typename LHS_T, typename RHS_T >
00076 auto
00077 kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00078 RHS_T
00079 {
00080 {
00081 const auto rhs_s1 = rhs.size1();
00082 const auto rhs_s2 = rhs.size2();
00083 auto result = RHS_T(lhs.size1()*rhs_s1, lhs.size2()*rhs_s2);
00084 result.clear();
00085
00086 for (auto
00087 lhs_it1 = lhs.begin1();
00088 lhs_it1 != lhs.end1();
00089 ++lhs_it1)
00090 for (auto
00091 lhs_it2 = lhs_it1.begin();
00092 lhs_it2 != lhs_it1.end();
00093 ++lhs_it2)

```



```

00094 {
00095 const auto start1 = rhs_s1 * lhs_it2.index1();
00096 const auto start2 = rhs_s2 * lhs_it2.index2();
00097 const auto& lhs_val = *lhs_it2;
00098 for (auto
00099 rhs_it1 = rhs.begin1();
00100 rhs_it1 != rhs.end1();
00101 ++rhs_it1)
00102 for (auto
00103 rhs_it2 = rhs_it1.begin();
00104 rhs_it2 != rhs_it1.end();
00105 ++rhs_it2)
00106 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00107 }
00108 return result;
00109 }
00110
00111 template< typename LHS_T, typename RHS_T >
00112 auto
00113 mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00114 RHS_T
00115 {
00116 {
00117 const auto rhs_s1 = rhs.size1();
00118 const auto rhs_s2 = rhs.size2();
00119 const auto dim = lhs.size1()*rhs_s1;
00120 auto result = RHS_T(dim, dim, dim);
00121 result.clear();
00122
00123 for (auto
00124 lhs_it1 = lhs.begin1();
00125 lhs_it1 != lhs.end1();
00126 ++lhs_it1)
00127 {
00128 const auto lhs_it2 = lhs_it1.begin();
00129 const auto start1 = rhs_s1 * lhs_it2.index1();
00130 const auto start2 = rhs_s2 * lhs_it2.index2();
00131 const auto& lhs_val = *lhs_it2;
00132 for (auto
00133 rhs_it1 = rhs.begin1();
00134 rhs_it1 != rhs.end1();
00135 ++rhs_it1)
00136 {
00137 const auto rhs_it2 = rhs_it1.begin();
00138 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00139 }
00140 }
00141 return result;
00142 }
00143 }
00144
00145 template< typename LHS_T, typename RHS_T >
00146 void
00147 nork_range(RHS_T& result,
00148 const typename LHS_T::const_iterator2 lhs_it2,
00149 const RHS_T& rhs,
00150 const typename RHS_T::size_type res_s1,
00151 const typename RHS_T::size_type res_s2)
00152 {
00153 // Definition matches [v] Section 4, Theorem 4.1.
00154 const auto start1 = res_s1 * lhs_it2.index1();
00155 const auto start2 = res_s2 * lhs_it2.index2();
00156 using ublas::range;
00157 const auto& range1 = range(start1, start1 + res_s1);
00158 const auto& range2 = range(start2, start2 + res_s2);
00159 using matrix_range_t = ublas::matrix_range<const RHS_T>;
00160 const auto& rhs_range = matrix_range_t(rhs, range1, range2);
00161 using Scalar_T = typename RHS_T::value_type;
00162 const auto lhs_val = numeric_traits<Scalar_T>::to_scalar_t(*lhs_it2);
00163 for (auto
00164 rhs_it1 = rhs_range.begin1();
00165 rhs_it1 != rhs_range.end1();
00166 ++rhs_it1)
00167 for (auto
00168 rhs_it2 = rhs_it1.begin();
00169 rhs_it2 != rhs_it1.end();
00170 ++rhs_it2)
00171 result(rhs_it2.index1(), rhs_it2.index2()) += lhs_val * *rhs_it2;
00172 }
00173
00174 template< typename LHS_T, typename RHS_T >
00175 auto
00176 nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono) -> const
00177 RHS_T
00178 {
00179 {
00180 // nork(A, kron(A, B)) is close to B
00181 // Definition matches [v] Section 4, Theorem 4.1.
00182 const auto lhs_s1 = lhs.size1();
00183 const auto lhs_s2 = lhs.size2();

```

```

00184 const auto rhs_s1 = rhs.size1();
00185 const auto rhs_s2 = rhs.size2();
00186 const auto res_s1 = rhs_s1 / lhs_s1;
00187 const auto res_s2 = rhs_s2 / lhs_s2;
00188 using Scalar_T = typename RHS_T::value_type;
00189 const auto norm_frob2_lhs = norm_frob2(lhs);
00190 if (!mono)
00191 {
00192 using error_t = error<RHS_T>;
00193 if (rhs_s1 == 0)
00194 throw error_t("matrix", "nork: number of rows must not be 0");
00195 if (rhs_s2 == 0)
00196 throw error_t("matrix", "nork: number of cols must not be 0");
00197 if (res_s1 * lhs_s1 != rhs_s1)
00198 throw error_t("matrix", "nork: incompatible numbers of rows");
00199 if (res_s2 * lhs_s2 != rhs_s2)
00200 throw error_t("matrix", "nork: incompatible numbers of cols");
00201 if (norm_frob2_lhs == Scalar_T(0))
00202 throw error_t("matrix", "nork: LHS must not be 0");
00203 }
00204 auto result = RHS_T(res_s1, res_s2);
00205 result.clear();
00206 for (auto
00207 lhs_it1 = lhs.begin1();
00208 lhs_it1 != lhs.end1();
00209 ++lhs_it1)
00210 {
00211 for (auto
00212 lhs_it2 = lhs_it1.begin();
00213 lhs_it2 != lhs_it1.end();
00214 ++lhs_it2)
00215 if (*lhs_it2 != Scalar_T(0))
00216 nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00217 }
00218 result /= norm_frob2_lhs;
00219 return result;
00220 }
00221
00222 template< typename LHS_T, typename RHS_T >
00223 auto
00224 signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00225 RHS_T
00226 {
00227 // signed_perm_nork(A, kron(A, B)) is close to B
00228 // Definition matches [v] Section 4, Theorem 4.1.
00229 const auto lhs_s1 = lhs.size1();
00230 const auto lhs_s2 = lhs.size2();
00231 const auto rhs_s1 = rhs.size1();
00232 const auto rhs_s2 = rhs.size2();
00233 const auto res_s1 = rhs_s1 / lhs_s1;
00234 const auto res_s2 = rhs_s2 / lhs_s2;
00235 using Scalar_T = typename RHS_T::value_type;
00236 const auto norm_frob2_lhs = Scalar_T(double(lhs_s1));
00237 auto result = RHS_T(res_s1, res_s2);
00238 result.clear();
00239 for (auto
00240 lhs_it1 = lhs.begin1();
00241 lhs_it1 != lhs.end1();
00242 ++lhs_it1)
00243 {
00244 const auto lhs_it2 = lhs_it1.begin();
00245 nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00246 }
00247 result /= norm_frob2_lhs;
00248 return result;
00249 }
00250
00251 template< typename Matrix_T >
00252 auto
00253 nnz(const Matrix_T& m) -> typename Matrix_T::size_type
00254 {
00255 using size_t = typename Matrix_T::size_type;
00256 auto result = size_t(0);
00257 for (auto
00258 it1 = m.begin1();
00259 it1 != m.end1();
00260 ++it1)
00261 {
00262 for (auto& entry : it1)
00263 if (entry != 0)
00264 ++result;
00265 }
00266 return result;
00267 }
00268
00269 template< typename Matrix_T >
00270 auto
00271 isinf(const Matrix_T& m) -> bool
00272 {
00273 using Scalar_T = typename Matrix_T::value_type;
00274 for (auto

```

```

00274 it1 = m.begin1();
00275 it1 != m.end1();
00276 ++it1)
00277 for (auto& entry : it1)
00278 if (numeric_traits<Scalar_T>::isInf(entry))
00279 return true;
00280
00281 return false;
00282 }
00283
00284 template< typename Matrix_T >
00285 auto
00286 isnan(const Matrix_T& m) -> bool
00287 {
00288 using Scalar_T = typename Matrix_T::value_type;
00289 for (auto
00290 it1 = m.begin1();
00291 it1 != m.end1();
00292 ++it1)
00293 for (auto& entry : it1)
00294 if (numeric_traits<Scalar_T>::isNaN(entry))
00295 return true;
00296
00297 return false;
00298 }
00299
00300 template< typename Matrix_T >
00301 inline
00302 auto
00303 unit(const typename Matrix_T::size_type dim) -> const
00304 Matrix_T
00305 {
00306 using Scalar_T = typename Matrix_T::value_type;
00307 return ublas::identity_matrix<Scalar_T>(dim);
00308 }
00309
00310 template< typename LHS_T, typename RHS_T >
00311 auto
00312 mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00313 const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00314 {
00315 using rhs_expression_t = const RHS_T;
00316 using matrix_row_t = typename ublas::matrix_row<rhs_expression_t>;
00317
00318 const auto dim = lhs().size1();
00319 // The following assumes that RHS_T is a sparse matrix type.
00320 auto result = RHS_T(dim, dim, dim);
00321 for (auto
00322 lhs_row = lhs().begin1();
00323 lhs_row != lhs().end1();
00324 ++lhs_row)
00325 {
00326 const auto& lhs_it = lhs_row.begin();
00327 if (lhs_it != lhs_row.end())
00328 {
00329 const auto& rhs_row = matrix_row_t(rhs(), lhs_it.index2());
00330 const auto& rhs_it = rhs_row.begin();
00331 if (rhs_it != rhs_row.end())
00332 result(lhs_it.index1(), rhs_it.index()) = (*lhs_it) * (*rhs_it);
00333 }
00334 }
00335 return result;
00336 }
00337
00338 template< typename LHS_T, typename RHS_T >
00339 inline
00340 auto
00341 sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00342 const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00343 {
00344 using expression_t = typename RHS_T::expression_type;
00345 return ublas::sparse_prod<expression_t>(lhs(), rhs());
00346 }
00347
00348 template< typename LHS_T, typename RHS_T >
00349 inline
00350 auto
00351 prod(const ublas::matrix_expression<LHS_T>& lhs,
00352 const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00353 {
00354 const auto dim = lhs().size1();
00355 RHS_T result(dim, dim);
00356 ublas::axpy_prod(lhs, rhs, result, true);
00357 return result;
00358 }
00359
00360 template< typename Scalar_T, typename LHS_T, typename RHS_T >

```

```

00367 auto
00368 inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T
00369 {
00370 auto result = Scalar_T(0);
00371 for (auto
00372 lhs_it1 = lhs.begin1();
00373 lhs_it1 != lhs.end1();
00374 ++lhs_it1)
00375 for (auto
00376 lhs_it2 = lhs_it1.begin();
00377 lhs_it2 != lhs_it1.end();
00378 ++lhs_it2)
00379 {
00380 const auto& rhs_val = rhs(lhs_it2.index1(), lhs_it2.index2());
00381 if (rhs_val != Scalar_T(0))
00382 result += (*lhs_it2) * rhs_val;
00383 }
00384 return result / lhs.size1();
00385 }
00386
00387 template< typename Matrix_T >
00388 auto
00389 norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type
00390 {
00391 using Scalar_T = typename Matrix_T::value_type;
00392
00393 auto result = Scalar_T(0);
00394 for (auto
00395 val_it1 = val.begin1();
00396 val_it1 != val.end1();
00397 ++val_it1)
00398 for (auto& val_entry : val_it1)
00399 {
00400 if (numeric_traits<Scalar_T>::isNaN(val_entry))
00401 return numeric_traits<Scalar_T>::NaN();
00402 result += val_entry * val_entry;
00403 }
00404 return result;
00405 }
00406
00407 template< typename Matrix_T >
00408 auto
00409 trace(const Matrix_T& val) -> typename Matrix_T::value_type
00410 {
00411 using Scalar_T = typename Matrix_T::value_type;
00412
00413 auto result = Scalar_T(0);
00414 auto dim = val.size1();
00415 for (auto
00416 ndx = decltype(dim)(0);
00417 ndx != dim;
00418 ++ndx)
00419 {
00420 const Scalar_T crd = val(ndx, ndx);
00421 if (numeric_traits<Scalar_T>::isNaN(crd))
00422 return numeric_traits<Scalar_T>::NaN();
00423 result += crd;
00424 }
00425 return result;
00426 }
00427
00428 #if defined(_GLUCAT_USE_BLAZE)
00429 template< typename Matrix_T >
00430 static
00431 auto
00432 to_blaze(const Matrix_T& val) -> blaze::DynamicMatrix<double, blaze::rowMajor>
00433 {
00434 const auto s1 = val.size1();
00435 const auto s2 = val.size2();
00436
00437 using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00438 auto result = blaze_matrix_t(s1, s2);
00439
00440 using Scalar_T = typename Matrix_T::value_type;
00441 using traits_t = numeric_traits<Scalar_T>;
00442
00443 for (auto
00444 val_it1 = val.begin1();
00445 val_it1 != val.end1();
00446 ++val_it1)
00447 for (auto
00448 val_it2 = val_it1.begin();
00449 val_it2 != val_it1.end();
00450 ++val_it2)
00451 result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00452
00453 return result;
00454 }

```

```

00457 }
00458
00459 #endif
00460
00461 template< typename Matrix_T >
00462 auto
00463 eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >
00464 {
00465 using complex_t = std::complex<double>;
00466 using complex_vector_t = typename std::vector<complex_t>;
00467
00468 const auto dim = val.size1();
00469 auto lambda = complex_vector_t(dim);
00470
00471 #if defined(_GLUCAT_USE_BLAZE)
00472 using complex_t = std::complex<double>;
00473 using blaze_complex_vector_t = blaze::DynamicVector<complex_t, blaze::columnVector>;
00474
00475 auto blaze_val = to_blaze(val);
00476 auto blaze_lambda = blaze_complex_vector_t(dim);
00477 blaze::geev(blaze_val, blaze_lambda);
00478
00479 for (auto
00480 k = decltype(dim)(0);
00481 k != dim;
00482 ++k)
00483 lambda[k] = blaze_lambda[k];
00484 #endif
00485 return lambda;
00486 }
00487
00488 template< typename Matrix_T >
00489 auto
00490 classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>
00491 {
00492 using Scalar_T = typename Matrix_T::value_type;
00493 eig_genus<Matrix_T> result;
00494
00495 auto lambda = eigenvalues(val);
00496
00497 std::set<double> arg_set;
00498
00499 const auto dim = lambda.size();
00500 static const auto epsilon =
00501 std::max(std::numeric_limits<double>::epsilon(),
00502 numeric_traits<Scalar_T>::to_double(std::numeric_limits<Scalar_T>::epsilon()));
00503 static const auto zero_eig_tol = 4096.0*epsilon;
00504
00505 bool neg_real_eig_found = false;
00506 bool imag_eig_found = false;
00507 bool zero_eig_found = false;
00508
00509 for (auto
00510 k = decltype(dim)(0);
00511 k != dim;
00512 ++k)
00513 {
00514 const auto lambda_k = lambda[k];
00515 arg_set.insert(std::arg(lambda_k));
00516
00517 const auto real_lambda_k = std::real(lambda_k);
00518 const auto imag_lambda_k = std::imag(lambda_k);
00519 const auto norm_tol = 4096.0*epsilon*std::norm(lambda_k);
00520
00521 if (!neg_real_eig_found &&
00522 real_lambda_k < -epsilon &&
00523 (imag_lambda_k == 0.0 ||
00524 imag_lambda_k * imag_lambda_k < norm_tol))
00525 neg_real_eig_found = true;
00526 if (!imag_eig_found &&
00527 imag_lambda_k > epsilon &&
00528 (real_lambda_k == 0.0 ||
00529 real_lambda_k * real_lambda_k < norm_tol))
00530 imag_eig_found = true;
00531 if (!zero_eig_found &&
00532 std::norm(lambda_k) < zero_eig_tol)
00533 zero_eig_found = true;
00534 }
00535
00536 if (zero_eig_found)
00537 result.m_is_singular = true;
00538
00539 static const auto pi = numeric_traits<double>::pi();
00540 if (neg_real_eig_found)
00541 {
00542 if (imag_eig_found)
00543 result.m_eig_case = both_eigs;
00544 }

```

```

00546 else
00547 {
00548 result.m_eig_case = neg_real_eigs;
00549 result.m_safe_arg = Scalar_T(-pi / 2.0);
00550 }
00551 }
00552
00553 if (result.m_eig_case == both_eigs)
00554 {
00555 auto arg_it = arg_set.begin();
00556 auto first_arg = *arg_it;
00557 auto best_arg = first_arg;
00558 auto best_diff = 0.0;
00559 auto previous_arg = first_arg;
00560 for (++arg_it;
00561 arg_it != arg_set.end();
00562 ++arg_it)
00563 {
00564 const auto arg_diff = *arg_it - previous_arg;
00565 if (arg_diff > best_diff)
00566 {
00567 best_diff = arg_diff;
00568 best_arg = previous_arg;
00569 }
00570 previous_arg = *arg_it;
00571 }
00572 const auto arg_diff = first_arg + 2.0*pi - previous_arg;
00573 if (arg_diff > best_diff)
00574 {
00575 best_diff = arg_diff;
00576 best_arg = previous_arg;
00577 }
00578 result.m_safe_arg = Scalar_T(pi - (best_arg + best_diff / 2.0));
00579 }
00580 return result;
00581 }
00582 } }
00583
00584 #endif // _GLUCAT_MATRIX_IMP_H

```

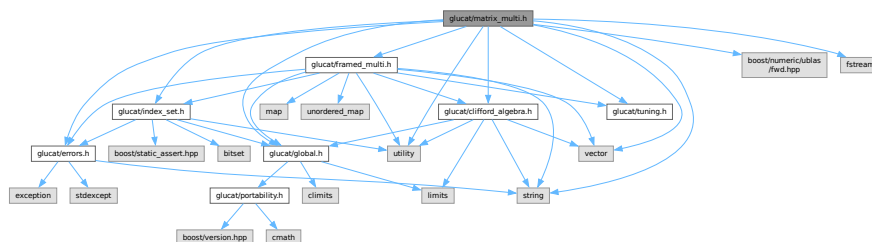
## 9.35 glucat/matrix\_multi.h File Reference

```

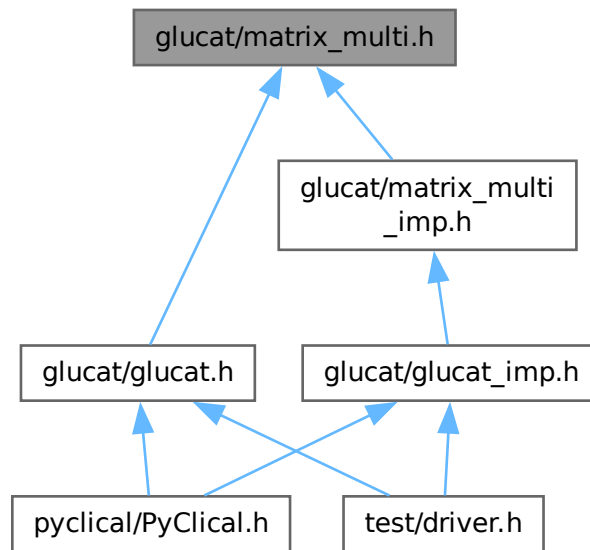
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <fstream>
#include <string>
#include <utility>
#include <vector>

```

Include dependency graph for matrix\_multi.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*A `matrix_multi< Scalar_T, LO, HI, Tune_P >` is a matrix approximation to a multivector.*
- struct `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`  
*Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.*

## Namespaces

- namespace `glucat`
- namespace `std`

## Functions

- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
`auto glucat::operator* (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
`auto glucat::operator^ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P>  
`auto glucat::operator& (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator% (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::star (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator/ (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator| (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator>> (std::istream &s, matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`  
*Read multivector from input.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::operator<< (std::ostream &os, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::reframe (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs, matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix\_multi< Scalar_T, LO, HI, Tune_P > &rhs_reframed) -> const index\_set< LO, HI >`  
*Find a common frame for operands of a binary operator.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::matrix\_sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::matrix\_log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P>`  
`auto glucat::exp (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*

## 9.36 [matrix\\_multi.h](#)

[Go to the documentation of this file.](#)



```

00001 #ifndef _GLUCAT_MATRIX_MULTI_H
00002 #define _GLUCAT_MATRIX_MULTI_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 matrix_multi.h : Declare a class for the matrix representation of a multivector
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039 #include "glucat/framed_multi.h"
00040
00041 #include <boost/numeric/ublas/fwd.hpp>
00042
00043 #include <fstream>
00044 #include <string>
00045 #include <utility>
00046 #include <vector>
00047
00048 namespace glucat
00049 {
00050 namespace ublas = boost::numeric::ublas;
00051
00052 // Forward declarations for friends
00053
00054 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00055 class framed_multi; // forward
00056
00057 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00058 class matrix_multi; // forward
00059
00060 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00061 auto
00062 operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00064
00065 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00066 auto
00067 operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00069
00070 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00071 auto
00072 operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00074
00075 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076 auto
00077 operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00079
00080 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00081 auto
00082 star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00084
00085 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00086 auto
00087 operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const

```

```

matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00089
00091 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092 auto
00093 operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00094
00096 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097 auto
00098 operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00101 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00102 auto
00103 operator« (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00104
00106 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00107 auto
00108 reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>;
00109
00110 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111 auto
00112 sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00113
00115 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00116 auto
00117 matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00118
00119 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00120 auto
00121 log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00122
00124 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00125 auto
00126 log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00127
00129 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00130 auto
00131 matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00132
00133 template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<> >
00134
00137 class matrix_multi :
00138 public clifford_algebra< Scalar_T, index_set<LO,HI>, matrix_multi<Scalar_T,LO,HI,Tune_P> >
00139 {
00140 public:
00141 using multivector_t = matrix_multi;
00142 using matrix_multi_t = multivector_t;
00143 using scalar_t = Scalar_T;
00144 using tune_p = Tune_P;
00145 using index_set_t = index_set<LO, HI>;
00146 using term_t = std::pair<const index_set_t, Scalar_T>;
00147 using vector_t = std::vector<Scalar_T>;
00148 using error_t = error<multivector_t>;
00149 using framed_multi_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00150 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00151 friend class framed_multi;
00152 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00153 friend class matrix_multi;
00154
00155 private:
00156 using orientation_t = ublas::row_major;
00157 using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>;
00158 using matrix_t = ublas::matrix<Scalar_T, orientation_t>;
00159 using matrix_index_t = typename matrix_t::size_type;
00160
00161 public:
00163 static auto classname() -> const std::string;
00165 ~matrix_multi() override = default;
00167 matrix_multi();
00169 template< typename Other_Scalar_T >
00170 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00172 template< typename Other_Scalar_T >
00173 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
const index_set_t frm, const bool prechecked = false);
00174
00176 matrix_multi(const multivector_t& val,
const index_set_t frm, const bool prechecked = false);
00177
00179 matrix_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00181 matrix_multi(const index_set_t ist, const Scalar_T& crd,
const index_set_t frm, const bool prechecked = false);
00182
00184 matrix_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());

```

```

00186 matrix_multi(const int scr, const index_set_t frm = index_set_t());
00188 matrix_multi(const vector_t& vec,
00189 const index_set_t frm, const bool prechecked = false);
00191 matrix_multi(const std::string& str);
00193 matrix_multi(const std::string& str,
00194 const index_set_t frm, const bool prechecked = false);
00196 matrix_multi(const char* str)
00197 { *this = matrix_multi(std::string(str)); };
00199 matrix_multi(const char* str,
00200 const index_set_t frm, const bool prechecked = false)
00201 { *this = matrix_multi(std::string(str), frm, prechecked); };
00203 template< typename Other_Scalar_T >
00204 matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00206 template< typename Other_Scalar_T >
00207 matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00208 const index_set_t frm, const bool prechecked = false);
00210 auto fast_matrix_multi(const index_set_t frm) const -> const matrix_multi_t;
00212 template< typename Other_Scalar_T >
00213 auto fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00214
00215 private:
00217 template< typename Matrix_T >
00218 matrix_multi(const Matrix_T& mtx, const index_set_t frm);
00220 matrix_multi(const matrix_t& mtx, const index_set_t frm);
00222 auto basis_element(const index_set<LO,HI>& ist) const -> const basis_matrix_t;
00223
00224 public:
00225 _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00226
00228 auto operator= (const multivector_t& rhs) -> multivector_t&;
00229
00231 static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t;
00232
00233 // Friend declarations
00234
00235 friend auto
00236 operator* <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00237 friend auto
00238 operator^ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00239 friend auto
00240 operator& <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00241 friend auto
00242 operator% <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00243 friend auto
00244 star <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> Scalar_T;
00245 friend auto
00246 operator/ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00247 friend auto
00248 operator| <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00249
00250 friend auto
00251 operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00252 friend auto
00253 operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00254 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00255 Other_Tune_P >
00256 friend auto
00257 reframe (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs, const
00258 matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs,
00259 matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs_reframed,
00260 matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs_reframed) -> const
00261 index_set<Other_LO,Other_HI>;
00262 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00263 Other_Tune_P >
00264 friend auto
00265 matrix_sqrt (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00266 const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00267 const index_t level)
00268 -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00269 template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00270 Other_Tune_P >
00271 friend auto
00272 matrix_log (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00273 const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00274 const index_t level)
00275 -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00276
00277 auto operator+= (const term_t& rhs) -> multivector_t&;
00278
00279 private:
00280 // Data members
00281 index_set_t m_frame;
00282 matrix_t m_matrix;
00283 };
00284
00285 // Non-members

```

```

00284
00286 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00287 auto
00288 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00289
00290 }
00291
00292 namespace std
00293 {
00295 template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00296 struct numeric_limits< glucat::matrix_multi<Scalar_T,LO,HI,Tune_P> > :
00297 public numeric_limits<Scalar_T>
00298 { };
00299 }
00300 #endif // _GLUCAT_MATRIX_MULTI_H

```

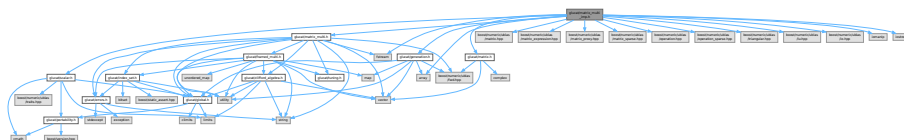
## 9.37 glucat/matrix\_multi\_imp.h File Reference

```

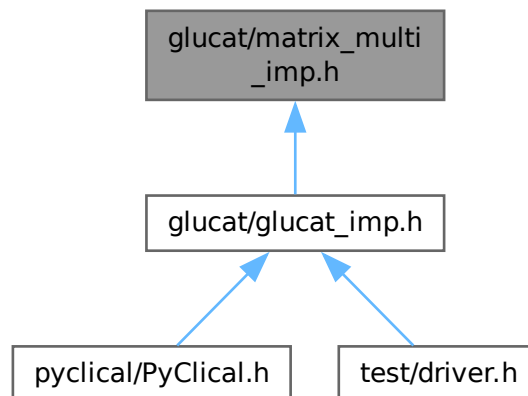
#include "glucat/matrix_multi.h"
#include "glucat/scalar.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/ublas/triangular.hpp>
#include <boost/numeric/ublas/lu.hpp>
#include <boost/numeric/ublas/io.hpp>
#include <fstream>
#include <iomanip>
#include <array>
#include <iostream>

```

Include dependency graph for matrix\_multi\_imp.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::basis\\_table< Scalar\\_T, LO, HI, Matrix\\_T >](#)  
*Table of basis elements used as a cache by basis\_element().*
- struct [pade::pade\\_sqrt\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).*
- struct [pade::pade\\_sqrt\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).*
- struct [pade::pade\\_sqrt\\_numer< float >](#)
- struct [pade::pade\\_sqrt\\_denom< float >](#)
- struct [pade::pade\\_sqrt\\_numer< long double >](#)
- struct [pade::pade\\_sqrt\\_denom< long double >](#)
- struct [pade::pade\\_sqrt\\_numer< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_numer< qd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< qd\\_real >](#)
- struct [pade::pade\\_log\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).*
- struct [pade::pade\\_log\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).*
- struct [pade::pade\\_log\\_numer< float >](#)
- struct [pade::pade\\_log\\_denom< float >](#)
- struct [pade::pade\\_log\\_numer< long double >](#)
- struct [pade::pade\\_log\\_denom< long double >](#)
- struct [pade::pade\\_log\\_numer< dd\\_real >](#)
- struct [pade::pade\\_log\\_denom< dd\\_real >](#)
- struct [pade::pade\\_log\\_numer< qd\\_real >](#)
- struct [pade::pade\\_log\\_denom< qd\\_real >](#)

## Namespaces

- namespace [glucat](#)
- namespace [pade](#)

## Functions

- auto [glucat::offset\\_level](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> [index\\_t](#)  
*Determine the log2 dim corresponding to signature p, q.*
- template<typename Matrix\_Index\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [glucat::folded\\_dim](#) (const [index\\_set](#)< LO, HI > &sub) -> Matrix\_Index\_T  
*Determine the matrix dimension of the fold of a subalgebra.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::reframe](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs\_reframed, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs\_reframed) -> const [index\\_set](#)< LO, HI >  
*Find a common frame for operands of a binary operator.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator\\*](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator^](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator&](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator%](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::star](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator/](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator|](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator<<](#) (std::ostream &os, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::operator>>](#) (std::istream &s, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*

- template<typename Multivector\_T, typename Matrix\_T, typename Basis\_Matrix\_T>  
static auto [glucat::fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T  
*Inverse generalized Fast Fourier Transform.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P, const size\_t Size>  
static auto [glucat::pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Pade' approximation.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static void [glucat::db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)  
*Single step of product form of Denman-Beavers square root iteration.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Product form of Denman-Beavers square root iteration.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y←tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Cyclic reduction square root iteration.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::matrix\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Square root of multivector with specified complexifier.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Square root of multivector with specified complexifier.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Pade' approximation of log.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
static auto [glucat::cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Incomplete square root cascade and Pade' approximation of log.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::matrix\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Natural logarithm of multivector with specified complexifier.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Natural logarithm of multivector with specified complexifier.*
- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P>  
auto [glucat::exp](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Exponential of multivector.*

## Variables

- template<typename Scalar\_T>  
const [pade\\_sqrt\\_numer](#)< Scalar\_T >::array [pade::pade\\_sqrt\\_numer](#)< Scalar\_T >::numer

- `template<typename Scalar_T>`  
`const pade_sqrt_denom< Scalar_T >::array pade::pade_sqrt_denom< Scalar_T >::denom`
- `const pade_sqrt_numer< float >::array pade::pade_sqrt_numer< float >::numer`
- `const pade_sqrt_denom< float >::array pade::pade_sqrt_denom< float >::denom`
- `const pade_sqrt_numer< longdouble >::array pade::pade_sqrt_numer< longdouble >::numer`
- `const pade_sqrt_denom< longdouble >::array pade::pade_sqrt_denom< longdouble >::denom`
- `const pade_sqrt_numer< dd_real >::array pade::pade_sqrt_numer< dd_real >::numer`
- `const pade_sqrt_denom< dd_real >::array pade::pade_sqrt_denom< dd_real >::denom`
- `const pade_sqrt_numer< qd_real >::array pade::pade_sqrt_numer< qd_real >::numer`
- `const pade_sqrt_denom< qd_real >::array pade::pade_sqrt_denom< qd_real >::denom`
- `template<typename Scalar_T>`  
`const pade_log_numer< Scalar_T >::array pade::pade_log_numer< Scalar_T >::numer`
- `template<typename Scalar_T>`  
`const pade_log_denom< Scalar_T >::array pade::pade_log_denom< Scalar_T >::denom`
- `const pade_log_numer< float >::array pade::pade_log_numer< float >::numer`
- `const pade_log_denom< float >::array pade::pade_log_denom< float >::denom`
- `const pade_log_numer< longdouble >::array pade::pade_log_numer< longdouble >::numer`
- `const pade_log_denom< longdouble >::array pade::pade_log_denom< longdouble >::denom`
- `const pade_log_numer< dd_real >::array pade::pade_log_numer< dd_real >::numer`
- `const pade_log_denom< dd_real >::array pade::pade_log_denom< dd_real >::denom`
- `const pade_log_numer< qd_real >::array pade::pade_log_numer< qd_real >::numer`
- `const pade_log_denom< qd_real >::array pade::pade_log_denom< qd_real >::denom`

## 9.38 matrix\_multi\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_MULTI_IMP_H
00002 #define _GLUCAT_MATRIX_MULTI_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 matrix_multi_imp.h : Implement the matrix representation of a multivector
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/matrix_multi.h"
00035
00036 #include "glucat/scalar.h"
00037 #include "glucat/generation.h"
00038 #include "glucat/matrix.h"
00039
00040 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00041 # pragma GCC diagnostic push
00042 # pragma GCC diagnostic ignored "-Wunused-local-typedefs"

```



```

00043 # endif
00044 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00045 # include <boost/serialization/array_wrapper.hpp>
00046 # endif
00047 #include <boost/numeric/ublas/matrix.hpp>
00048 #include <boost/numeric/ublas/matrix_expression.hpp>
00049 #include <boost/numeric/ublas/matrix_proxy.hpp>
00050 #include <boost/numeric/ublas/matrix_sparse.hpp>
00051 #include <boost/numeric/ublas/operation.hpp>
00052 #include <boost/numeric/ublas/operation_sparse.hpp>
00053 #include <boost/numeric/ublas/triangular.hpp>
00054 #include <boost/numeric/ublas/lu.hpp>
00055 #include <boost/numeric/ublas/io.hpp>
00056 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00057 # pragma GCC diagnostic pop
00058 # endif
00059
00060 #include <fstream>
00061 #include <iomanip>
00062 #include <array>
00063 #include <iostream>
00064
00065 namespace glucat
00066 {
00067 // References for algorithms:
00068 // [CHKL]:
00069 // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00070 // [MB]: Beatrice Meini, "The Matrix Square Root From a New Functional Perspective:
00071 // Theoretical Results and Computational Issues", SIAM Journal on
00072 // Matrix Analysis and Applications 26(2):362-376, 2004.
00073 // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00074
00075 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076 auto
00077 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00078 classname() -> const std::string
00079 { return "matrix_multi"; }
00080
00081 // Reference: [P] Table 15.27, p 133
00082 inline
00083 auto
00084 offset_level(const index_t p, const index_t q) -> index_t
00085 {
00086 // Offsets between the log2 of the matrix dimension for the current signature
00087 // and that of the real superalgebra
00088 static const std::array<int, 8> offset_log2_dim = {0, 1, 0, 1, 1, 2, 1, 1};
00089 const index_t bott = pos_mod(p-q, 8);
00090 return (p+q)/2 + offset_log2_dim[bott];
00091 }
00092
00093 // Reference: [P] Table 15.27, p 133
00094 template< typename Matrix_Index_T, const index_t LO, const index_t HI >
00095 inline
00096 static
00097 auto
00098 folded_dim(const index_set<LO,HI>& sub) -> Matrix_Index_T
00099 { return 1 « offset_level(sub.count_pos(), sub.count_neg()); }
00100
00101 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00102 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00103 matrix_multi()
00104 : m_frame(index_set_t()),
00105 m_matrix(matrix_t(1, 1))
00106 { this->m_matrix.clear(); }
00107
00108 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00109 template< typename Other_Scalar_T >
00110 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00111 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00112 : m_frame(val.m_frame), m_matrix(val.m_matrix.size1(), val.m_matrix.size2())
00113 {
00114 this->m_matrix.clear();
00115 for (auto
00116 val_it1 = val.m_matrix.begin1();
00117 val_it1 != val.m_matrix.end1();
00118 ++val_it1)
00119 for (auto
00120 val_it2 = val_it1.begin();
00121 val_it2 != val_it1.end();
00122 ++val_it2)
00123 this->m_matrix(val_it2.index1(), val_it2.index2()) =
00124 numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00125 }
00126
00127 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00128 template< typename Other_Scalar_T >
00129 matrix_multi<Scalar_T,LO,HI,Tune_P>::

```

```

00135 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val, const index_set_t frm, const bool
prechecked)
00136 : m_frame(frm)
00137 {
00138 if (frm != val.m_frame)
00139 *this = multivector_t(framed_multi_t(val), frm);
00140 else
00141 {
00142 const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00143 this->m_matrix.resize(dim, dim, false);
00144 this->m_matrix.clear();
00145 for (auto
00146 val_it1 = val.m_matrix.begin1();
00147 val_it1 != val.m_matrix.end1();
00148 ++val_it1)
00149 for (auto
00150 val_it2 = val_it1.begin();
00151 val_it2 != val_it1.end();
00152 ++val_it2)
00153 this->m_matrix(val_it2.index1(), val_it2.index2()) =
numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00154 }
00155 }
00156
00157 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00158 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00159 matrix_multi(const multivector_t& val, const index_set_t frm, const bool prechecked)
00160 : m_frame(frm)
00161 {
00162 if (frm != val.m_frame)
00163 *this = multivector_t(framed_multi_t(val), frm);
00164 else
00165 this->m_matrix = val.m_matrix;
00166 }
00167
00168 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00169 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00170 matrix_multi(const index_set_t ist, const Scalar_T& crd)
00171 : m_frame(ist)
00172 {
00173 const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00174 this->m_matrix.resize(dim, dim, false);
00175 this->m_matrix.clear();
00176 *this += term_t(ist, crd);
00177 }
00178
00179 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00180 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00181 matrix_multi(const index_set_t ist, const Scalar_T& crd, const index_set_t frm, const bool
prechecked)
00182 : m_frame(frm)
00183 {
00184 if (!prechecked && (ist | frm) != frm)
00185 throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00186 const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00187 this->m_matrix.resize(dim, dim, false);
00188 this->m_matrix.clear();
00189 *this += term_t(ist, crd);
00190 }
00191
00192 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00193 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00194 matrix_multi(const Scalar_T& scr, const index_set_t frm)
00195 : m_frame(frm)
00196 {
00197 const auto dim = folded_dim<matrix_index_t>(frm);
00198 this->m_matrix.resize(dim, dim, false);
00199 this->m_matrix.clear();
00200 *this += term_t(index_set_t(), scr);
00201 }
00202
00203 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00204 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00205 matrix_multi(const int scr, const index_set_t frm)
00206 { *this = multivector_t(Scalar_T(scr), frm); }
00207
00208 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00209 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00210 matrix_multi(const vector_t& vec,
00211 const index_set_t frm, const bool prechecked)
00212 : m_frame(frm)
00213 {
00214 if (!prechecked && index_t(vec.size()) != frm.count())
00215 throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00216 const auto dim = folded_dim<matrix_index_t>(frm);
00217 this->m_matrix.resize(dim, dim, false);
00218 this->m_matrix.clear();

```

```

00225 auto idx = frm.min();
00226 const auto frm_end = frm.max()+1;
00227 for (auto& crd : vec)
00228 {
00229 *this += term_t(index_set_t(idx), crd);
00230 for (
00231 ++idx;
00232 idx != frm_end && !frm[idx];
00233 ++idx)
00234 ;
00235 }
00236 }
00237
00238 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00239 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00240 matrix_multi(const std::string& str)
00241 { *this = framed_multi_t(str); }
00242
00243 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00244 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00245 matrix_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00246 { *this = multivector_t(framed_multi_t(str), frm, prechecked); }
00247
00248 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00249 template< typename Other_Scalar_T >
00250 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00251 matrix_multi(const framed_multi_t<Other_Scalar_T,LO,HI,Tune_P>& val)
00252 : m_frame(val.frame())
00253 {
00254 if (val.size() >= Tune_P::fast_size_threshold)
00255 {
00256 try
00257 {
00258 *this = val.template fast_matrix_multi<Scalar_T>(this->m_frame);
00259 return;
00260 }
00261 catch (const glucat_error& e)
00262 { }
00263 }
00264 const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00265 this->m_matrix.resize(dim, dim, false);
00266 this->m_matrix.clear();
00267 for (auto& val_term : val)
00268 *this += val_term;
00269 }
00270
00271 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00272 template< typename Other_Scalar_T >
00273 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00274 matrix_multi(const framed_multi_t<Other_Scalar_T,LO,HI,Tune_P>& framed_val, const index_set_t frm,
00275 const bool prechecked)
00276 {
00277 const auto val = framed_val.truncated();
00278 const auto our_frame = val.frame() | frm;
00279 if (val.size() >= Tune_P::fast_size_threshold)
00280 {
00281 try
00282 {
00283 *this = val.template fast_matrix_multi<Scalar_T>(our_frame);
00284 return;
00285 }
00286 catch (const glucat_error& e)
00287 { }
00288 }
00289 this->m_frame = our_frame;
00290 const auto dim = folded_dim<matrix_index_t>(our_frame);
00291 this->m_matrix.resize(dim, dim, false);
00292 this->m_matrix.clear();
00293 for (auto& val_term : val)
00294 *this += val_term;
00295 }
00296
00297 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00298 template< typename Matrix_T >
00299 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00300 matrix_multi(const Matrix_T& mtx, const index_set_t frm)
00301 : m_frame(frm), m_matrix(mtx.size1(), mtx.size2())
00302 {
00303 this->m_matrix.clear();
00304 for (auto
00305 mtx_it1 = mtx.begin1();
00306 mtx_it1 != mtx.end1();
00307 ++mtx_it1)
00308 {
00309 for (auto
00310 mtx_it2 = mtx_it1.begin();
00311 mtx_it2 != mtx_it1.end();
00312 ++mtx_it2)
00313 this->m_matrix(mtx_it2.index1(), mtx_it2.index2()) =

```

```

 numeric_traits<Scalar_T>::to_scalar_t(*mtx_it2);
00316 }
00317
00319 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00320 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00321 matrix_multi(const matrix_t& mtx, const index_set_t frm)
00322 : m_frame(frm), m_matrix(mtx)
00323 { }
00324
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 auto
00328 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00329 operator=(const multivector_t& rhs) -> multivector_t&
00330 {
00331 // Check for assignment to self
00332 if (this == &rhs)
00333 return *this;
00334 this->m_frame = rhs.m_frame;
00335 this->m_matrix = rhs.m_matrix;
00336 return *this;
00337 }
00338
00340 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00341 inline
00342 auto
00343 reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00344 matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>
00345 {
00346 using index_set_t = index_set<LO, HI>;
00347 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00348 using framed_multi_t = typename multivector_t::framed_multi_t;
00349 // Determine the initial common frame
00350 index_set_t our_frame = lhs.m_frame | rhs.m_frame;
00351 framed_multi_t framed_lhs;
00352 framed_multi_t framed_rhs;
00353 if ((lhs.m_frame != our_frame) || (rhs.m_frame != our_frame))
00354 {
00355 // The common frame may expand as a result of the transform to framed_multi_t
00356 framed_lhs = framed_multi_t(lhs);
00357 framed_rhs = framed_multi_t(rhs);
00358 our_frame |= framed_lhs.frame() | framed_rhs.frame();
00359 }
00360 // Do the reframing only where necessary
00361 if (lhs.m_frame != our_frame)
00362 lhs_reframed = multivector_t(framed_lhs, our_frame, true);
00363 if (rhs.m_frame != our_frame)
00364 rhs_reframed = multivector_t(framed_rhs, our_frame, true);
00365 return our_frame;
00366 }
00367
00369 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00370 auto
00371 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00372 operator==(const multivector_t& rhs) const -> bool
00373 {
00374 // Ensure that there is no aliasing
00375 if (this == &rhs)
00376 return true;
00377
00378 // Operate only within a common frame
00379 multivector_t lhs_reframed;
00380 multivector_t rhs_reframed;
00381 const index_set_t our_frame = reframe(*this, rhs, lhs_reframed, rhs_reframed);
00382 const multivector_t& lhs_ref = (this->m_frame == our_frame)
? *this
: lhs_reframed;
00383 const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
? rhs
: rhs_reframed;
00384 return ublas::norm_inf(lhs_ref.m_matrix - rhs_ref.m_matrix) == 0;
00385 }
00386
00388 // Test for equality of multivector and scalar
00389 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00390 inline
00391 auto
00392 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00393 operator==(const Scalar_T& scr) const -> bool
00394 {
00395 if (scr != Scalar_T(0))
00396 return *this == multivector_t(framed_multi_t(scr), this->m_frame, true);
00397 else if (ublas::norm_inf(this->m_matrix) != 0)
00398 return false;
00399 else
00400 return true;
00401 }
00402
00403

```

```

00404 {
00405 const matrix_index_t dim = this->m_matrix.size1();
00406 return !(dim == 1 && this->isnan());
00407 }
00408 }
00409
00410 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00411 inline
00412 auto
00413 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00414 operator+= (const Scalar_T& scr) -> multivector_t&
00415 { return *this += term_t(index_set_t(), scr); }
00416
00417 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00418 inline
00419 auto
00420 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00421 operator+= (const multivector_t& rhs) -> multivector_t&
00422 {
00423 // Ensure that there is no aliasing
00424 if (this == &rhs)
00425 return *this *= Scalar_T(2);
00426
00427 // Operate only within a common frame
00428 multivector_t rhs_reframed;
00429 const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00430 const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00431 ? rhs
00432 : rhs_reframed;
00433
00434 noalias(this->m_matrix) += rhs_ref.m_matrix;
00435 return *this;
00436 }
00437
00438 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00439 inline
00440 auto
00441 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00442 operator-= (const Scalar_T& scr) -> multivector_t&
00443 { return *this += term_t(index_set_t(), -scr); }
00444
00445 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00446 inline
00447 auto
00448 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00449 operator-= (const multivector_t& rhs) -> multivector_t&
00450 {
00451 // Ensure that there is no aliasing
00452 if (this == &rhs)
00453 return *this = Scalar_T(0);
00454
00455 // Operate only within a common frame
00456 multivector_t rhs_reframed;
00457 const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00458 const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00459 ? rhs
00460 : rhs_reframed;
00461
00462 noalias(this->m_matrix) -= rhs_ref.m_matrix;
00463 return *this;
00464 }
00465
00466 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00467 inline
00468 auto
00469 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00470 operator- () const -> const multivector_t
00471 { return multivector_t(-(this->m_matrix), this->m_frame); }
00472
00473 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00474 inline
00475 auto
00476 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00477 operator*= (const Scalar_T& scr) -> multivector_t&
00478 { // multiply coordinates of all terms by scalar
00479
00480 using traits_t = numeric_traits<Scalar_T>;
00481 if (traits_t::isNaN_or_isInf(scr) || this->isnan())
00482 return *this = traits_t::NaN();
00483 if (scr == Scalar_T(0))
00484 *this = Scalar_T(0);
00485 else
00486 this->m_matrix *= scr;
00487 return *this;
00488 }
00489
00490 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```

```

00498 inline
00499 auto
00500 operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00501 {
00502 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00503 using index_set_t = typename multivector_t::index_set_t;
00504
00505 if (lhs.isnan() || rhs.isnan())
00506 return numeric_traits<Scalar_T>::NaN();
00507
00508 // Operate only within a common frame
00509 multivector_t lhs_reframed;
00510 multivector_t rhs_reframed;
00511 const index_set_t our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00512 const multivector_t& lhs_ref = (lhs.m_frame == our_frame)
00513 ? lhs
00514 : lhs_reframed;
00515 const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00516 ? rhs
00517 : rhs_reframed;
00518
00519 using matrix_t = typename multivector_t::matrix_t;
00520 using matrix_index_t = typename matrix_t::size_type;
00521
00522 const matrix_index_t dim = lhs_ref.m_matrix.size();
00523 multivector_t result = multivector_t(matrix_t(dim, dim), our_frame);
00524 result.m_matrix.clear();
00525 ublas::axpy_prod(lhs_ref.m_matrix, rhs_ref.m_matrix, result.m_matrix, true);
00526 return result;
00527 }
00528
00529 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00530 inline
00531 auto
00532 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00533 operator*= (const multivector_t& rhs) -> multivector_t&
00534 { return *this = *this * rhs; }
00535
00536 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00537 inline
00538 auto
00539 operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00540 {
00541 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00542 using framed_multi_t = typename multivector_t::framed_multi_t;
00543 return framed_multi_t(lhs) ^ framed_multi_t(rhs);
00544 }
00545
00546 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00547 inline
00548 auto
00549 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00550 operator^= (const multivector_t& rhs) -> multivector_t&
00551 { return *this = *this ^ rhs; }
00552
00553 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00554 inline
00555 auto
00556 operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00557 {
00558 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00559 using framed_multi_t = typename multivector_t::framed_multi_t;
00560 return framed_multi_t(lhs) & framed_multi_t(rhs);
00561 }
00562
00563 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00564 inline
00565 auto
00566 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00567 operator&= (const multivector_t& rhs) -> multivector_t&
00568 { return *this = *this & rhs; }
00569
00570 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00571 inline
00572 auto
00573 operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00574 {
00575 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00576 using framed_multi_t = typename multivector_t::framed_multi_t;
00577 return framed_multi_t(lhs) % framed_multi_t(rhs);
00578 }
00579
00580 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```

```

00588 inline
00589 auto
00590 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00591 operator%=(const multivector_t& rhs) -> multivector_t&
00592 { return *this = *this % rhs; }
00593
00594 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00595 inline
00596 auto
00597 star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00598 -> Scalar_T
00599 { return (lhs * rhs).scalar(); }
00600
00601 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00602 inline
00603 auto
00604 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00605 operator/=(const Scalar_T& scr) -> multivector_t&
00606 { return *this *= Scalar_T(1)/scr; }
00607
00608 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00609 auto
00610 00612 operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00613 {
00614 using traits_t = numeric_traits<Scalar_T>;
00615
00616 if (lhs.isnan() || rhs.isnan())
00617 return traits_t::NaN();
00618
00619 if (rhs == Scalar_T(0))
00620 return traits_t::NaN();
00621
00622 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00623
00624 // Operate only within a common frame
00625 multivector_t lhs_reframed;
00626 multivector_t rhs_reframed;
00627 const auto our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00628 const auto& lhs_ref = (lhs.m_frame == our_frame)
00629 ? lhs
00630 : lhs_reframed;
00631 const auto& rhs_ref = (rhs.m_frame == our_frame)
00632 ? rhs
00633 : rhs_reframed;
00634
00635 // Solve result == lhs_ref/rhs_ref <=> result*rhs_ref == lhs_ref
00636 // We now solve X == B/A
00637 // (where X == result, B == lhs_ref.m_matrix and A == rhs_ref.m_matrix)
00638 // X == B/A <=> X*A == B <=> AT*XT == BT
00639 // So, we solve AT*XT == BT
00640
00641 using matrix_t = typename multivector_t::matrix_t;
00642 using matrix_index_t = typename matrix_t::size_type;
00643
00644 const auto& AT = matrix_t(ublas::trans(rhs_ref.m_matrix));
00645 auto LU = AT;
00646
00647 using permutation_t = ublas::permutation_matrix<matrix_index_t>;
00648
00649 auto pvector = permutation_t(AT.size1());
00650 if (!ublas::lu_factorize(LU, pvector))
00651 {
00652 const auto& BT = matrix_t(ublas::trans(lhs_ref.m_matrix));
00653 auto XT = BT;
00654 ublas::lu_substitute(LU, pvector, XT);
00655 if (matrix::isnan(XT))
00656 return traits_t::NaN();
00657
00658 // Iterative refinement.
00659 // Reference: Nicholas J. Higham, "Accuracy and Stability of Numerical Algorithms",
00660 // SIAM, 1996, ISBN 0-89871-355-2, Chapter 11
00661 if (Tune_P::div_max_steps > 0)
00662 {
00663 // matrix_t R = ublas::prod(AT, XT) - BT;
00664 auto R = matrix_t(-BT);
00665 ublas::axpy_prod(AT, XT, R, false);
00666 if (matrix::isnan(R))
00667 return traits_t::NaN();
00668
00669 auto nr = Scalar_T(ublas::norm_inf(R));
00670 if (nr != Scalar_T(0) && !traits_t::isNan_or_isInf(nr))
00671 {
00672 auto XTnew = XT;
00673 auto nrold = nr + Scalar_T(1);
00674 for (auto
00675 step = 0;

```

```

00676 step != Tune_P::div_max_steps &&
00677 nr < nroid &&
00678 nr != Scalar_T(0) &&
00679 nr == nr;
00680 ++step)
00681 {
00682 nroid = nr;
00683 if (step != 0)
00684 XT = XTnew;
00685 auto& D = R;
00686 ublas::lu_substitute(LU, pvector, D);
00687 XTnew -= D;
00688 // noalias(R) = ublas::prod(AT, XTnew) - BT;
00689 R = -BT;
00690 ublas::axpy_prod(AT, XTnew, R, false);
00691 nr = ublas::norm_inf(R);
00692 }
00693 }
00694 }
00695 return multivector_t(ublas::trans(XT), our_frame);
00696 }
00697 else
00698 // AT is singular. Return NaN
00699 return traits_t::NaN();
00700 }
00701
00702 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00703 inline
00704 auto
00705 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00706 operator/=(const multivector_t& rhs) -> multivector_t&
00707 { return *this = *this / rhs; }
00708
00709 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00710 inline
00711 auto
00712 operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00713 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00714 { return rhs * lhs / rhs.involute(); }
00715
00716 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00717 inline
00718 auto
00719 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00720 operator|=(const multivector_t& rhs) -> multivector_t&
00721 { return *this = rhs * *this / rhs.involute(); }
00722
00723 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00724 inline
00725 auto
00726 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00727 inv() const -> const multivector_t
00728 { return multivector_t(Scalar_T(1), this->m_frame) / *this; }
00729
00730 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00731 inline
00732 auto
00733 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00734 pow(int m) const -> const multivector_t
00735 { return glucat::pow(*this, m); }
00736
00737 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00738 auto
00739 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00740 outer_pow(int m) const -> const multivector_t
00741 {
00742 if (m < 0)
00743 throw error_t("outer_pow(m): negative exponent");
00744 framed_multi_t a = *this;
00745 return a.outer_pow(m);
00746 }
00747
00748 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00749 inline
00750 auto
00751 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00752 grade() const -> index_t
00753 { return framed_multi_t(*this).grade(); }
00754
00755 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00756 inline
00757 auto
00758 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00759 frame() const -> const index_set_t
00760 { return this->m_frame; }
00761
00762 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```



```

00771 inline
00772 auto
00773 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00774 operator[] (const index_set_t ist) const -> Scalar_T
00775 {
00776 // Use matrix inner product only if ist is in frame
00777 if ((ist | this->m_frame) == this->m_frame)
00778 return matrix::inner<Scalar_T>(this->basis_element(ist), this->m_matrix);
00779 else
00780 return Scalar_T(0);
00781 }
00782
00783 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00784 inline
00785 auto
00786 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00787 operator() (index_t grade) const -> const multivector_t
00788 {
00789 if ((grade < 0) || (grade > HI-LO))
00790 return 0;
00791 else
00792 return (framed_multi_t(*this))(grade);
00793 }
00794
00795 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 inline
00797 auto
00798 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00799 scalar() const -> Scalar_T
00800 {
00801 const matrix_index_t dim = this->m_matrix.size1();
00802 return matrix::trace(this->m_matrix) / Scalar_T(double(dim));
00803 }
00804
00805 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00806 inline
00807 auto
00808 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00809 pure() const -> const multivector_t
00810 { return *this - this->scalar(); }
00811
00812 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00813 inline
00814 auto
00815 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00816 even() const -> const multivector_t
00817 { return framed_multi_t(*this).even(); }
00818
00819 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00820 inline
00821 auto
00822 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00823 odd() const -> const multivector_t
00824 { return framed_multi_t(*this).odd(); }
00825
00826 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00827 inline
00828 auto
00829 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00830 vector_part() const -> const vector_t
00831 { return this->vector_part(this->frame(), true); }
00832
00833 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00834 inline
00835 auto
00836 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00837 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00838 {
00839 if (!prechecked && (this->frame() | frm) != frm)
00840 throw error_t("vector_part(frm): value is outside of requested frame");
00841 vector_t result;
00842 // If we need to enlarge the frame we may as well use a framed_multi_t
00843 if (this->frame() != frm)
00844 return framed_multi_t(*this).vector_part(frm, true);
00845
00846 const auto begin_index = frm.min();
00847 const auto end_index = frm.max()+1;
00848 for (auto
00849 idx = begin_index;
00850 idx != end_index;
00851 ++idx)
00852 {
00853 if (frm[idx])
00854 // Frame may contain indices which do not correspond to a grade 1 term but
00855 // frame cannot omit any index corresponding to a grade 1 term
00856 result.push_back(
00857 matrix::inner<Scalar_T>(this->basis_element(index_set_t(idx)),
00858 this->m_matrix));
00859 }
00860 return result;
00861 }

```

```

00865
00866 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00867 inline
00868 auto
00869 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00870 involute() const -> const multivector_t
00871 { return framed_multi_t(*this).involute(); }
00872
00873
00874 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00875 inline
00876 auto
00877 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00878 reverse() const -> const multivector_t
00879 { return framed_multi_t(*this).reverse(); }
00880
00881
00882 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00883 inline
00884 auto
00885 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00886 conj() const -> const multivector_t
00887 { return framed_multi_t(*this).conj(); }
00888
00889
00890 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00891 inline
00892 auto
00893 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00894 quad() const -> Scalar_T
00895 { // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
00896 // Arvind Raja ref: "old clical: quadfunction(p:pterm):pterm in file compmod.pas"
00897 return framed_multi_t(*this).quad();
00898 }
00899
00900
00901 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00902 inline
00903 auto
00904 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00905 norm() const -> Scalar_T
00906 {
00907 const matrix_index_t dim = this->m_matrix.size1();
00908 return matrix::norm_frob2(this->m_matrix) / Scalar_T(double(dim));
00909 }
00910
00911
00912 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00913 inline
00914 auto
00915 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00916 max_abs() const -> Scalar_T
00917 { return framed_multi_t(*this).max_abs(); }
00918
00919
00920 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00921 auto
00922 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00923 random(const index_set<LO,HI> frm, Scalar_T fill) -> const multivector_t
00924 {
00925 return framed_multi<Scalar_T,LO,HI,Tune_P>::random(frm, fill);
00926 }
00927
00928
00929 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00930 inline
00931 void
00932 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00933 write(const std::string& msg) const
00934 { framed_multi_t(*this).write(msg); }
00935
00936
00937 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00938 inline
00939 void
00940 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00941 write(std::ofstream& ofile, const std::string& msg) const
00942 {
00943 if (!ofile)
00944 throw error_t("write(ofile,msg): cannot write to output file");
00945 framed_multi_t(*this).write(ofile, msg);
00946 }
00947
00948
00949 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00950 inline
00951 auto
00952 operator<< (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
00953 {
00954 os << typename matrix_multi<Scalar_T,LO,HI,Tune_P>::framed_multi_t(val);
00955 return os;
00956 }
00957
00958
00959 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00960 inline
00961 auto

```

```

00963 operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&
00964 { // Input looks like 1.0-2.0{1,2}+3.2{3,4}
00965 framed_multi<Scalar_T,LO,HI,Tune_P> local;
00966 s » local;
00967 // If s.bad() then we have a corrupt input
00968 // otherwise we are fine and can copy the resulting matrix_multi
00969 if (!s.bad())
00970 val = local;
00971 return s;
00972 }
00973
00975 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976 inline
00977 auto
00978 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00979 isinf() const -> bool
00980 {
00981 if (std::numeric_limits<Scalar_T>::has_infinity)
00982 return matrix::isinf(this->m_matrix);
00983 else
00984 return false;
00985 }
00986
00988 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00989 inline
00990 auto
00991 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00992 isnan() const -> bool
00993 {
00994 if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
00995 return matrix::isnan(this->m_matrix);
00996 else
00997 return false;
00998 }
00999
01001 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01002 inline
01003 auto
01004 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01005 truncated(const Scalar_T& limit) const -> const multivector_t
01006 { return framed_multi_t(*this).truncated(limit); }
01007
01009 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01010 inline
01011 auto
01012 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01013 operator+= (const term_t& term) -> multivector_t&
01014 {
01015 if (term.second != Scalar_T(0))
01016 this->m_matrix.plus_assign(matrix_t(this->basis_element(term.first)) * term.second);
01017 return *this;
01018 }
01019
01021 template< typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T >
01022 static
01023 auto
01024 fast(const Matrix_T& X, index_t level) -> Multivector_T
01025 {
01026 using framed_multi_t = Multivector_T;
01027
01028 using index_set_t = typename framed_multi_t::index_set_t;
01029 using Scalar_T = typename framed_multi_t::scalar_t;
01030 using matrix_t = Matrix_T;
01031 using basis_matrix_t = Basis_Matrix_T;
01032 using basis_scalar_t = typename basis_matrix_t::value_type;
01033 using traits_t = numeric_traits<Scalar_T>;
01034
01035 if (level == 0)
01036 return framed_multi_t(traits_t::to_scalar_t(X(0,0)));
01037
01038 if (ublas::norm_inf(X) == 0)
01039 return Scalar_T(0);
01040
01041 const basis_matrix_t& I = matrix::unit<basis_matrix_t>(2);
01042 basis_matrix_t J(2,2,2);
01043 J.clear();
01044 J(0,1) = basis_scalar_t(-1);
01045 J(1,0) = basis_scalar_t(1);
01046 basis_matrix_t K = J;
01047 K(0,1) = basis_scalar_t(1);
01048 basis_matrix_t JK = I;
01049 JK(0,0) = basis_scalar_t(-1);
01050
01051 using matrix::signed_perm_nork;
01052 const index_set_t ist_mn = index_set_t(-level);
01053 const index_set_t ist_pn = index_set_t(level);
01054 const index_set_t ist_mnpn = ist_mn | ist_pn;

```

```

01055 if (level == 1)
01056 {
01057 using term_t = typename framed_multi_t::term_t;
01058 const Scalar_T i_x = traits_t::to_scalar_t(signed_perm_nork(I, X)(0, 0));
01059 const Scalar_T j_x = traits_t::to_scalar_t(signed_perm_nork(J, X)(0, 0));
01060 const Scalar_T k_x = traits_t::to_scalar_t(signed_perm_nork(K, X)(0, 0));
01061 const Scalar_T jk_x = traits_t::to_scalar_t(signed_perm_nork(JK, X)(0, 0));
01062 framed_multi_t
01063 result = i_x;
01064 result += term_t(ist_mn, j_x); // j_x * mn;
01065 result += term_t(ist_pn, k_x); // k_x * pn;
01066 return result += term_t(ist_mnpn, jk_x); // jk_x * mnpn;
01067 }
01068 else
01069 {
01070 const framed_multi_t& mn = framed_multi_t(ist_mn);
01071 const framed_multi_t& pn = framed_multi_t(ist_pn);
01072 const framed_multi_t& mnpn = framed_multi_t(ist_mnpn);
01073 const framed_multi_t& i_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01074 (signed_perm_nork(I, X), level-1);
01075 const framed_multi_t& j_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01076 (signed_perm_nork(J, X), level-1);
01077 const framed_multi_t& k_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01078 (signed_perm_nork(K, X), level-1);
01079 const framed_multi_t& jk_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01080 (signed_perm_nork(JK, X), level-1);
01081 framed_multi_t
01082 result = i_x.even() - jk_x.odd();
01083 result += (j_x.even() - k_x.odd()) * mn;
01084 result += (k_x.even() - j_x.odd()) * pn;
01085 return result += (jk_x.even() - i_x.odd()) * mnpn;
01086 }
01087 }
01088
01090 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01091 inline
01092 auto
01093 matrix_multi<Scalar_T, LO, HI, Tune_P>::
01094 fast_matrix_multi(const index_set_t frm) const -> const multivector_t
01095 {
01096 if (this->m_frame == frm)
01097 return *this;
01098 else
01099 return (this->template fast_framed_multi<Scalar_T>()).template fast_matrix_multi<Scalar_T>(frm);
01100 }
01101
01103 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01104 template <typename Other_Scalar_T>
01105 auto
01106 matrix_multi<Scalar_T, LO, HI, Tune_P>::
01107 fast_framed_multi() const -> const framed_multi<Other_Scalar_T, LO, HI, Tune_P>
01108 {
01109 // Determine the amount of off-centering needed
01110 index_t p = this->m_frame.count_pos();
01111 index_t q = this->m_frame.count_neg();
01112
01113 const index_t bott = pos_mod(p-q, 8);
01114 p += std::max(gen::offset_to_super[bott], index_t(0));
01115 q -= std::min(gen::offset_to_super[bott], index_t(0));
01116
01117 const index_t orig_p = p;
01118 const index_t orig_q = q;
01119 while (p-q > 4)
01120 { p -= 4; q += 4; }
01121 while (p-q < -3)
01122 { p += 4; q -= 4; }
01123 if (p-q > 1)
01124 {
01125 index_t old_p = p;
01126 p = q+1;
01127 q = old_p-1;
01128 }
01129 const index_t level = (p+q)/2;
01130
01131 // Do the inverse fast transform
01132 using framed_multi_t = framed_multi<Other_Scalar_T, LO, HI, Tune_P>;
01133 framed_multi_t val = fast<framed_multi_t, matrix_t, basis_matrix_t>(this->m_matrix, level);
01134
01135 // Off-centre val
01136 switch (pos_mod(orig_p-orig_q, 8))
01137 {
01138 case 2:
01139 case 3:
01140 case 4:
01141 val.centred_qpl_pml(p, q);
01142 break;
01143 default:

```

```

01144 break;
01145 }
01146 if (orig_p-orig_q > 4)
01147 while (p != orig_p)
01148 val.centred_pp4_qm4(p, q);
01149 if (orig_p-orig_q < -3)
01150 while (p != orig_p)
01151 val.centred_pm4_qp4(p, q);
01152
01153 // Return unfolded val
01154 return val.unfold(this->m_frame);
01155 }
01156
01157 template< typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T >
01158 class basis_table :
01159 public std::map< std::pair< const index_set<LO,HI>, const index_set<LO,HI> >,
01160 Matrix_T* >
01161 {
01162 public:
01163 static auto basis() -> basis_table& { static basis_table b; return b;}
01164 private:
01165 friend class friend_for_private_destructor;
01166 // Enforce singleton
01167 // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
01168 basis_table() = default;
01169 ~basis_table() = default;
01170 public:
01171 basis_table(const basis_table&) = delete;
01172 auto operator= (const basis_table&) -> basis_table& = delete;
01173 };
01174
01175 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01176 auto
01177 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01178 basis_element(const index_set_t& ist) const -> const basis_matrix_t
01179 {
01180 using index_set_pair_t = std::pair<const index_set_t, const index_set_t>;
01181 const auto& unfolded_pair = index_set_pair_t(ist, this->m_frame);
01182
01183 using basis_table_t = basis_table<Scalar_T, LO, HI, basis_matrix_t>;
01184 auto& basis_cache = basis_table_t::basis();
01185
01186 const auto frame_count = this->m_frame.count();
01187 const auto use_cache = frame_count <= index_t(Tune_P::basis_max_count);
01188
01189 if (use_cache)
01190 {
01191 const auto basis_it = basis_cache.find(unfolded_pair);
01192 if (basis_it != basis_cache.end())
01193 return *(basis_it->second);
01194 }
01195 const auto folded_set = ist.fold(this->m_frame);
01196 const auto folded_frame = this->m_frame.fold();
01197 const auto& folded_pair = index_set_pair_t(folded_set, folded_frame);
01198 using basis_pair_t = std::pair<const index_set_pair_t, basis_matrix_t*>;
01199 if (use_cache)
01200 {
01201 const auto basis_it = basis_cache.find(folded_pair);
01202 if (basis_it != basis_cache.end())
01203 {
01204 auto* result_ptr = basis_it->second;
01205 basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01206 return *result_ptr;
01207 }
01208 }
01209 const auto folded_max = folded_frame.max();
01210 const auto folded_min = folded_frame.min();
01211 const auto p = std::max(folded_max, index_t(0));
01212 const auto q = std::max(index_t(-folded_min), index_t(0));
01213 const auto* e = (gen::generator_table<basis_matrix_t>::generator())(p, q);
01214 const auto dim = matrix_index_t(1) << offset_level(p, q);
01215 auto result = matrix::unit<basis_matrix_t>(dim);
01216 for (auto
01217 k = folded_min;
01218 k <= folded_max;
01219 ++k)
01220 {
01221 if (folded_set[k])
01222 result = matrix::mono_prod(result, e[k]);
01223 }
01224 if (use_cache)
01225 {
01226 auto* result_ptr = new basis_matrix_t(result);
01227 basis_cache.insert(basis_pair_t(folded_pair, result_ptr));
01228 basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01229 }
01230 return result;
01231 }
01232
01233 }
01234
01235 }
01236

```

```

01238 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size
01239 >
01239 inline
01240 static
01241 auto
01242 pade_approx(
01243 const std::array<Scalar_T, Size>& numer,
01244 const std::array<Scalar_T, Size>& denom,
01245 const matrix_multi<Scalar_T,LO,HI,Tune_P>& X) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01246 {
01247 // Pade' approximation
01248 // Reference: [GW], Section 4.3, pp318-322
01249 // Reference: [GL], Section 11.3, p572-576.
01250
01251 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01252 using traits_t = numeric_traits<Scalar_T>;
01253
01254 if (X.isnan())
01255 return traits_t::NaN();
01256
01257 // Array size is assumed to be even
01258 const auto nbr_even_powers = Size/2 - 1;
01259
01260 // Create an array of even powers
01261 auto XX = std::vector<multivector_t>(nbr_even_powers);
01262 XX[0] = X * X;
01263 XX[1] = XX[0] * XX[0];
01264 for (auto
01265 k = size_t(2);
01266 k != nbr_even_powers;
01267 ++k)
01268 XX[k] = XX[k-2] * XX[1];
01269
01270 // Calculate numerator N and denominator D
01271 auto N = multivector_t(numer[1]);
01272 for (auto
01273 k = size_t(0);
01274 k != nbr_even_powers;
01275 ++k)
01276 N += XX[k] * numer[2*k + 3];
01277 N *= X;
01278 N += numer[0];
01279 for (auto
01280 k = size_t(0);
01281 k != nbr_even_powers;
01282 ++k)
01283 N += XX[k] * numer[2*k + 2];
01284 auto D = multivector_t(denom[1]);
01285 for (auto
01286 k = size_t(0);
01287 k != nbr_even_powers;
01288 ++k)
01289 D += XX[k] * denom[2*k + 3];
01290 D *= X;
01291 D += denom[0];
01292 for (auto
01293 k = size_t(0);
01294 k != nbr_even_powers;
01295 ++k)
01296 D += XX[k] * denom[2*k + 2];
01297 return N / D;
01298 }
01299
01301 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01302 inline
01303 static
01304 void
01305 db_step(matrix_multi<Scalar_T,LO,HI,Tune_P>& M, matrix_multi<Scalar_T,LO,HI,Tune_P>& Y)
01306 {
01307 // Reference: [CHKL]
01308 const auto& invM = inv(M);
01309 M = ((M + invM)/Scalar_T(2) + Scalar_T(1)) / Scalar_T(2);
01310 Y *= (invM + Scalar_T(1)) / Scalar_T(2);
01311 }
01312
01314 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01315 static
01316 auto
01317 db_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01318 Scalar_T norm_tol=std::numeric_limits<Scalar_T>::epsilon(), 4)) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01319 {
01320 // Reference: [CHKL]
01321 if (val == Scalar_T(0))
01322 return val;
01323
01324 static const auto sqrt_max_steps = Tune_P::db_sqrt_max_steps;

```

```

01325 auto M = val;
01326 auto Y = val;
01327
01328 for (auto
01329 step = 0;
01330 step != sqrt_max_steps && norm(M - Scalar_T(1)) > norm_tol;
01331 ++step)
01332 {
01333 if (Y.isnan())
01334 return numeric_traits<Scalar_T>::NaN();
01335 db_step(M, Y);
01336 }
01337 return Y;
01338 }
01339
01340 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01341 static
01342 auto
01343 cr_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01344 Scalar_T norm_Y_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) -> const
01345 matrix_multi<Scalar_T,LO,HI,Tune_P>
01346 {
01347 // Reference: [MB]
01348 if (val == Scalar_T(0))
01349 return val;
01350
01351 static const auto sqrt_max_steps = Tune_P::cr_sqrt_max_steps;
01352 auto Z = Scalar_T(2) * (Scalar_T(1) + val);
01353 auto Y = Scalar_T(1) - val;
01354 auto norm_Y = norm(Y);
01355 for (auto
01356 step = 0;
01357 step != sqrt_max_steps && norm_Y > norm_Y_tol;
01358 ++step)
01359 {
01360 const auto old_norm_Y = norm_Y;
01361 Y = (-Y / Z) * Y;
01362 norm_Y = norm(Y);
01363 if (Y.isnan() || (norm_Y > old_norm_Y * Scalar_T(2)))
01364 return numeric_traits<Scalar_T>::NaN();
01365
01366 Z += Y * Scalar_T(2);
01367 }
01368 return Z / Scalar_T(4);
01369 }
01370 }
01371
01372 namespace pade {
01373 // Reference: [Z], Padel
01374 template< typename Scalar_T >
01375 struct pade_sqrt_numer
01376 {
01377 using array = std::array<Scalar_T, 14>;
01378 static const array numer;
01379 };
01380
01381 template< typename Scalar_T >
01382 const typename pade_sqrt_numer<Scalar_T>::array pade_sqrt_numer<Scalar_T>::numer =
01383 {
01384 1.0, 27.0/4.0, 81.0/4.0, 2277.0/64.0,
01385 10395.0/256.0, 32319.0/1024.0, 8721.0/512.0, 26163.0/4096.0,
01386 53703.0/32768.0, 36465.0/131072.0, 3861.0/131072.0, 7371.0/4194304.0,
01387 819.0/16777216.0, 27.0/67108864.0
01388 };
01389
01390 // Reference: [Z], Padel
01391 template< typename Scalar_T >
01392 struct pade_sqrt_denom
01393 {
01394 using array = std::array<Scalar_T, 14>;
01395 static const array denom;
01396 };
01397
01398 template< typename Scalar_T >
01399 const typename pade_sqrt_denom<Scalar_T>::array pade_sqrt_denom<Scalar_T>::denom =
01400 {
01401 1.0, 25.0/4.0, 69.0/4.0, 1771.0/64.0,
01402 7315.0/256.0, 20349.0/1024.0, 4845.0/512.0, 12597.0/4096.0,
01403 21879.0/32768.0, 12155.0/131072.0, 1001.0/131072.0, 1365.0/4194304.0,
01404 91.0/16777216.0, 1.0/67108864.0
01405 };
01406
01407 template< >
01408 struct pade_sqrt_numer<float>
01409 {
01410 using array = std::array<float, 10>;
01411 static const array numer;
01412 };
01413 const typename pade_sqrt_numer<float>::array pade_sqrt_numer<float>::numer =

```

```

01414 {
01415 1.0, 19.0/4.0, 19.0/2.0, 665.0/64.0,
01416 1729.0/256.0, 2717.0/1024.0, 627.0/1024.0, 627.0/8192.0,
01417 285.0/65536.0, 19.0/262144.0
01418 };
01419 template< >
01420 struct pade_sqrt_denom<float>
01421 {
01422 using array = std::array<float, 10>;
01423 static const array denom;
01424 };
01425 const typename pade_sqrt_denom<float>::array pade_sqrt_denom<float>::denom =
01426 {
01427 1.0, 17.0/4.0, 15.0/2.0, 455.0/64.0,
01428 1001.0/256.0, 1287.0/1024.0, 231.0/1024.0, 165.0/8192.0,
01429 45.0/65536, 1.0/262144.0
01430 };
01431
01432 template< >
01433 struct pade_sqrt_number<long double>
01434 {
01435 using array = std::array<long double, 18>;
01436 static const array number;
01437 };
01438 const typename pade_sqrt_number<long double>::array pade_sqrt_number<long double>::number =
01439 {
01440 1.0L, 35.0L/4.0L, 35.0L, 5425.0L/64.0L,
01441 35525.0L/256.0L, 166257.0L/1024.0L, 143325.0L/1024.0L, 740025.0L/8192.0L,
01442 2877875.0L/65536.0L, 4206125.0L/262144.0L, 572033.0L/131072.0L, 1820105.0L/2097152.0L,
01443 1028755.0L/8388608.0L, 395675.0L/33554432.0L, 24225.0L/33554432.0L, 6783.0L/268435456.0L,
01444 1785.0L/4294967296.0L, 35.0L/17179869184.0L
01445 };
01446 template< >
01447 struct pade_sqrt_denom<long double>
01448 {
01449 using array = std::array<long double, 18>;
01450 static const array denom;
01451 };
01452 const typename pade_sqrt_denom<long double>::array pade_sqrt_denom<long double>::denom =
01453 {
01454 1.0L, 33.0L/4.0L, 31.0L, 4495.0L/64.0L,
01455 27405.0L/256.0L, 118755.0L/1024.0L, 94185.0L/1024.0L, 444015.0L/8192.0L,
01456 1562275.0L/65536.0L, 2042975.0L/262144.0L, 245157.0L/131072.0L, 676039.0L/2097152.0L,
01457 323323.0L/8388608.0L, 101745.0L/33554432.0L, 4845.0L/33554432.0L, 969.0L/268435456.0L,
01458 153.0L/4294967296.0L, 1.0L/17179869184.0L
01459 };
01460
01461 #if defined(_GLUCAT_USE_QD)
01462 template< >
01463 struct pade_sqrt_number<dd_real>
01464 {
01465 using array = std::array<dd_real, 22>;
01466 static const array number;
01467 };
01468 const typename pade_sqrt_number<dd_real>::array pade_sqrt_number<dd_real>::number =
01469 {
01470 dd_real("1"), dd_real("43")/dd_real("4"),
01471 dd_real("215")/dd_real("4"), dd_real("10621")/dd_real("64"),
01472 dd_real("90687")/dd_real("256"), dd_real("567987")/dd_real("1024"),
01473 dd_real("168861")/dd_real("256"), dd_real("1246355")/dd_real("2048"),
01474 dd_real("7228859")/dd_real("16384"), dd_real("16583853")/dd_real("65536"),
01475 dd_real("7538115")/dd_real("65536"), dd_real("173376645")/dd_real("4194304"),
01476 dd_real("195747825")/dd_real("16777216"), dd_real("171655785")/dd_real("67108864"),
01477 dd_real("14375115")/dd_real("33554432"), dd_real("14375115")/dd_real("268435456"),
01478 dd_real("20764055")/dd_real("4294967296"), dd_real("5167525")/dd_real("17179869184"),
01479 dd_real("206701")/dd_real("17179869184"), dd_real("76153")/dd_real("274877906944"),
01480 dd_real("3311")/dd_real("1099511627776"), dd_real("43")/dd_real("4398046511104")
01481 };
01482 template< >
01483 struct pade_sqrt_denom<dd_real>
01484 {
01485 using array = std::array<dd_real, 22>;
01486 static const array denom;
01487 };
01488 const typename pade_sqrt_denom<dd_real>::array pade_sqrt_denom<dd_real>::denom =
01489 {
01490 dd_real("1"), dd_real("41")/dd_real("4"),
01491 dd_real("195")/dd_real("4"), dd_real("9139")/dd_real("64"),
01492 dd_real("73815")/dd_real("256"), dd_real("435897")/dd_real("1024"),
01493 dd_real("121737")/dd_real("256"), dd_real("840565")/dd_real("2048"),
01494 dd_real("4539051")/dd_real("16384"), dd_real("9641775")/dd_real("65536"),
01495 dd_real("4032015")/dd_real("65536"), dd_real("84672315")/dd_real("4194304"),
01496 dd_real("86493225")/dd_real("16777216"), dd_real("67863915")/dd_real("67108864"),
01497 dd_real("5014575")/dd_real("33554432"), dd_real("4345965")/dd_real("268435456"),
01498 dd_real("5311735")/dd_real("4294967296"), dd_real("1081575")/dd_real("17179869184"),
01499 dd_real("33649")/dd_real("17179869184"), dd_real("8855")/dd_real("274877906944"),
01500 dd_real("231")/dd_real("1099511627776"), dd_real("1")/dd_real("4398046511104")

```



```

01501 };
01502
01503 template< >
01504 struct pade_sqrt_numer<qd_real>
01505 {
01506 using array = std::array<qd_real, 34>;
01507 static const array number;
01508 };
01509 const typename pade_sqrt_numer<qd_real>::array pade_sqrt_numer<qd_real>::number =
01510 {
01511 qd_real("1"),
01512 qd_real("134"),
01513 qd_real("633485")/qd_real("256"),
01514 qd_real("15246721")/qd_real("1024"),
01515 qd_real("2518145487")/qd_real("65536"),
01516 qd_real("12301285425")/qd_real("262144"),
01517 qd_real("6344873535")/qd_real("131072"),
01518 qd_real("89075432355")/qd_real("2097152"),
01519 qd_real("267226297065")/qd_real("8388608"),
01520 qd_real("687479618945")/qd_real("33554432"),
01521 qd_real("379874182975")/qd_real("33554432"),
01522 qd_real("1443521895305")/qd_real("268435456"),
01523 qd_real("9425348845815")/qd_real("4294967296"),
01524 qd_real("13195488384141")/qd_real("17179869184"),
01525 qd_real("987417498133")/qd_real("4294967296"),
01526 qd_real("8055248011085")/qd_real("137438953472"),
01527 qd_real("6958363175533")/qd_real("549755813888"),
01528 qd_real("5056698705201")/qd_real("219902325552"),
01529 qd_real("766166470485")/qd_real("219902325552"),
01530 qd_real("766166470485")/qd_real("17592186044416"),
01531 qd_real("623623871325")/qd_real("140737488355328"),
01532 qd_real("203123203803")/qd_real("562949953421312"),
01533 qd_real("6478601247")/qd_real("281474976710656"),
01534 qd_real("5038912081")/qd_real("4503599627370496"),
01535 qd_real("719844583")/qd_real("18014398509481984"),
01536 qd_real("71853815")/qd_real("72057594037927936"),
01537 qd_real("1165197")/qd_real("72057594037927936"),
01538 qd_real("87703")/qd_real("576460752303423488"),
01539 qd_real("12529")/qd_real("18446744073709551616"),
01540 qd_real("67")/qd_real("73786976294838206464")
01541 };
01542
01543 template< >
01544 struct pade_sqrt_denom<qd_real>
01545 {
01546 using array = std::array<qd_real, 34>;
01547 static const array denom;
01548 };
01549 const typename pade_sqrt_denom<qd_real>::array pade_sqrt_denom<qd_real>::denom =
01550 {
01551 qd_real("1"),
01552 qd_real("126"),
01553 qd_real("557845")/qd_real("256"),
01554 qd_real("12515965")/qd_real("1024"),
01555 qd_real("1916797311")/qd_real("65536"),
01556 qd_real("8996462475")/qd_real("262144"),
01557 qd_real("4450881435")/qd_real("131072"),
01558 qd_real("59826782925")/qd_real("2097152"),
01559 qd_real("171503444385")/qd_real("8388608"),
01560 qd_real("420696483235")/qd_real("33554432"),
01561 qd_real("221120793075")/qd_real("33554432"),
01562 qd_real("797168807855")/qd_real("268435456"),
01563 qd_real("4923689695575")/qd_real("4294967296"),
01564 qd_real("6499270398159")/qd_real("17179869184"),
01565 qd_real("456864812569")/qd_real("4294967296"),
01566 qd_real("3486599885395")/qd_real("137438953472"),
01567 qd_real("2804116503573")/qd_real("549755813888"),
01568 qd_real("1886827875075")/qd_real("219902325552"),
01569 qd_real("263012370465")/qd_real("219902325552"),
01570 qd_real("240141729555")/qd_real("17592186044416"),
01571 qd_real("176848560525")/qd_real("140737488355328"),
01572 qd_real("51538723353")/qd_real("562949953421312"),
01573 qd_real("1450433115")/qd_real("281474976710656"),
01574 qd_real("977699359")/qd_real("4503599627370496"),
01575 qd_real("118183439")/qd_real("18014398509481984"),
01576 qd_real("9652005")/qd_real("72057594037927936"),
01577 qd_real("121737")/qd_real("72057594037927936"),
01578 qd_real("6545")/qd_real("576460752303423488"),
01579 qd_real("561")/qd_real("18446744073709551616"),
01580 qd_real("1")/qd_real("73786976294838206464")
01581 };
01582
01583 #endif
01584 }
01585
01586 namespace glucat
01587 {
01588 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01589 auto

```

```

01563 matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01564 const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01565 const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01566 {
01567 // Reference: [GW], Section 4.3, pp318-322
01568 // Reference: [GL], Section 11.3, p572-576
01569 // Reference: [Z], Padel
01570
01571 using traits_t = numeric_traits<Scalar_T>;
01572
01573 if (val.isnan())
01574 return traits_t::NaN();
01575
01576 const auto scr_val = val.scalar();
01577 if (val == scr_val)
01578 {
01579 if (scr_val < Scalar_T(0))
01580 return i * traits_t::sqrt(-scr_val);
01581 else
01582 return traits_t::sqrt(scr_val);
01583 }
01584
01585 // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01586 const auto scale =
01587 (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < Scalar_T(1))
01588 ? scr_val
01589 : (scr_val < Scalar_T(0))
01590 ? -abs(val)
01591 : abs(val);
01592 const auto sqrt_scale = traits_t::sqrt(traits_t::abs(scale));
01593 if (traits_t::isNaN_or_isInf(sqrt_scale))
01594 return traits_t::NaN();
01595
01596 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01597 auto rescale = multivector_t(sqrt_scale);
01598 if (scale < Scalar_T(0))
01599 rescale = i * sqrt_scale;
01600
01601 const auto& unitval = val / scale;
01602 static const auto max_norm = Scalar_T(1.0/4.0);
01603 auto use_approx_sqrt = true;
01604 auto use_cr_sqrt = false;
01605 auto scaled_result = multivector_t();
01606 #if defined(_GLUCAT_USE_EIGENVALUES)
01607 static const auto sqrt_2 = traits_t::sqrt(Scalar_T(2));
01608 if (level == 0)
01609 {
01610 // What kind of eigenvalues does the matrix contain?
01611 const auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
01612 const index_t next_level =
01613 (genus.m_is_singular)
01614 ? level
01615 : level + 1;
01616 switch (genus.m_eig_case)
01617 {
01618 case matrix::neg_real_eigs:
01619 scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01620 use_approx_sqrt = false;
01621 break;
01622 case matrix::both_eigs:
01623 {
01624 const auto safe_arg = genus.m_safe_arg;
01625 scaled_result = matrix_sqrt(exp(i*safe_arg) * unitval, i, next_level) * exp(-i*safe_arg /
01626 Scalar_T(2));
01627 }
01628 use_approx_sqrt = false;
01629 break;
01630 default:
01631 break;
01632 }
01633 use_cr_sqrt = genus.m_is_singular;
01634 }
01635 #endif
01636 if (use_approx_sqrt)
01637 {
01638 scaled_result =
01639 (norm(unitval - Scalar_T(1)) < max_norm)
01640 // Pade' approximation of square root
01641 ? pade_approx(pade::pade_sqrt_numer<Scalar_T>::numer,
01642 pade::pade_sqrt_denom<Scalar_T>::denom,
01643 unitval - Scalar_T(1))
01644 // Product form of Denman-Beavers square root iteration
01645 : (use_cr_sqrt)
01646 ? cr_sqrt(unitval)
01647 : db_sqrt(unitval);
01648 }
01649 return (scaled_result.isnan() ||

```

```

01649 !approx_equal(pow(scaled_result, 2), unitval))
01650 ? traits_t::NaN()
01651 : scaled_result * rescale;
01652 }
01653
01654 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01655 auto
01656 sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01657 bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01658 {
01659 // Reference: [GW], Section 4.3, pp318-322
01660 // Reference: [GL], Section 11.3, p572-576
01661 // Reference: [Z], Padel
01662
01663 using traits_t = numeric_traits<Scalar_T>;
01664
01665 if (val.isnan())
01666 return traits_t::NaN();
01667
01668 check_complex(val, i, prechecked);
01669
01670 switch (Tune_P::function_precision)
01671 {
01672 case precision_demoted:
01673 {
01674 using demoted_scalar_t = typename traits_t::demoted::type;
01675 using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
01676
01677 const auto& demoted_val = demoted_multivector_t(val);
01678 const auto& demoted_i = demoted_multivector_t(i);
01679
01680 return matrix_sqrt(demoted_val, demoted_i, 0);
01681 }
01682 break;
01683 case precision_promoted:
01684 {
01685 using promoted_scalar_t = typename traits_t::promoted::type;
01686 using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
01687
01688 const auto& promoted_val = promoted_multivector_t(val);
01689 const auto& promoted_i = promoted_multivector_t(i);
01690
01691 return matrix_sqrt(promoted_val, promoted_i, 0);
01692 }
01693 break;
01694 default:
01695 return matrix_sqrt(val, i, 0);
01696 }
01697 }
01698 }
01699
01700 namespace pade {
01701 // Reference: [Z], Padel
01702 template< typename Scalar_T >
01703 struct pade_log_number
01704 {
01705 using array = std::array<Scalar_T, 14>;
01706 static const array number;
01707 };
01708
01709 template< typename Scalar_T >
01710 const typename pade_log_number<Scalar_T>::array pade_log_number<Scalar_T>::number =
01711 {
01712 0.0, 1.0, 6.0, 4741.0/300.0,
01713 1441.0/60.0, 107091.0/4600.0, 8638.0/575.0, 263111.0/40250.0,
01714 153081.0/80500.0, 395243.0/1101240.0, 28549.0/688275.0, 605453.0/228813200.0,
01715 785633.0/10296594000.0, 1145993.0/1873980108000.0
01716 };
01717
01718 // Reference: [Z], Padel
01719 template< typename Scalar_T >
01720 struct pade_log_denom
01721 {
01722 using array = std::array<Scalar_T, 14>;
01723 static const array denom;
01724 };
01725
01726 template< typename Scalar_T >
01727 const typename pade_log_denom<Scalar_T>::array pade_log_denom<Scalar_T>::denom =
01728 {
01729 1.0, 13.0/2.0, 468.0/25.0, 1573.0/50.0,
01730 1573.0/46.0, 11583.0/460.0, 10296.0/805.0, 2574.0/575.0,
01731 11583.0/10925.0, 143.0/874.0, 572.0/37145.0, 117.0/148580.0,
01732 13.0/742900.0, 1.0/10400600.0
01733 };
01734
01735 template< >
01736 struct pade_log_number<float>
01737 {

```

```

01738 using array = std::array<float, 10>;
01739 static const array number;
01740 };
01741 const typename pade_log_number<float>::array pade_log_number<float>::number =
01742 {
01743 0.0, 1.0, 4.0, 1337.0/204.0,
01744 385.0/68.0, 1879.0/680.0, 193.0/255.0, 197.0/1820.0,
01745 419.0/61880.0, 7129.0/61261200.0
01746 };
01747 template< >
01748 struct pade_log_denom<float>
01749 {
01750 using array = std::array<float, 10>;
01751 static const array denom;
01752 };
01753 const typename pade_log_denom<float>::array pade_log_denom<float>::denom =
01754 {
01755 1.0, 9.0/2.0, 144.0/17.0, 147.0/17.0,
01756 441.0/85.0, 63.0/34.0, 84.0/221.0, 9.0/221.0,
01757 9.0/4862.0, 1.0/48620.0
01758 };
01759
01760 template< >
01761 struct pade_log_number<long double>
01762 {
01763 using array = std::array<long double, 18>;
01764 static const array number;
01765 };
01766 const typename pade_log_number<long double>::array pade_log_number<long double>::number =
01767 {
01768 0.0L, 1.0L, 8.0L,
01769 3835.0L/132.0L, 8365.0L/132.0L, 11363807.0L/122760.0L, 162981.0L/1705.0L,
01770 9036157.0L/125860.0L,
01771 18009875.0L/453096.0L, 44211925.0L/2718576.0L, 4149566.0L/849555.0L,
01772 16973929.0L/16020180.0L,
01773 172459.0L/1068012.0L, 116317061.0L/7025382936.0L, 19679783.0L/18441630207.0L,
01774 23763863.0L/614721006900.0L,
01775 50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
01776 };
01777 template< >
01778 struct pade_log_denom<long double>
01779 {
01780 using array = std::array<long double, 18>;
01781 static const array denom;
01782 };
01783 const typename pade_log_denom<long double>::array pade_log_denom<long double>::denom =
01784 {
01785 1.0L, 17.0L/2.0L, 1088.0L/33.0L,
01786 850.0L/11.0L, 41650.0L/341.0L, 140777.0L/1023.0L, 1126216.0L/9889.0L,
01787 63206.0L/899.0L,
01788 790075.0L/24273.0L, 60775.0L/5394.0L, 38896.0L/13485.0L,
01789 21658.0L/40455.0L,
01790 21658.0L/310155.0L, 4165.0L/682341.0L, 680.0L/2047023.0L,
01791 34.0L/3411705.0L,
01792 17.0L/129644790.0L, 1.0L/2333606220
01793 };
01794 #if defined(_GLUCAT_USE_QD)
01795 template< >
01796 struct pade_log_number<dd_real>
01797 {
01798 using array = std::array<dd_real, 22>;
01799 static const array number;
01800 };
01801 const typename pade_log_number<dd_real>::array pade_log_number<dd_real>::number =
01802 {
01803 dd_real("0"), dd_real("1"),
01804 dd_real("10"), dd_real("22781")/dd_real("492"),
01805 dd_real("21603")/dd_real("164"), dd_real("5492649")/dd_real("21320"),
01806 dd_real("978724")/dd_real("2665"), dd_real("4191605")/dd_real("10619"),
01807 dd_real("12874933")/dd_real("39442"), dd_real("11473457")/dd_real("54612"),
01808 dd_real("2406734")/dd_real("22755"), dd_real("166770367")/dd_real("4004880"),
01809 dd_real("30653165")/dd_real("2402928"), dd_real("647746389")/dd_real("215195552"),
01810 dd_real("25346331")/dd_real("47074027"), dd_real("278270613")/dd_real("3900419380"),
01811 dd_real("105689791")/dd_real("15601677520"), dd_real("606046475")/dd_real("1379188292768"),
01812 dd_real("969715")/dd_real("53502994116"), dd_real("11098301")/dd_real("26204577562592"),
01813 dd_real("118999")/dd_real("26204577562592"), dd_real("18858053")/dd_real("1392249205900512960")
01814 };
01815 template< >
01816 struct pade_log_denom<dd_real>
01817 {
01818 using array = std::array<dd_real, 22>;
01819 static const array denom;
01820 };
01821 const typename pade_log_denom<dd_real>::array pade_log_denom<dd_real>::denom =
01822 {

```

```

01817 dd_real("1"), dd_real("21")/dd_real("2"),
01818 dd_real("2100")/dd_real("41"), dd_real("12635")/dd_real("82"),
01819 dd_real("341145")/dd_real("1066"), dd_real("1037799")/dd_real("2132"),
01820 dd_real("11069856")/dd_real("19721"), dd_real("9883800")/dd_real("19721"),
01821 dd_real("6918660")/dd_real("19721"), dd_real("293930")/dd_real("1517"),
01822 dd_real("1410864")/dd_real("16687"), dd_real("88179")/dd_real("3034"),
01823 dd_real("734825")/dd_real("94054"), dd_real("305235")/dd_real("188108"),
01824 dd_real("348840")/dd_real("1363783"), dd_real("40698")/dd_real("1363783"),
01825 dd_real("6783")/dd_real("2727566"), dd_real("9975")/dd_real("70916716"),
01826 dd_real("266")/dd_real("53187537"), dd_real("7")/dd_real("70916716"),
01827 dd_real("7")/dd_real("8155422340"), dd_real("1")/dd_real("538257874440")
01828 };
01829
01830 template< >
01831 struct pade_log_number<qd_real>
01832 {
01833 using array = std::array<qd_real, 34>;
01834 static const array number;
01835 };
01836 const typename pade_log_number<qd_real>::array pade_log_number<qd_real>::number =
01837 {
01838 qd_real("0"), qd_real("1"),
01839 qd_real("16"),
01840 qd_real("95201")/qd_real("780"),
01841 qd_real("30721")/qd_real("52"),
01842 qd_real("7416257")/qd_real("3640"),
01843 qd_real("1039099")/qd_real("195"),
01844 qd_real("6097772319")/qd_real("555100"),
01845 qd_real("1564058073")/qd_real("85400"),
01846 qd_real("30404640205")/qd_real("1209264"),
01847 qd_real("725351278")/qd_real("25193"),
01848 qd_real("4092322670789")/qd_real("147429436"),
01849 qd_real("4559713849589")/qd_real("201040140"),
01850 qd_real("5049361751189")/qd_real("320023080"),
01851 qd_real("74979677195")/qd_real("8000577"),
01852 qd_real("16569850691873")/qd_real("3481514244"),
01853 qd_real("1065906022369")/qd_real("515779888"),
01854 qd_real("335956770855841")/qd_real("438412904800"),
01855 qd_real("1462444287585964")/qd_real("6041877844275"),
01856 qd_real("397242326339851")/qd_real("6122436215532"),
01857 qd_real("64211291334131")/qd_real("4373168725380"),
01858 qd_real("142322343550859")/qd_real("51080680851480"),
01859 qd_real("154355972958659")/qd_real("351179680853925"),
01860 qd_real("167483568676259")/qd_real("2937139148960100"),
01861 qd_real("4230788929433")/qd_real("704913395750424"),
01862 qd_real("197968763176019")/qd_real("392923948371995600"),
01863 qd_real("10537522306718")/qd_real("319250708052246425"),
01864 qd_real("236648286272519")/qd_real("144249197475035425500"),
01865 qd_real("260715545088119")/qd_real("4375558990076074573500"),
01866 qd_real("289596255666839")/qd_real("192874640282553367199880"),
01867 qd_real("880262510547")/qd_real("361639950529787563499775"),
01868 qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
01869 qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
01870 qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
01871 };
01872
01873 template< >
01874 struct pade_log_denom<qd_real>
01875 {
01876 using array = std::array<qd_real, 34>;
01877 static const array denom;
01878 };
01879 const typename pade_log_denom<qd_real>::array pade_log_denom<qd_real>::denom =
01880 {
01881 qd_real("1"),
01882 qd_real("33")/qd_real("2"),
01883 qd_real("8448")/qd_real("65"),
01884 qd_real("42284")/qd_real("65"),
01885 qd_real("211420")/qd_real("91"),
01886 qd_real("573562")/qd_real("91"),
01887 qd_real("32119472")/qd_real("2379"),
01888 qd_real("92917044")/qd_real("3965"),
01889 qd_real("603960786")/qd_real("17995"),
01890 qd_real("144626625")/qd_real("3599"),
01891 qd_real("2776831200")/qd_real("68381"),
01892 qd_real("16692542100")/qd_real("478667"),
01893 qd_real("12241197540")/qd_real("478667"),
01894 qd_real("1098569010")/qd_real("68381"),
01895 qd_real("31387686000")/qd_real("3624193"),
01896 qd_real("9939433900")/qd_real("2479711"),
01897 qd_real("67091178825")/qd_real("42155087"),
01898 qd_real("2683647153")/qd_real("4959422"),
01899 qd_real("19083713088")/qd_real("121505839"),
01900 qd_real("4708152900")/qd_real("121505839"),
01901 qd_real("941630580")/qd_real("116546417"),
01902 qd_real("88704330")/qd_real("62755763"),
01903 qd_real("12902448")/qd_real("62755763"),
01904 qd_real("1542684")/qd_real("62755763"),

```

```

01876 qd_real("6427850")/qd_real("2698497809"),
qd_real("3471039")/qd_real("18889484663"),
01877 qd_real("8544096")/qd_real("774468871183"),
qd_real("39556")/qd_real("79027435835"),
01878 qd_real("118668")/qd_real("7191496660985"),
qd_real("10230")/qd_real("27327687311743"),
01879 qd_real("5456")/qd_real("1011124430534491"),
qd_real("44")/qd_real("1011124430534491"),
01880 qd_real("11")/qd_real("70778710137414370"),
qd_real("1")/qd_real("7219428434016265740")
01881 };
01882 #endif
01883 }
01884
01885 namespace glucat{
01887 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01888 static
01889 auto
01890 pade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01891 {
01892 // Reference: [GW], Section 4.3, pp318-322
01893 // Reference: [CHKL]
01894 // Reference: [GL], Section 11.3, p572-576
01895 // Reference: [Z], Padel
01896
01897 using traits_t = numeric_traits<Scalar_T>;
01898 if (val == Scalar_T(0) || val.isnan())
01899 return traits_t::NaN();
01900 else
01901 return pade_approx(pade::pade_log_number<Scalar_T>::number,
pade::pade_log_denom<Scalar_T>::denom,
01902 val - Scalar_T(1));
01903 }
01904
01905 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01906 static
01907 auto
01908 cascade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01909 {
01910 // Reference: [CHKL]
01911 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01912 using traits_t = numeric_traits<Scalar_T>;
01913 if (val == Scalar_T(0) || val.isnan())
01914 return traits_t::NaN();
01915
01916 using limits_t = std::numeric_limits<Scalar_T>;
01917 static const auto epsilon = limits_t::epsilon();
01918 static const auto max_inner_norm = traits_t::pow(epsilon, 2);
01919 static const auto max_outer_norm = Scalar_T(6.0/limits_t::digits);
01920 auto Y = val;
01921 auto E = multivector_t(Scalar_T(0));
01922 Scalar_T norm_Y_1;
01923 auto pow_2_outer_step = Scalar_T(1);
01924 auto pow_4_outer_step = Scalar_T(1);
01925 int outer_step;
01926 for (outer_step = 0, norm_Y_1 = norm(Y - Scalar_T(1));
outer_step != Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm;
01927 ++outer_step, norm_Y_1 = norm(Y - Scalar_T(1)))
01928 {
01929 if (Y == Scalar_T(0) || Y.isnan())
01930 return traits_t::NaN();
01931
01932 // Incomplete product form of Denman-Beavers square root iteration
01933 auto M = Y;
01934 for (auto
inner_step = 0;
01935 inner_step != Tune_P::log_max_inner_steps &&
norm(M - Scalar_T(1)) * pow_4_outer_step > max_inner_norm;
01936 ++inner_step)
01937 db_step(M, Y);
01938
01939 E += (M - Scalar_T(1)) * pow_2_outer_step;
01940 pow_2_outer_step *= Scalar_T(2);
01941 pow_4_outer_step *= Scalar_T(4);
01942 }
01943 if (outer_step == Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm)
01944 return traits_t::NaN();
01945 else
01946 return pade_log(Y) * pow_2_outer_step - E;
01947 }
01948
01949 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01950 auto
01951 matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01952

```

```

01959 const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01960 {
01961 // Scaled incomplete square root cascade and scaled Pade' approximation of log
01962 // Reference: [CHKL]
01963
01964 using traits_t = numeric_traits<Scalar_T>;
01965 if (val == Scalar_T(0) || val.isnan())
01966 return traits_t::NaN();
01967
01968 static const auto pi = traits_t::pi();
01969 const auto scr_val = val.scalar();
01970 if (val == scr_val)
01971 {
01972 if (scr_val < Scalar_T(0))
01973 return i * pi + traits_t::log(-scr_val);
01974 else
01975 return traits_t::log(scr_val);
01976 }
01977
01978 // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01979 const auto max_norm = Scalar_T(1.0/9.0);
01980 const auto scale =
01981 (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < max_norm)
01982 ? scr_val
01983 : (scr_val < Scalar_T(0))
01984 ? -abs(val)
01985 : abs(val);
01986 if (scale == Scalar_T(0))
01987 return traits_t::NaN();
01988
01989 using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01990 const auto log_scale = traits_t::log(traits_t::abs(scale));
01991 auto rescale = multivector_t(log_scale);
01992 if (scale < Scalar_T(0))
01993 rescale = i * pi + log_scale;
01994 const auto unitval = val/scale;
01995 if (inv(unitval).isnan())
01996 return traits_t::NaN();
01997
01998 #if defined(_GLUCAT_USE_EIGENVALUES)
01999 auto scaled_result = multivector_t();
02000 if (level == 0)
02001 {
02002 // What kind of eigenvalues does the matrix contain?
02003 auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
02004 switch (genus.m_eig_case)
02005 {
02006 case matrix::neg_real_eigs:
02007 scaled_result = matrix_log(-i * unitval, i, level + 1) + i * pi/Scalar_T(2);
02008 break;
02009 case matrix::both_eigs:
02010 {
02011 const Scalar_T safe_arg = genus.m_safe_arg;
02012 scaled_result = matrix_log(exp(i*safe_arg) * unitval, i, level + 1) - i * safe_arg;
02013 }
02014 break;
02015 default:
02016 scaled_result = cascade_log(unitval);
02017 break;
02018 }
02019 }
02020 else
02021 scaled_result = cascade_log(unitval);
02022 #else
02023 auto scaled_result = cascade_log(unitval);
02024 #endif
02025 return (scaled_result.isnan())
02026 ? traits_t::NaN()
02027 : scaled_result + rescale;
02028 }
02029
02030 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02031 auto
02032 log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
02033 bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02034 {
02035 using traits_t = numeric_traits<Scalar_T>;
02036
02037 if (val == Scalar_T(0) || val.isnan())
02038 return traits_t::NaN();
02039
02040 check_complex(val, i, prechecked);
02041
02042 switch (Tune_P::function_precision)
02043 {
02044 case precision_demoted:
02045 {

```

```

02046 using demoted_scalar_t = typename traits_t::demoted::type;
02047 using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02048
02049 const auto& demoted_val = demoted_multivector_t(val);
02050 const auto& demoted_i = demoted_multivector_t(i);
02051
02052 return matrix_log(demoted_val, demoted_i, 0);
02053 }
02054 break;
02055 case precision_promoted:
02056 {
02057 using promoted_scalar_t = typename traits_t::promoted::type;
02058 using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02059
02060 const auto& promoted_val = promoted_multivector_t(val);
02061 const auto& promoted_i = promoted_multivector_t(i);
02062
02063 return matrix_log(promoted_val, promoted_i, 0);
02064 }
02065 break;
02066 default:
02067 return matrix_log(val, i, 0);
02068 }
02069 }
02070
02071 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02072 auto
02073 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02074 {
02075 {
02076 using traits_t = numeric_traits<Scalar_T>;
02077 if (val.isnan())
02078 return traits_t::NaN();
02079
02080 const auto scr_val = val.scalar();
02081 if (val == scr_val)
02082 return traits_t::exp(scr_val);
02083
02084 switch (Tune_P::function_precision)
02085 {
02086 case precision_demoted:
02087 {
02088 using demoted_scalar_t = typename traits_t::demoted::type;
02089 using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02090
02091 const auto& demoted_val = demoted_multivector_t(val);
02092 return clifford_exp(demoted_val);
02093 }
02094 break;
02095 case precision_promoted:
02096 {
02097 using promoted_scalar_t = typename traits_t::promoted::type;
02098 using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02099
02100 const auto& promoted_val = promoted_multivector_t(val);
02101 return clifford_exp(promoted_val);
02102 }
02103 break;
02104 default:
02105 return clifford_exp(val);
02106 }
02107 }
02108 }
02109 #endif // _GLUCAT_MATRIX_MULTI_IMP_H

```

## 9.39 glucat/portability.h File Reference

```

#include <boost/version.hpp>
#include <cmath>

```



- `#define _GLUCAT_ISNAN(x)`
- `#define _GLUCAT_ISINF(x)`
- `#define UBLAS_ABS abs`
- `#define UBLAS_SQRT sqrt`

#### 9.39.1.1 \_GLUCAT\_ISINF

**Value:**

Definition at line 43 of file [portability.h](#).

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### 9.39.1.2 \_GLUCAT\_ISNAN

```
#define _GLUCAT_ISNAN(
 x)
```

#### Value:

```
(x != x)
```

Definition at line 42 of file [portability.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#).

### 9.39.1.3 UBLAS\_ABS

```
#define UBLAS_ABS abs
```

Definition at line 51 of file [portability.h](#).

### 9.39.1.4 UBLAS\_SQRT

```
#define UBLAS_SQRT sqrt
```

Definition at line 52 of file [portability.h](#).

## 9.40 portability.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_PORTABILITY_H
00002 #define _GLUCAT_PORTABILITY_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 portability.h : Work around non-standard compilers and libraries
00006 -----
00007 begin : Sun 2001-08-18
00008 copyright : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include <boost/version.hpp>
00035 #include <cmath>
00036
00037 // Workaround for isnan and isinf
```

```

00038 #if __cplusplus > 199711L
00039 # define _GLUCAT_ISNAN(x) (std::isnan(x))
00040 # define _GLUCAT_ISINF(x) (std::isinf(x))
00041 #else
00042 # define _GLUCAT_ISNAN(x) (x != x)
00043 # define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
00044 #endif
00045
00046 // Workaround for abs and sqrt
00047 #if BOOST_VERSION >= 103400
00048 # define UBLAS_ABS type_abs
00049 # define UBLAS_SQRT type_sqrt
00050 #else
00051 # define UBLAS_ABS abs
00052 # define UBLAS_SQRT sqrt
00053 #endif
00054
00055 // Use with Cygwin gcc to obtain __WORDSIZE
00056 #if defined(HAVE_BITS_WORDSIZE_H)
00057 # include <bits/wordsize.h>
00058 #endif
00059
00060 #endif // _GLUCAT_PORTABILITY_H

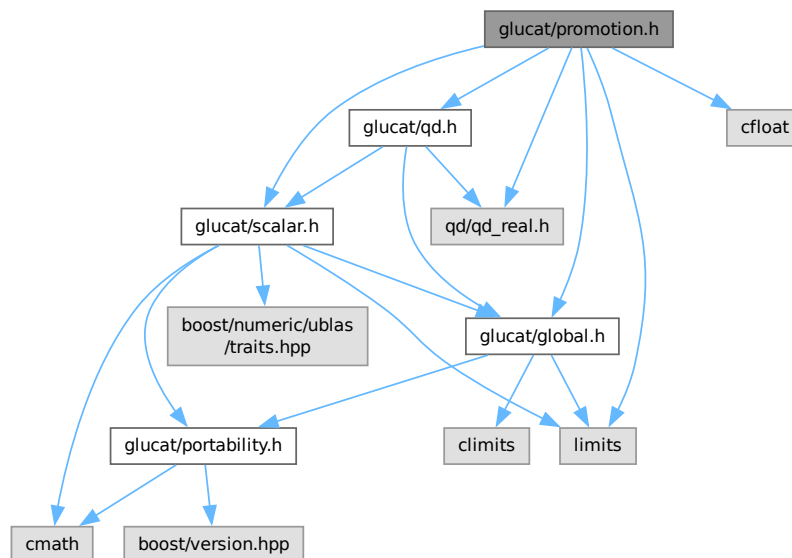
```

## 9.41 glucat/promotion.h File Reference

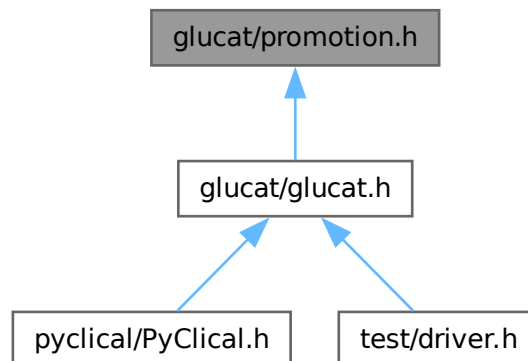
```

#include "glucat/global.h"
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <cfloat>
#include <limits>
#include <qd/qd_real.h>
Include dependency graph for promotion.h:

```



This graph shows which files directly or indirectly include this file:



## Classes

- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

## Namespaces

- namespace [glucat](#)

## 9.42 promotion.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_PROMOTION_H
00002 #define _GLUCAT_PROMOTION_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 promotion.h : Define promotion and demotion for specific scalar types
00006 -----
00007 begin : 2021-11-13
00008 copyright : (C) 2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was

```

```

00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036 #include "glucat/qd.h"
00037
00038 #include <cfloat>
00039 #include <limits>
00040
00041 #if defined(_GLUCAT_USE_QD)
00042 # include <qd/qd_real.h>
00043 #endif
00044
00045 namespace glucat
00046 {
00047 // Reference: [AA], 2.4, p. 30-31
00048
00049 #if !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00050 # if DBL_MANT_DIG < LDBL_MANT_DIG
00051 #
00052 template<>
00053 struct
00054 numeric_traits<double>::
00055 promoted {using type = long double;};
00056
00057 template<>
00058 struct
00059 numeric_traits<long double>::
00060 demoted {using type = double;};
00061
00062 # else
00063 template<>
00064 struct
00065 numeric_traits<double>::
00066 promoted {using type = double;};
00067
00068 template<>
00069 struct
00070 numeric_traits<long double>::
00071 demoted {using type = float;};
00072
00073 # endif // DBL_MANT_DIG < LDBL_MANT_DIG
00074
00075 # else
00076 template<>
00077 struct
00078 numeric_traits<long double>::
00079 promoted {using type = long double;};
00080
00081 # else
00082 # if (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00083 #
00084 template<>
00085 struct
00086 numeric_traits<double>::
00087 promoted {using type = long double;};
00088
00089 template<>
00090 struct
00091 numeric_traits<long double>::
00092 demoted {using type = double;};
00093
00094 template<>
00095 struct
00096 numeric_traits<long double>::
00097 promoted {using type = dd_real;};
00098
00099 template<>
00100 struct
00101 numeric_traits<dd_real>::
00102 demoted {using type = long double;};
00103
00104 template<>
00105 struct
00106 numeric_traits<dd_real>::
00107 promoted {using type = qd_real;};
00108
00109 template<>
00110 struct
00111 numeric_traits<qd_real>::
00112 promoted {using type = qd_real;};
00113
00114 template<>
00115 struct
00116 numeric_traits<qd_real>::
00117 promoted {using type = qd_real;};
00118
00119 template<>
00120 struct
00121 numeric_traits<qd_real>::
00122 promoted {using type = qd_real;};
00123
00124 template<>
00125 struct

```

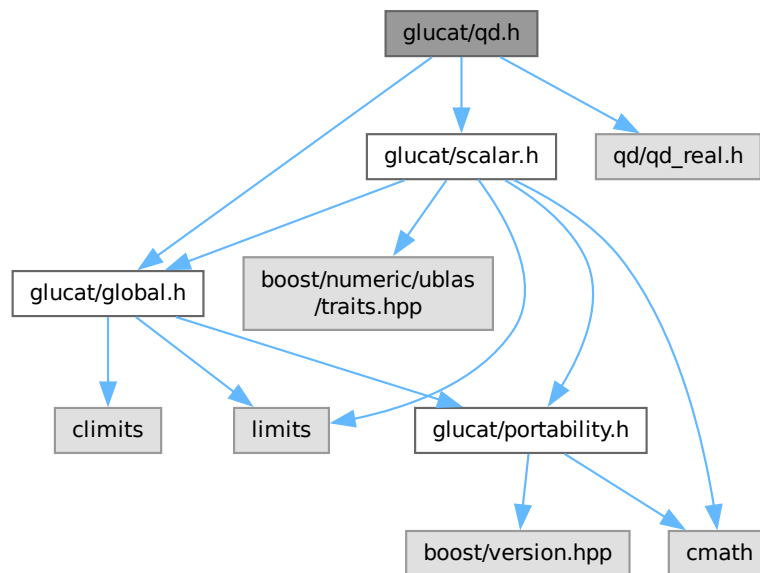
```

00125 numeric_traits<qd_real>::
00126 demoted {using type = dd_real;};
00127
00128 # elif (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00129
00131 template<>
00132 struct
00133 numeric_traits<double>::
00134 promoted {using type = dd_real;};
00135
00137 template<>
00138 struct
00139 numeric_traits<long double>::
00140 demoted {using type = float;};
00141
00143 template<>
00144 struct
00145 numeric_traits<long double>::
00146 promoted {using type = dd_real;};
00147
00149 template<>
00150 struct
00151 numeric_traits<dd_real>::
00152 demoted {using type = double;};
00153
00155 template<>
00156 struct
00157 numeric_traits<dd_real>::
00158 promoted {using type = qd_real;};
00159
00161 template<>
00162 struct
00163 numeric_traits<qd_real>::
00164 demoted {using type = dd_real;};
00165
00166 # else
00167
00169 template<>
00170 struct
00171 numeric_traits<double>::
00172 promoted {using type = dd_real;};
00173
00175 template<>
00176 struct
00177 numeric_traits<dd_real>::
00178 demoted {using type = double;};
00179
00181 template<>
00182 struct
00183 numeric_traits<dd_real>::
00184 promoted {using type = long double;};
00185
00187 template<>
00188 struct
00189 numeric_traits<long double>::
00190 demoted {using type = dd_real;};
00191
00193 template<>
00194 struct
00195 numeric_traits<long double>::
00196 promoted {using type = qd_real;};
00197
00199 template<>
00200 struct
00201 numeric_traits<qd_real>::
00202 demoted {using type = long double;};
00203
00204 # endif // (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00205
00207 template<>
00208 struct
00209 numeric_traits<qd_real>::
00210 promoted {using type = qd_real;};
00211
00212 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00213
00214 } // namespace glucat
00215
00216 #endif // _GLUCAT_PROMOTION_H

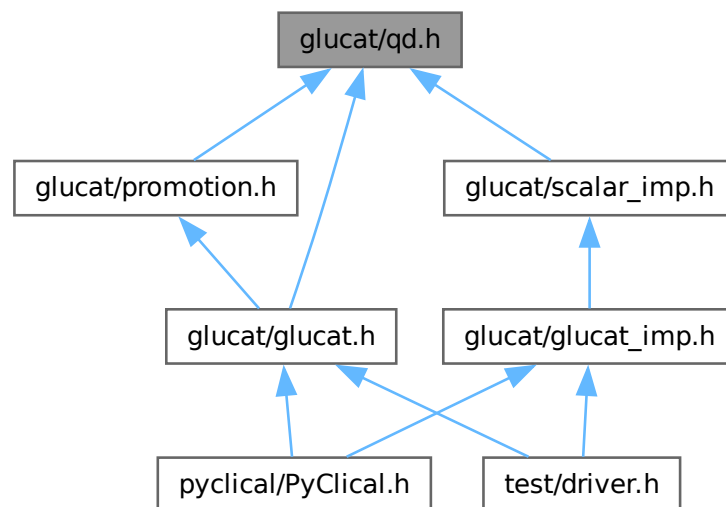
```

## 9.43 glucat/qd.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
#include <qd/qd_real.h>
Include dependency graph for qd.h:
```



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## 9.44 qd.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_QD_H
00002 #define _GLUCAT_QD_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 qd.h : Define functions for dd_real and qd_real as scalar_t
00006
00007 begin : 2010-03-23
00008 copyright : (C) 2010-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 #if defined(_GLUCAT_USE_QD)
00038 # include <qd/qd_real.h>
00039 #endif
00040
00041 namespace glucat
00042 {
00043 // Reference: [AA], 2.4, p. 30-31
00044
00045 #if defined(_GLUCAT_USE_QD) && defined(QD_API)
00046 # define _GLUCAT_QD_F(_T, _F) \
00047 template<> \
00048 inline \
00049 auto \
00050 numeric_traits<_T>:: \
00051 _F(const _T& val) -> _T \
00052 { return ::_F(val); }
00053
00054 template<>
00055 inline
00056 auto
00057 numeric_traits<dd_real>::
00058 isNaN(const dd_real& val) -> bool
00059 { return val.isnan(); }
00060
00061 template<>
00062 inline
00063 auto
00064 numeric_traits<dd_real>::
00065 isInf(const dd_real& val) -> bool
00066 { return val.isinf(); }
00067
00068 template<>
00069 inline
00070 auto
00071 numeric_traits<dd_real>::
00072 isNaN_or_isInf(const dd_real& val) -> bool

```



```

00079 { return val.isnan() || val.isinf(); }
00080
00082 template<>
00083 inline
00084 auto
00085 numeric_traits<dd_real>::
00086 to_int(const dd_real& val) -> int
00087 { return ::to_int(val); }
00088
00090 template<>
00091 inline
00092 auto
00093 numeric_traits<dd_real>::
00094 to_double(const dd_real& val) -> double
00095 { return ::to_double(val); }
00096
00098 template<>
00099 inline
00100 auto
00101 numeric_traits<dd_real>::
00102 fmod(const dd_real& lhs, const dd_real& rhs) -> dd_real
00103 { return ::fmod(lhs, rhs); }
00104
00106 template<>
00107 inline
00108 auto
00109 numeric_traits<dd_real>::
00110 pow(const dd_real& val, int n) -> dd_real
00111 {
00112 if (val == dd_real(0))
00113 {
00114 return
00115 (n < 0)
00116 ? NaN()
00117 : (n == 0)
00118 ? dd_real(1)
00119 : dd_real(0);
00120 }
00121 auto result = dd_real(1);
00122 auto power =
00123 (n < 0)
00124 ? dd_real(1)/val
00125 : val;
00126 for (auto
00127 k = std::abs(n);
00128 k != 0;
00129 k /= 2)
00130 {
00131 if (k % 2)
00132 result *= power;
00133 power *= power;
00134 }
00135 return result;
00136 }
00137
00139 template<>
00140 inline
00141 auto
00142 numeric_traits<dd_real>::
00143 pi() -> dd_real
00144 { return dd_real::_pi; }
00145
00147 template<>
00148 inline
00149 auto
00150 numeric_traits<dd_real>::
00151 ln_2() -> dd_real
00152 { return dd_real::_log2; }
00153
00155 _GLUCAT_QD_F(dd_real, exp)
00156
00157
00158 _GLUCAT_QD_F(dd_real, log)
00159
00160
00161 _GLUCAT_QD_F(dd_real, cos)
00162
00163
00164 _GLUCAT_QD_F(dd_real, acos)
00165
00166
00167 _GLUCAT_QD_F(dd_real, cosh)
00168
00169
00170 _GLUCAT_QD_F(dd_real, sin)
00171
00172

```

```

00173 _GLUCAT_QD_F(dd_real, asin)
00174
00175
00176 _GLUCAT_QD_F(dd_real, sinh)
00177
00178
00179 _GLUCAT_QD_F(dd_real, tan)
00180
00181
00182 _GLUCAT_QD_F(dd_real, atan)
00183
00184
00185 _GLUCAT_QD_F(dd_real, tanh)
00186
00187
00188 template<>
00189 inline
00190 auto
00191 numeric_traits<qd_real>::
00192 isNaN(const qd_real& val) -> bool
00193 { return val.isnan(); }
00194
00196 template<>
00197 inline
00198 auto
00199 numeric_traits<qd_real>::
00200 isInf(const qd_real& val) -> bool
00201 { return val.isinf(); }
00202
00204 template<>
00205 inline
00206 auto
00207 numeric_traits<qd_real>::
00208 isNaN_or_isInf(const qd_real& val) -> bool
00209 { return val.isnan() || val.isinf(); }
00210
00212 template<>
00213 inline
00214 auto
00215 numeric_traits<qd_real>::
00216 to_int(const qd_real& val) -> int
00217 { return ::to_int(val); }
00218
00220 template<>
00221 inline
00222 auto
00223 numeric_traits<qd_real>::
00224 to_double(const qd_real& val) -> double
00225 { return ::to_double(val); }
00226
00228 template<>
00229 inline
00230 auto
00231 numeric_traits<qd_real>::
00232 fmod(const qd_real& lhs, const qd_real& rhs) -> qd_real
00233 { return ::fmod(lhs, rhs); }
00234
00236 template<>
00237 inline
00238 auto
00239 numeric_traits<qd_real>::
00240 pow(const qd_real& val, int n) -> qd_real
00241 {
00242 if (val == qd_real(0))
00243 {
00244 return
00245 (n < 0)
00246 ? NaN()
00247 : (n == 0)
00248 ? qd_real(1)
00249 : qd_real(0);
00250 }
00251 auto result = qd_real(1);
00252 auto power =
00253 (n < 0)
00254 ? qd_real(1)/val
00255 : val;
00256 for (auto
00257 k = std::abs(n);
00258 k != 0;
00259 k /= 2)
00260 {
00261 if (k % 2)
00262 result *= power;
00263 power *= power;
00264 }
00265 return result;

```

```

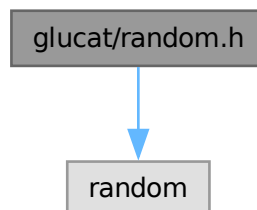
00266 }
00267
00269 template<>
00270 inline
00271 auto
00272 numeric_traits<qd_real>::
00273 pi() -> qd_real
00274 { return qd_real::_pi; }
00275
00277 template<>
00278 inline
00279 auto
00280 numeric_traits<qd_real>::
00281 ln_2() -> qd_real
00282 { return qd_real::_log2; }
00283
00285 _GLUCAT_QD_F(qd_real, exp)
00286
00287
00288 _GLUCAT_QD_F(qd_real, log)
00289
00290
00291 _GLUCAT_QD_F(qd_real, cos)
00292
00293
00294 _GLUCAT_QD_F(qd_real, acos)
00295
00296
00297 _GLUCAT_QD_F(qd_real, cosh)
00298
00299
00300 _GLUCAT_QD_F(qd_real, sin)
00301
00302
00303 _GLUCAT_QD_F(qd_real, asin)
00304
00305
00306 _GLUCAT_QD_F(qd_real, sinh)
00307
00308
00309 _GLUCAT_QD_F(qd_real, tan)
00310
00311
00312 _GLUCAT_QD_F(qd_real, atan)
00313
00314
00315 _GLUCAT_QD_F(qd_real, tanh)
00316
00317 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00318
00319 } // namespace glucat
00320
00321 #endif // _GLUCAT_QD_H

```

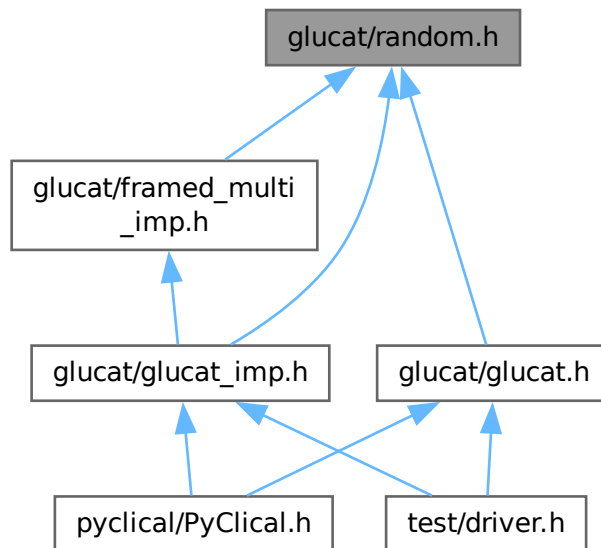
## 9.45 glucat/random.h File Reference

```
#include <random>
```

Include dependency graph for random.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::random_generator< Scalar_T >`  
Random number generator with single instance per `Scalar_T`.

## Namespaces

- namespace `glucat`

## 9.46 random.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_RANDOM_H
00002 #define _GLUCAT_RANDOM_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 random.h : Random number generator with single instance per Scalar_T
00006 -----
00007 begin : 2010-03-28
00008 copyright : (C) 2001-2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020

```

```

00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include <random>
00035
00036 namespace glucat
00037 {
00038 // Enforce singleton
00039 // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00040 template< typename Scalar_T >
00041 class random_generator
00042 {
00043 private:
00044 friend class friend_for_private_destructor;
00045 public:
00046 static auto generator() -> random_generator& { static random_generator g; return g; }
00047 random_generator(const random_generator&) = delete;
00048 auto operator= (const random_generator&) -> random_generator& = delete;
00049 private:
00050 static const unsigned long seed = 19590921UL;
00051
00052 std::mt19937 uint_gen;
00053 std::uniform_real_distribution<double> uniform_dist;
00054 std::normal_distribution<double> normal_dist;
00055
00056 random_generator() :
00057 uint_gen(), uniform_dist(0.0, 1.0), normal_dist(0.0, 1.0)
00058 { this->uint_gen.seed(seed); }
00059
00060 ~random_generator() = default;
00061
00062 public:
00063 auto uniform() -> Scalar_T
00064 { return Scalar_T(this->uniform_dist(this->uint_gen)); }
00065 auto normal() -> Scalar_T
00066 { return Scalar_T(this->normal_dist(this->uint_gen)); }
00067 };
00068 }
00069 #endif // _GLUCAT_RANDOM_H

```

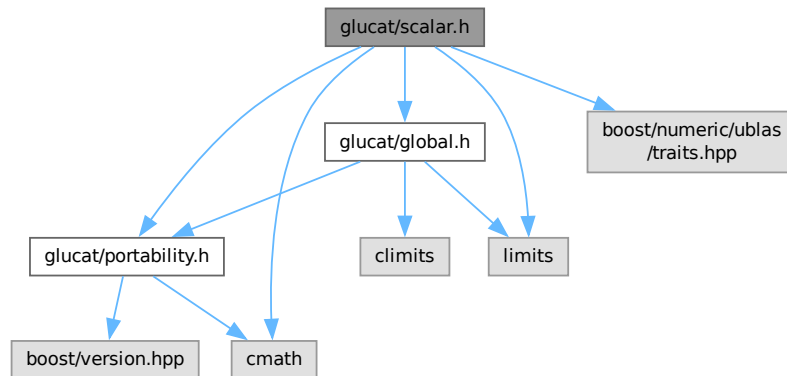
## 9.47 glucat/scalar.h File Reference

```

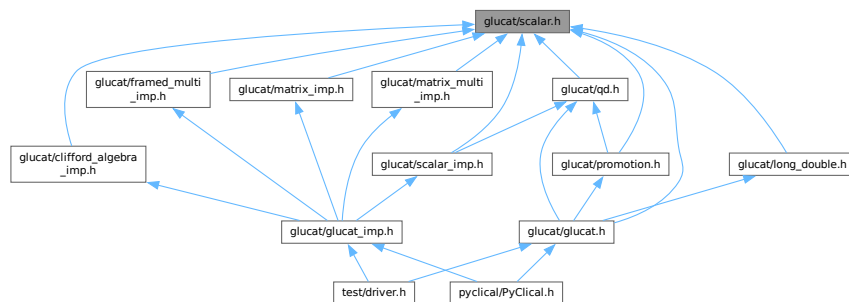
#include "glucat/portability.h"
#include "glucat/global.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::numeric\\_traits< Scalar\\_T >](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

## Namespaces

- namespace [glucat](#)

## Functions

- template<typename [Scalar\\_T](#)>  
auto [glucat::log2](#) (const [Scalar\\_T](#) &x) -> [Scalar\\_T](#)  
*Log base 2 of scalar.*

## 9.48 scalar.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_H
00002 #define _GLUCAT_SCALAR_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 scalar.h : Define functions for scalar_t
00006 -----
00007 begin : 2001-12-20
00008 copyright : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/portability.h"
00035 #include "glucat/global.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044 // Reference: [AA], 2.4, p. 30-31
00045 template< typename Scalar_T >
00046 class numeric_traits
00047 {
00048 private:
00049 inline
00050 static
00051 auto
00052 isInf(const Scalar_T& val, bool_to_type<false>) -> bool
00053 { return false; }
00054
00055 inline
00056 static
00057 auto
00058 isInf(const Scalar_T& val, bool_to_type<true>) -> bool
00059 { return _GLUCAT_ISINF(val); }
00060
00061 inline
00062 static
00063 auto
00064 isNaN(const Scalar_T& val, bool_to_type<false>) -> bool
00065 { return false; }
00066
00067 inline
00068 static
00069 auto
00070 isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00071 { return _GLUCAT_ISNAN(val); }
00072
00073 public:
00074 inline
00075 static
00076 auto
00077 isInf(const Scalar_T& val) -> bool
00078 {
00079 return isInf(val,
00080 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >());
00081 }
00082 }
00083 }

```

```

00090 inline
00091 static
00092 auto
00093 isNaN(const Scalar_T& val) -> bool
00094 {
00095 return isNaN(val,
00096 bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >());
00097 }
00098
00099 inline
00100 static
00101 auto
00102 isNaN_or_isInf(const Scalar_T& val) -> bool
00103 {
00104 return isNaN(val,
00105 bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >())
00106 || isInf(val,
00107 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >());
00108 }
00109
00110 inline
00111 static
00112 auto
00113 NaN() -> Scalar_T
00114 {
00115 return std::numeric_limits<Scalar_T>::has_quiet_NaN
00116 ? std::numeric_limits<Scalar_T>::quiet_NaN()
00117 : Scalar_T(std::log(0.0));
00118 }
00119
00120 inline
00121 static
00122 auto
00123 to_int(const Scalar_T& val) -> int
00124 { return static_cast<int>(val); }
00125
00126 inline
00127 static
00128 auto
00129 to_double(const Scalar_T& val) -> double
00130 { return static_cast<double>(val); }
00131
00132 template <typename Other_Scalar_T >
00133 inline
00134 static
00135 auto
00136 to_scalar_t(const Other_Scalar_T& val) -> Scalar_T
00137 { return static_cast<Scalar_T>(val); }
00138
00139 struct promoted {using type = double;};
00140
00141 struct demoted {using type = float;};
00142
00143 inline
00144 static
00145 auto
00146 fmod(const Scalar_T& lhs, const Scalar_T& rhs) -> Scalar_T
00147 { return std::fmod(lhs, rhs); }
00148
00149 inline
00150 static
00151 auto
00152 conj(const Scalar_T& val) -> Scalar_T
00153 { return val; }
00154
00155 inline
00156 static
00157 auto
00158 real(const Scalar_T& val) -> Scalar_T
00159 { return val; }
00160
00161 inline
00162 static
00163 auto
00164 imag(const Scalar_T& val) -> Scalar_T
00165 { return Scalar_T(0); }
00166
00167 inline
00168 static
00169 auto
00170 abs(const Scalar_T& val) -> Scalar_T
00171 { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_ABS(val); }
00172
00173 inline
00174 static
00175 auto
00176 pi() -> Scalar_T

```



Generated by Doxygen

```

00292 static
00293 auto
00294 tanh(const Scalar_T& val) -> Scalar_T
00295 { return std::tanh(val); }
00296
00297 };
00298
00299 template< typename Scalar_T >
00300 inline
00301 auto
00302 log2(const Scalar_T& x) -> Scalar_T
00303 { return numeric_traits<Scalar_T>::log2(x); }
00304 }
00305
00306 #endif // _GLUCAT_SCALAR_H

```

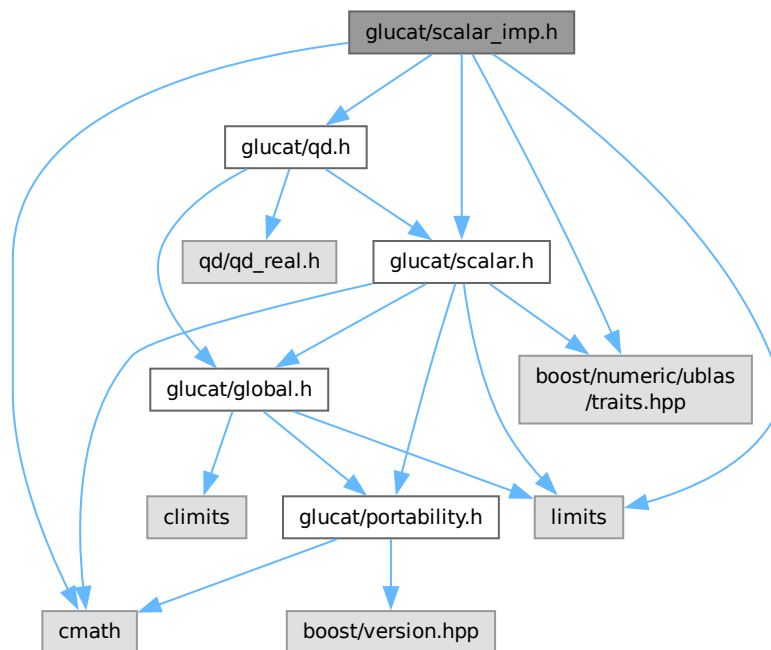
## 9.49 glucat/scalar\_imp.h File Reference

```

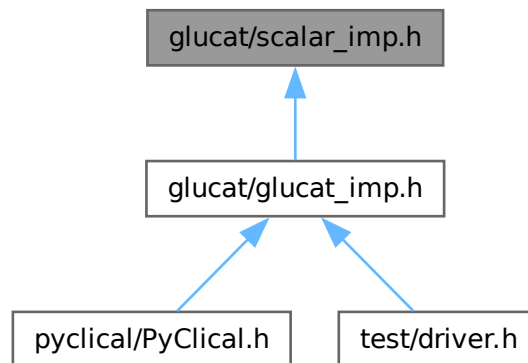
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## Functions

- `template<typename Scalar_T>`  
`auto glucat::to\_promote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::promoted::type`  
*Cast to promote.*
- `template<typename Scalar_T>`  
`auto glucat::to\_demote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::demoted::type`  
*Cast to demote.*

## 9.50 scalar\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_IMP_H
00002 #define _GLUCAT_SCALAR_IMP_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 scalar_imp.h : Define functions for scalar_t
00006 -----
00007 begin : 2001-12-20
00008 copyright : (C) 2001-2014 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.

```

```

00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/scalar.h"
00035 #include "glucat/qd.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044 // Reference: [AA], 2.4, p. 30-31
00045
00046 template< >
00047 template< typename Other_Scalar_T >
00048 inline
00049 auto
00050 numeric_traits<float>::
00051 to_scalar_t(const Other_Scalar_T& val) -> float
00052 { return static_cast<float>(numeric_traits<Other_Scalar_T>::to_double(val)); }
00053
00054 template< >
00055 template< typename Other_Scalar_T >
00056 inline
00057 auto
00058 numeric_traits<double>::
00059 to_scalar_t(const Other_Scalar_T& val) -> double
00060 { return numeric_traits<Other_Scalar_T>::to_double(val); }
00061
00062 #if defined(_GLUCAT_USE_QD)
00063 template< >
00064 template< >
00065 inline
00066 auto
00067 numeric_traits<long double>::
00068 to_scalar_t(const dd_real& val) -> long double
00069 { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00070
00071 template< >
00072 template< >
00073 inline
00074 auto
00075 numeric_traits<long double>::
00076 to_scalar_t(const qd_real& val) -> long double
00077 { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00078
00079 template< >
00080 template< >
00081 inline
00082 auto
00083 numeric_traits<dd_real>::
00084 to_scalar_t(const long double& val) -> dd_real
00085 { return {double(val), double(val - static_cast<long double>(double(val)))}; }
00086
00087 template< >
00088 template< >
00089 inline
00090 auto
00091 numeric_traits<dd_real>::
00092 to_scalar_t(const qd_real& val) -> dd_real
00093 { return {val.x[0], val.x[1]}; }
00094
00095 template< >
00096 template< >
00097 inline
00098 auto
00099 numeric_traits<qd_real>::
00100 to_scalar_t(const long double& val) -> qd_real
00101 { return {double(val), double(val - static_cast<long double>(double(val))), 0.0, 0.0}; }
00102
00103 template< >
00104 template< >
00105 inline
00106 auto
00107 numeric_traits<qd_real>::
00108 to_scalar_t(const dd_real& val) -> qd_real
00109 { return {val.x[0], val.x[1], 0.0, 0.0}; }
00110
00111
00112
00113
00114
00115
00116
00117
00118

```

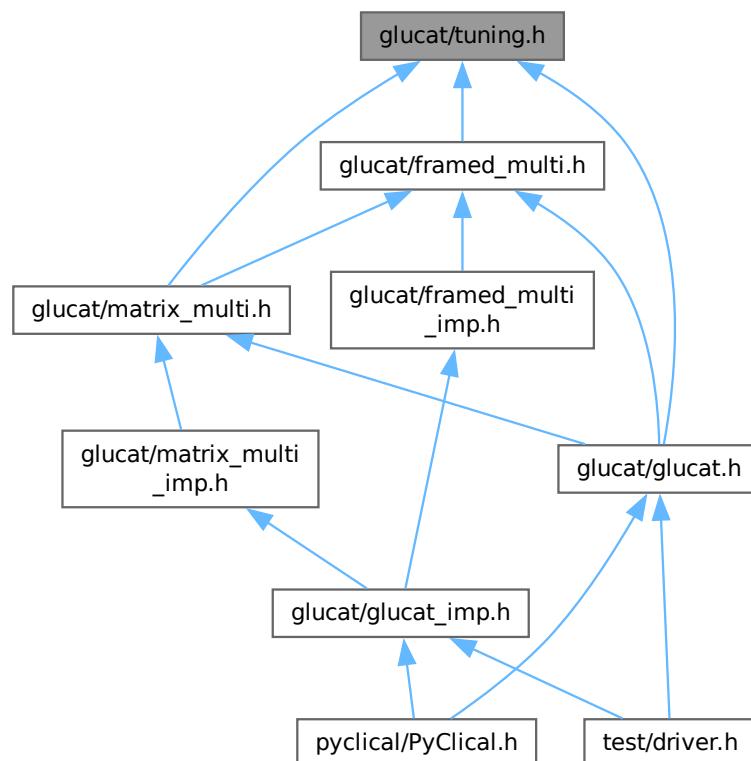
```

00119 #endif
00120
00122 template< typename Scalar_T >
00123 inline
00124 auto
00125 to_promote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::promoted::type
00126 {
00127 using promoted_scalar_t = typename numeric_traits<Scalar_T>::promoted::type;
00128 return numeric_traits<promoted_scalar_t>::to_scalar_t(val);
00129 }
00130
00132 template< typename Scalar_T >
00133 inline
00134 auto
00135 to_demote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::demoted::type
00136 {
00137 using demoted_scalar_t = typename numeric_traits<Scalar_T>::demoted::type;
00138 return numeric_traits<demoted_scalar_t>::to_scalar_t(val);
00139 }
00140 }
00141
00142 #endif // _GLUCAT_SCALAR_IMP_H

```

## 9.51 glucat/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Functions

- [\\_GLUCAT\\_CTAssert](#) (std::numeric\_limits< unsigned int >::radix==2, CannotSetThresholds) namespace [glucat](#)

## 9.51.1 Function Documentation

### 9.51.1.1 `_GLUCAT_CTAssert()`

```
_GLUCAT_CTAssert (
 std::numeric_limits< unsigned int >::radix == 2,
 CannotSetThresholds)
```

Base class for policies

Precision policy

Tuning policy

Minimum index count needed to invoke matrix multiplication algorithm

Maximum steps of iterative refinement in division algorithm

Maximum number of steps in cyclic reduction square root iteration

Maximum number of steps in Denman-Beavers square root iteration

Maximum number of incomplete square roots in cascade log algorithm

Maximum number of steps in incomplete square root within cascade log algorithm

Maximum index count of folded frames in basis cache

Minimum map size needed to invoke generalized FFT

Minimum matrix dimension needed to invoke inverse generalized FFT

Minimum size needed for to invoke faster products algorithms

Denominator of proportion of different bits allowed in approximate equality

Extra number of different bits allowed in approximate equality

Precision used for exp, log and sqrt functions

Definition at line 35 of file [tuning.h](#).

References [\\_GLUCAT\\_CTAssert](#).

## 9.52 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TUNING_H
00002 #define GLUCAT_TUNING_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 tuning.h : Policy classes to control tuning
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 // If radix of int is not 2, we can't easily set thresholds
00035 _GLUCAT_CTAssert(std::numeric_limits<unsigned int>::radix == 2, CannotSetThresholds)
00036
00037 namespace glucat
00038 {
00039 struct policy{};
00040
00041 enum precision_t
00042 {
00043 precision_demoted,
00044 precision_same,
00045 precision_promoted
00046 };
00047
00048 // Tuning policy default constants
00049 const unsigned int Tuning_Default_Mult_Matrix_Threshold = 8;
00050 const unsigned int Tuning_Default_Div_Max_Steps = 4;
00051 const unsigned int Tuning_Default_CR_Sqrt_Max_Steps = 256;
00052 const unsigned int Tuning_Default_DB_Sqrt_Max_Steps = 256;
00053 const unsigned int Tuning_Default_Log_Max_Outer_Steps = 256;
00054 const unsigned int Tuning_Default_Log_Max_Inner_Steps = 32;
00055 const unsigned int Tuning_Default_Basis_Max_Count = 12;
00056 const unsigned int Tuning_Default_Fast_Size_Threshold = 1 << 6;
00057 const unsigned int Tuning_Default_Inv_Fast_Dim_Threshold = 1 << 3;
00058 const unsigned int Tuning_Default_Products_Size_Threshold = 1 << 22;
00059 const unsigned int Tuning_Default_Denom_Different_Bits = 8;
00060 const unsigned int Tuning_Default_Extra_Different_Bits = 8;
00061 const precision_t Tuning_Default_Function_Precision = precision_same;
00062
00063 template
00064 <
00065 unsigned int Mult_Matrix_Threshold = Tuning_Default_Mult_Matrix_Threshold,
00066 unsigned int Div_Max_Steps = Tuning_Default_Div_Max_Steps,
00067 unsigned int CR_Sqrt_Max_Steps = Tuning_Default_CR_Sqrt_Max_Steps,
00068 unsigned int DB_Sqrt_Max_Steps = Tuning_Default_DB_Sqrt_Max_Steps,
00069 unsigned int Log_Max_Outer_Steps = Tuning_Default_Log_Max_Outer_Steps,
00070 unsigned int Log_Max_Inner_Steps = Tuning_Default_Log_Max_Inner_Steps,
00071 unsigned int Basis_Max_Count = Tuning_Default_Basis_Max_Count,
00072 unsigned int Fast_Size_Threshold = Tuning_Default_Fast_Size_Threshold,
00073 unsigned int Inv_Fast_Dim_Threshold = Tuning_Default_Inv_Fast_Dim_Threshold,
00074 unsigned int Products_Size_Threshold = Tuning_Default_Products_Size_Threshold,
00075 unsigned int Denom_Different_Bits = Tuning_Default_Denom_Different_Bits,
00076 unsigned int Extra_Different_Bits = Tuning_Default_Extra_Different_Bits,
00077 precision_t Function_Precision = Tuning_Default_Function_Precision
00078 >
00079 struct tuning : policy
00080 {
00081 using tune_p = tuning
00082 <

```

```

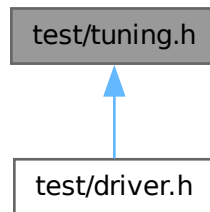
00086 Mult_Matrix_Threshold,
00087 Div_Max_Steps,
00088 CR_Sqrt_Max_Steps,
00089 DB_Sqrt_Max_Steps,
00090 Log_Max_Outer_Steps,
00091 Log_Max_Inner_Steps,
00092 Basis_Max_Count,
00093 Fast_Size_Threshold,
00094 Inv_Fast_Dim_Threshold,
00095 Products_Size_Threshold,
00096 Denom_Different_Bits,
00097 Extra_Different_Bits,
00098 Function_Precision
00099 >;
00100 // Tuning for multiplication
00102 enum { mult_matrix_threshold = Mult_Matrix_Threshold };
00103 // Tuning for division
00105 enum { div_max_steps = Div_Max_Steps };
00106 // Tuning for sqrt
00108 enum { cr_sqrt_max_steps = CR_Sqrt_Max_Steps };
00110 enum { db_sqrt_max_steps = DB_Sqrt_Max_Steps };
00111 // Tuning for log
00113 enum { log_max_outer_steps = Log_Max_Outer_Steps };
00115 enum { log_max_inner_steps = Log_Max_Inner_Steps };
00116 // Tuning for basis cache
00118 enum { basis_max_count = Basis_Max_Count };
00119 // Tuning for FFT
00121 enum { fast_size_threshold = Fast_Size_Threshold };
00123 enum { inv_fast_dim_threshold = Inv_Fast_Dim_Threshold };
00124 // Tuning for products (other than geometric product)
00126 enum { products_size_threshold = Products_Size_Threshold };
00127 // Tuning for precision of exp, log and sqrt functions
00129 enum { denom_different_bits = Denom_Different_Bits };
00131 enum { extra_different_bits = Extra_Different_Bits };
00133 static const precision_t function_precision = Function_Precision;
00134 };
00135
00136 using tuning_demoted = tuning
00137 <
00138 Tuning_Default_Mult_Matrix_Threshold,
00139 Tuning_Default_Div_Max_Steps,
00140 Tuning_Default_CR_Sqrt_Max_Steps,
00141 Tuning_Default_DB_Sqrt_Max_Steps,
00142 Tuning_Default_Log_Max_Outer_Steps,
00143 Tuning_Default_Log_Max_Inner_Steps,
00144 Tuning_Default_Basis_Max_Count,
00145 Tuning_Default_Fast_Size_Threshold,
00146 Tuning_Default_Inv_Fast_Dim_Threshold,
00147 Tuning_Default_Products_Size_Threshold,
00148 Tuning_Default_Denom_Different_Bits,
00149 Tuning_Default_Extra_Different_Bits,
00150 precision_demoted
00151 >;
00152
00153 using tuning_promoted = tuning
00154 <
00155 Tuning_Default_Mult_Matrix_Threshold,
00156 Tuning_Default_Div_Max_Steps,
00157 Tuning_Default_CR_Sqrt_Max_Steps,
00158 Tuning_Default_DB_Sqrt_Max_Steps,
00159 Tuning_Default_Log_Max_Outer_Steps,
00160 Tuning_Default_Log_Max_Inner_Steps,
00161 Tuning_Default_Basis_Max_Count,
00162 Tuning_Default_Fast_Size_Threshold,
00163 Tuning_Default_Inv_Fast_Dim_Threshold,
00164 Tuning_Default_Products_Size_Threshold,
00165 Tuning_Default_Denom_Different_Bits,
00166 Tuning_Default_Extra_Different_Bits,
00167 precision_promoted
00168 >;
00169 }
00170
00171 #endif // GLUCAT_TUNING_H

```



## 9.53 test/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- using [glucat::tuning\\_slow](#)
- using [glucat::tuning\\_naive](#)
- using [glucat::tuning\\_fast](#)

### Variables

- const unsigned int [glucat::Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [glucat::Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [glucat::Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [glucat::Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [glucat::Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 9.54 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_TUNING_H
00002 #define GLUCAT_TEST_TUNING_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 tuning.h : Class definitions to control test tuning
00006
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00036 const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits;
00037 const unsigned int Tuning_Max_Threshold = 1 << Tuning_Int_Digits;
00038
00039 // Specific tuning policy constants and tuning policies
00040
00041 const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold;
00042 const unsigned int Tuning_Slow_Basis_Max_Count = 0;
00043 const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold;
00044 const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00045 const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold;
00046
00047 using tuning_slow = tuning
00048 <
00049 Tuning_Slow_Mult_Matrix_Threshold,
00050 Tuning_Default_Div_Max_Steps,
00051 Tuning_Default_CR_Sqrt_Max_Steps,
00052 Tuning_Default_DB_Sqrt_Max_Steps,
00053 Tuning_Default_Log_Max_Outer_Steps,
00054 Tuning_Default_Log_Max_Inner_Steps,
00055 Tuning_Slow_Basis_Max_Count,
00056 Tuning_Slow_Fast_Size_Threshold,
00057 Tuning_Slow_Inv_Fast_Dim_Threshold,
00058 Tuning_Slow_Products_Size_Threshold,
00059 Tuning_Default_Denom_Different_Bits,
00060 Tuning_Default_Extra_Different_Bits,
00061 Tuning_Default_Function_Precision
00062 >;
00063
00064 const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0;
00065 const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold;
00066 const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold;
00067 const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00068
00069 using tuning_naive = tuning
00070 <
00071 Tuning_Naive_Mult_Matrix_Threshold,
00072 Tuning_Default_Div_Max_Steps,
00073 Tuning_Default_CR_Sqrt_Max_Steps,
00074 Tuning_Default_DB_Sqrt_Max_Steps,
00075 Tuning_Default_Log_Max_Outer_Steps,
00076 Tuning_Default_Log_Max_Inner_Steps,
00077 Tuning_Naive_Basis_Max_Count,
00078 Tuning_Naive_Fast_Size_Threshold,
00079 Tuning_Naive_Inv_Fast_Dim_Threshold,
00080 Tuning_Default_Products_Size_Threshold,
00081 Tuning_Default_Denom_Different_Bits,
00082 Tuning_Default_Extra_Different_Bits,

```

```

00083 Tuning_Default_Function_Precision
00084 >;
00085
00086 const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0;
00087 const unsigned int Tuning_Fast_Div_Max_Steps = 0;
00088 const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256;
00089 const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256;
00090 const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16;
00091 const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8;
00092 const unsigned int Tuning_Fast_Basis_Max_Count = 1;
00093 const unsigned int Tuning_Fast_Fast_Size_Threshold = 0;
00094 const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0;
00095 const unsigned int Tuning_Fast_Products_Size_Threshold = 0;
00096
00097 using tuning_fast = tuning
00098 <
00099 Tuning_Fast_Mult_Matrix_Threshold,
00100 Tuning_Fast_Div_Max_Steps,
00101 Tuning_Fast_CR_Sqrt_Max_Steps,
00102 Tuning_Fast_DB_Sqrt_Max_Steps,
00103 Tuning_Fast_Log_Max_Outer_Steps,
00104 Tuning_Fast_Log_Max_Inner_Steps,
00105 Tuning_Fast_Basis_Max_Count,
00106 Tuning_Fast_Fast_Size_Threshold,
00107 Tuning_Fast_Inv_Fast_Dim_Threshold,
00108 Tuning_Fast_Products_Size_Threshold,
00109 Tuning_Default_Denom_Different_Bits,
00110 Tuning_Default_Extra_Different_Bits,
00111 Tuning_Default_Function_Precision
00112 >;
00113 }
00114 #endif // GLUCAT_TEST_TUNING_H

```

## 9.55 pyclical/glucat.pxd File Reference

### Namespaces

- namespace [glucat](#)

## 9.56 glucat.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 # Generic library of universal Clifford algebra templates
00006 #
00007 # glucat.pxd: Basic Cython definitions
00008 # corresponding to C++ definitions from PyClical.h.
00009 # Kept as a separate module from PyClical.pxd to avoid namespace clashes.
00010 #
00011 # copyright : (C) 2008-2012 by Paul C. Leopardi
00012 #
00013 # This library is free software: you can redistribute it and/or modify
00014 # it under the terms of the GNU Lesser General Public License as published
00015 # by the Free Software Foundation, either version 3 of the License, or
00016 # (at your option) any later version.
00017 #
00018 # This library is distributed in the hope that it will be useful,
00019 # but WITHOUT ANY WARRANTY; without even the implied warranty of
00020 # MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00021 # GNU Lesser General Public License for more details.
00022 #
00023 # You should have received a copy of the GNU Lesser General Public License
00024 # along with this library. If not, see <http://www.gnu.org/licenses/>.
00025
00026 from libcpp.vector cimport vector
00027
00028 cdef extern from "PyClical.h":
00029
00030 cdef cppclass String:
00031 char* c_str()
00032

```

```

00033 cdef cppclass IndexSet:
00034 IndexSet ()
00035 IndexSet (IndexSet Ist) except+
00036 IndexSet (int idx) except+
00037 IndexSet (char* str) except+
00038 inline bint operator==(IndexSet Rhs)
00039 inline bint operator!=(IndexSet Rhs)
00040 inline bint operator<(IndexSet Rhs)
00041 inline IndexSet invert "operator~"()
00042 inline bint getitem "operator[]"(int idx)
00043 inline IndexSet set()
00044 inline IndexSet set(int idx) except+
00045 inline IndexSet set(int idx, int val) except+
00046 inline IndexSet reset()
00047 inline IndexSet reset(int idx) except+
00048 int count()
00049 int count_pos()
00050 int count_neg()
00051 int min()
00052 int max()
00053 int sign_of_mult(IndexSet Rhs)
00054 int sign_of_square()
00055 int hash_fn()
00056
00057 int compare(IndexSet Lhs, IndexSet Rhs)
00058 int min_neg(IndexSet Ist)
00059 int max_pos(IndexSet Ist)
00060
00061 ctypedef double scalar_t
00062
00063 cdef cppclass Clifford:
00064 Clifford ()
00065 Clifford (Clifford Clf) except+
00066 Clifford (Clifford Clf, IndexSet ist) except+
00067 Clifford (scalar_t scr) except+
00068 Clifford (char* str) except+
00069 Clifford (IndexSet ist, scalar_t scr) except+
00070 Clifford (vector[scalar_t] vec, IndexSet ist) except+
00071 bint operator==(Clifford Rhs)
00072 bint operator!=(Clifford Rhs)
00073 Clifford neg "operator-"()
00074 scalar_t getitem "operator[]"(IndexSet Ist)
00075 Clifford call "operator()"(int grade)
00076 scalar_t scalar()
00077 Clifford pure()
00078 Clifford even()
00079 Clifford odd()
00080 vector[scalar_t] vector_part()
00081 vector[scalar_t] vector_part(IndexSet frm) except+
00082 Clifford involute()
00083 Clifford reverse()
00084 Clifford conj()
00085 Clifford random(IndexSet Ist, scalar_t fill)
00086 scalar_t norm()
00087 scalar_t quad()
00088 IndexSet frame()
00089 scalar_t max_abs()
00090 Clifford inv()
00091 Clifford pow(int m)
00092 Clifford outer_pow(int m)
00093 Clifford truncated(scalar_t limit)
00094 bint isinf()
00095 bint isnan()
00096 void write(char* msg)
00097
00098 scalar_t error_squared_tol(Clipford Clf)
00099 scalar_t error_squared(Clipford Lhs, Clifford Rhs, scalar_t threshold)
00100 bint approx_equal(Clipford Lhs, Clifford Rhs, scalar_t threshold, scalar_t tol)
00101 scalar_t scalar(Clipford Clf)
00102 scalar_t real(Clipford Clf)
00103 scalar_t imag(Clipford Clf)
00104 Clifford pure(Clipford Clf)
00105 Clifford even(Clipford Clf)
00106 Clifford odd(Clipford Clf)
00107 Clifford involute(Clipford Clf)
00108 Clifford reverse(Clipford Clf)
00109 Clifford conj(Clipford Clf)
00110 scalar_t norm(Clipford Clf)
00111 scalar_t abs(Clipford Clf)
00112 scalar_t max_abs(Clipford Clf)
00113 scalar_t quad(Clipford Clf)
00114 Clifford inv(Clipford Clf)
00115 Clifford pow(Clipford Clf, int m)
00116 Clifford outer_pow(Clipford Clf, int m)
00117
00118 Clifford complexifier(Clipford Clf)
00119 Clifford sqrt(Clipford Clf, Clifford I) except+

```

```

00120 Clifford sqrt(Clifford Clf)
00121 Clifford exp(Clifford Clf)
00122 Clifford log(Clifford Clf, Clifford I) except+
00123 Clifford log(Clifford Clf)
00124 Clifford cos(Clifford Clf, Clifford I) except+
00125 Clifford cos(Clifford Clf)
00126 Clifford acos(Clifford Clf, Clifford I) except+
00127 Clifford acos(Clifford Clf)
00128 Clifford cosh(Clifford Clf)
00129 Clifford acosh(Clifford Clf, Clifford I) except+
00130 Clifford acosh(Clifford Clf)
00131 Clifford sin(Clifford Clf, Clifford I) except+
00132 Clifford sin(Clifford Clf)
00133 Clifford asin(Clifford Clf, Clifford I) except+
00134 Clifford asin(Clifford Clf)
00135 Clifford sinh(Clifford Clf)
00136 Clifford asinh(Clifford Clf, Clifford I) except+
00137 Clifford asinh(Clifford Clf)
00138 Clifford tan(Clifford Clf, Clifford I) except+
00139 Clifford tan(Clifford Clf)
00140 Clifford atan(Clifford Clf, Clifford I) except+
00141 Clifford atan(Clifford Clf)
00142 Clifford tanh(Clifford Clf)
00143 Clifford atanh(Clifford Clf, Clifford I) except+
00144 Clifford atanh(Clifford Clf)
00145
00146 cdef extern from "PyClical.h" namespace "cga3":
00147 Clifford agc3(Clifford Clf)
00148 Clifford cga3(Clifford Clf)
00149 Clifford cga3std(Clifford Clf)

```

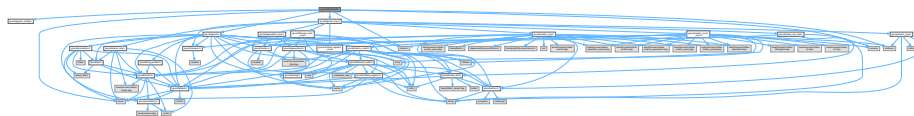
## 9.57 pyclical/PyClical.h File Reference

```

#include "glucat/glucat_config.h"
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include <iostream>
#include <sstream>
#include <iomanip>
#include <limits>

```

Include dependency graph for PyClical.h:



### Namespaces

- namespace `cga3`  
*Definitions for 3D Conformal Geometric Algebra [DL].*

### Typedefs

- using `String` = `std::string`
- using `IndexSet` = `index_set<lo_ndx, hi_ndx>`
- using `scalar_t` = `double`
- using `Clifford` = `matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>`

## Functions

- `template<typename Scalar_T>`  
`PyObject * PyFloat_FromDouble (Scalar_T v)`
- `template<typename Index_Set_T>`  
`String index_set_to_repr (const Index_Set_T &ist)`  
*The "official" string representation of Index\_Set\_T ist.*
- `template<typename Index_Set_T>`  
`String index_set_to_str (const Index_Set_T &ist)`  
*The "informal" string representation of Index\_Set\_T ist.*
- `template<typename Multivector_T>`  
`String clifford_to_repr (const Multivector_T &mv)`  
*The "official" string representation of Multivector\_T mv.*
- `template<typename Multivector_T>`  
`String clifford_to_str (const Multivector_T &mv)`  
*The "informal" string representation of Multivector\_T mv.*
- `template<typename Multivector_T>`  
`Multivector_T cga3::cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T>`  
`Multivector_T cga3::cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T>`  
`Multivector_T cga3::agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

## Variables

- `String glucat_package_version = GLUCAT_PACKAGE_VERSION`
- `const index_t lo_ndx = DEFAULT_LO`
- `const index_t hi_ndx = DEFAULT_HI`
- `const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()`

## 9.57.1 Typedef Documentation

### 9.57.1.1 Clifford

```
using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>
```

Definition at line 148 of file `PyClical.h`.

### 9.57.1.2 IndexSet

```
using IndexSet = index_set<lo_ndx, hi_ndx>
```

Definition at line 145 of file `PyClical.h`.

### 9.57.1.3 scalar\_t

using `scalar_t` = double

Definition at line 147 of file [PyClical.h](#).

### 9.57.1.4 String

using `String` = std::string

Definition at line 51 of file [PyClical.h](#).

## 9.57.2 Function Documentation

### 9.57.2.1 clifford\_to\_repr()

```
template<typename Multivector_T>
String clifford_to_repr (
 const Multivector_T & mv) [inline]
```

The “official” string representation of Multivector\_T mv.

Definition at line 75 of file [PyClical.h](#).

References [clifford\\_to\\_repr\(\)](#).

Referenced by [PyClical.clifford::\\_\\_repr\\_\\_\(\)](#), and [clifford\\_to\\_repr\(\)](#).

### 9.57.2.2 clifford\_to\_str()

```
template<typename Multivector_T>
String clifford_to_str (
 const Multivector_T & mv) [inline]
```

The “informal” string representation of Multivector\_T mv.

Definition at line 86 of file [PyClical.h](#).

References [glucat::abs\(\)](#), and [clifford\\_to\\_str\(\)](#).

Referenced by [PyClical.clifford::\\_\\_str\\_\\_\(\)](#), and [clifford\\_to\\_str\(\)](#).

### 9.57.2.3 index\_set\_to\_repr()

```
template<typename Index_Set_T>
String index_set_to_repr (
 const Index_Set_T & ist) [inline]
```

The “official” string representation of Index\_Set\_T ist.

Definition at line 57 of file [PyClical.h](#).

References [index\\_set\\_to\\_repr\(\)](#).

Referenced by [PyClical.index\\_set::\\_\\_repr\\_\\_\(\)](#), and [index\\_set\\_to\\_repr\(\)](#).

#### 9.57.2.4 index\_set\_to\_str()

```
template<typename Index_Set_T>
String index_set_to_str (
 const Index_Set_T & ist) [inline]
```

The "informal" string representation of Index\_Set\_T ist.

Definition at line 66 of file [PyClical.h](#).

References [index\\_set\\_to\\_str\(\)](#).

Referenced by [PyClical.index\\_set::\\_\\_str\\_\\_\(\)](#), and [index\\_set\\_to\\_str\(\)](#).

#### 9.57.2.5 PyFloat\_FromDouble()

```
template<typename Scalar_T>
PyObject * PyFloat_FromDouble (
 Scalar_T v) [inline]
```

Create a PyFloatObject object from Scalar\_T v. Needed because Scalar\_T might not be the same as double.

Definition at line 45 of file [PyClical.h](#).

References [PyFloat\\_FromDouble\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

Referenced by [PyFloat\\_FromDouble\(\)](#).

### 9.57.3 Variable Documentation

#### 9.57.3.1 epsilon

```
const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()
```

Definition at line 150 of file [PyClical.h](#).

Referenced by [glucat::cascade\\_log\(\)](#), and [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 9.57.3.2 glucat\_package\_version

```
String glucat_package_version = GLUCAT_PACKAGE_VERSION
```

Definition at line 53 of file [PyClical.h](#).

#### 9.57.3.3 hi\_ndx

```
const index_t hi_ndx = DEFAULT_HI
```

Definition at line 144 of file [PyClical.h](#).



## 9.57.3.4 lo\_ndx

```
const index_t lo_ndx = DEFAULT_LO
```

Definition at line 143 of file [PyClicl.h](#).

## 9.58 PyClicl.h

[Go to the documentation of this file.](#)

```
00001 /*****
00002 GluCat : Generic library of universal Clifford algebra templates
00003 PyClicl.h : C++ definitions needed by PyClicl
00004 -----
00005 copyright : (C) 2008-2021 by Paul C. Leopardi
00006 *****/
00007
00008 This library is free software: you can redistribute it and/or modify
00009 it under the terms of the GNU Lesser General Public License as published
00010 by the Free Software Foundation, either version 3 of the License, or
00011 (at your option) any later version.
00012
00013 This library is distributed in the hope that it will be useful,
00014 but WITHOUT ANY WARRANTY; without even the implied warranty of
00015 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00016 GNU Lesser General Public License for more details.
00017
00018 You should have received a copy of the GNU Lesser General Public License
00019 along with this library. If not, see <http://www.gnu.org/licenses/>.
00020
00021 *****/
00022 This library is based on a prototype written by Arvind Raja and was
00023 licensed under the LGPL with permission of the author. See Arvind Raja,
00024 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00025 in Ablamowicz, Lounesto and Parra (eds.)
00026 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00027 *****/
00028 See also Arvind Raja's original header comments in glucat/glucat.h
00029 *****/
00030 // References for algorithms:
00031 // [DL]:
00032 // C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00033
00034 #include "glucat/glucat_config.h"
00035 #include "glucat/glucat.h"
00036 #include "glucat/glucat_imp.h"
00037 #include <iostream>
00038 #include <sstream>
00039 #include <iomanip>
00040 #include <limits>
00041
00042 template<typename Scalar_T>
00043 inline PyObject* PyFloat_FromDouble(Scalar_T v)
00044 { return ::PyFloat_FromDouble(glucat::numeric_traits<Scalar_T>::to_double(v)); }
00045
00046 // String representations for use by PyClicl Python classes.
00047 using String = std::string;
00048
00049 String glucat_package_version = GLUCAT_PACKAGE_VERSION;
00050
00051 template<typename Index_Set_T>
00052 inline String index_set_to_repr(const Index_Set_T& ist)
00053 {
00054 std::ostringstream os;
00055 os << "index_set(" << ist << ")";
00056 return os.str();
00057 }
00058
00059 template<typename Index_Set_T>
00060 inline String index_set_to_str(const Index_Set_T& ist)
00061 {
00062 std::ostringstream os;
00063 os << ist;
00064 return os.str();
00065 }
00066
00067 template<typename Multivector_T>
```

```

00075 inline String clifford_to_repr(const Multivector_T& mv)
00076 {
00077 using scalar_t = typename Multivector_T::scalar_t;
00078 std::ostringstream os;
00079 os << std::setprecision(std::numeric_limits<scalar_t>::digits10 + 1);
00080 os << "clifford(\"" << mv << "\")";
00081 return os.str();
00082 }
00083
00085 template<typename Multivector_T>
00086 inline String clifford_to_str(const Multivector_T& mv)
00087 {
00088 using scalar_t = typename Multivector_T::scalar_t;
00089 std::ostringstream os;
00090 if (abs(mv) < std::numeric_limits<scalar_t>::epsilon())
00091 os << 0.0;
00092 else
00093 os << std::setprecision(4) << mv.truncated(scalar_t(1.0e-4));
00094 return os.str();
00095 }
00096
00097 namespace cga3
00098 {
00099 template<typename Multivector_T>
00100 inline Multivector_T cga3(const Multivector_T& x)
00101 {
00102 using cl = Multivector_T;
00103 using ist = typename cl::index_set_t;
00104 static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00105
00106 return (cl(ist(4)) - x) * ninf3 * (x - cl(ist(4)));
00107 }
00108
00109 template<typename Multivector_T>
00110 inline Multivector_T cga3std(const Multivector_T& X)
00111 {
00112 using cl = Multivector_T;
00113 using ist = typename cl::index_set_t;
00114 using scalar_t = typename cl::scalar_t;
00115 static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00116
00117 return scalar_t(-2.0) * X / (X & ninf3);
00118 }
00119
00120 template<typename Multivector_T>
00121 inline Multivector_T agc3(const Multivector_T& X)
00122 {
00123 using cl = Multivector_T;
00124 using ist = typename cl::index_set_t;
00125 using scalar_t = typename cl::scalar_t;
00126
00127 const cl& cga3stdX = cga3std(X);
00128 return (cl(ist(1))*cga3stdX[ist(1)] +
00129 cl(ist(2))*cga3stdX[ist(2)] +
00130 cl(ist(3))*cga3stdX[ist(3)]) / scalar_t(2.0);
00131 }
00132 }
00133
00134 // Specifications of the IndexSet and Clifford C++ classes for use with PyClical.
00135 using namespace glucat;
00136 const index_t lo_ndx = DEFAULT_LO;
00137 const index_t hi_ndx = DEFAULT_HI;
00138 using IndexSet = index_set<lo_ndx, hi_ndx>;
00139
00140 using scalar_t = double;
00141 using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>;
00142
00143 const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon();
00144
00145 // Do not warn about unused values. This affects clang++ as well as g++.
00146 #pragma GCC diagnostic ignored "-Wunused-value"
00147
00148 #if defined(__clang__)
00149 // Do not warn about unused functions. The affects clang++ only.
00150 #pragma clang diagnostic ignored "-Wunused-function"
00151
00152 // Do not warn about unneeded internal declarations. The affects clang++ only.
00153 #pragma clang diagnostic ignored "-Wunused-declaration"
00154 #endif

```

## 9.59 pyclical/PyClical.pxd File Reference

### Namespaces

- namespace [PyClical](#)

## 9.60 PyClical.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 # Generic library of universal Clifford algebra templates
00006 #
00007 # PyClical.pxd: Basic Cython definitions for PyClical
00008 # corresponding to C++ definitions from PyClical.h.
00009 #
00010 # copyright : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 # This library is free software: you can redistribute it and/or modify
00013 # it under the terms of the GNU Lesser General Public License as published
00014 # by the Free Software Foundation, either version 3 of the License, or
00015 # (at your option) any later version.
00016 #
00017 # This library is distributed in the hope that it will be useful,
00018 # but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 # MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 # GNU Lesser General Public License for more details.
00021 #
00022 # You should have received a copy of the GNU Lesser General Public License
00023 # along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 cimport glucat
00026 from glucat cimport IndexSet, String, Clifford, scalar_t, vector
00027 from libcpp.string cimport string
00028 #
00029 cdef extern from "PyClical.h":
00030 string glucat_package_version
00031 #
00032 IndexSet operator&(IndexSet Lhs, IndexSet Rhs)
00033 IndexSet operator|(IndexSet Lhs, IndexSet Rhs)
00034 IndexSet operator^(IndexSet Lhs, IndexSet Rhs)
00035 #
00036 string index_set_to_repr(IndexSet& Ist)
00037 string index_set_to_str(IndexSet& Ist)
00038 #
00039 Clifford operator+(Clifford Lhs, Clifford Rhs)
00040 Clifford operator-(Clifford Lhs, Clifford Rhs)
00041 Clifford operator*(Clifford Lhs, Clifford Rhs)
00042 Clifford operator&(Clifford Lhs, Clifford Rhs)
00043 Clifford operator%(Clifford Lhs, Clifford Rhs)
00044 Clifford operator^(Clifford Lhs, Clifford Rhs)
00045 Clifford operator/(Clifford Lhs, Clifford Rhs)
00046 Clifford operator|(Clifford Lhs, Clifford Rhs)
00047 #
00048 string clifford_to_repr(Clifford& Clf)
00049 string clifford_to_str(Clifford& Clf)
00050 #
00051 const scalar_t epsilon

```

## 9.61 pyclical/PyClical.pyx File Reference

### Classes

- class [PyClical.index\\_set](#)
- class [PyClical.clifford](#)

## Namespaces

- namespace [PyClical](#)

## Functions

- [PyClical.index\\_set\\_hidden\\_doctests](#) ()
- [PyClical.clifford\\_hidden\\_doctests](#) ()
- [PyClical.e](#) (obj)
- [PyClical.istpq](#) (p, q)
- [PyClical.\\_test](#) ()

## Variables

- [PyClical.\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [PyClical.lhs](#)
- [PyClical.rhs](#)
- [PyClical.threshold](#) = error\_squared\_tol([rhs](#)) if threshold is [None](#) else threshold
- [PyClical.None](#)
- [PyClical.tol](#) = error\_squared\_tol([rhs](#)) if tol is [None](#) else tol
- [PyClical.obj](#)
- [PyClical.i](#)
- [PyClical.ixt](#)
- [PyClical.fill](#)
- [PyClical.scalar\\_epsilon](#) = [epsilon](#)
- float [PyClical.pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [PyClical.tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [PyClical.cl](#) = [clifford](#)
- [PyClical.ist](#) = [index\\_set](#)
- [PyClical.ninf3](#) = [e](#)(4) + [e](#)(-1)
- [PyClical.nbar3](#) = [e](#)(4) - [e](#)(-1)

## 9.62 PyClical.pyx

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 # distutils: language = c++
00004 #
00005 # PyClical: Python interface to GluCat:
00006 # Generic library of universal Clifford algebra templates
00007 #
00008 # PyClical.pyx: Cython definitions visible from Python.
00009 #
00010 # copyright : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 # This library is free software: you can redistribute it and/or modify
00013 # it under the terms of the GNU Lesser General Public License as published
00014 # by the Free Software Foundation, either version 3 of the License, or
00015 # (at your option) any later version.
00016 #
00017 # This library is distributed in the hope that it will be useful,
00018 # but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 # MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 # GNU Lesser General Public License for more details.
00021 #
00022 # You should have received a copy of the GNU Lesser General Public License
00023 # along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 # References for definitions:

```

```

00026 # [DL]:
00027 # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00028
00029 import math
00030 import numbers
00031 import collections
00032
00033 from PyClical cimport *
00034
00035 __version__ = str(glucat_package_version, 'utf-8')
00036
00037 # Forward reference
00038 cdef class index_set
00039
00040 cdef inline IndexSet toIndexSet(obj):
00041 """
00042 Return the C++ IndexSet instance wrapped by index_set(obj).
00043 """
00044 return index_set(obj).instance[0]
00045
00046 cdef class index_set:
00047 """
00048 Python class index_set wraps C++ class IndexSet.
00049 """
00050 cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
00051
00052 cdef inline wrap(index_set self, IndexSet other):
00053 """
00054 Wrap an instance of the C++ class IndexSet.
00055 """
00056 self.instance[0] = other
00057 return self
00058
00059 cdef inline IndexSet unwrap(index_set self):
00060 """
00061 Return the wrapped C++ IndexSet instance.
00062 """
00063 return self.instance[0]
00064
00065 cpdef copy(index_set self):
00066 """
00067 Copy this index_set object.
00068
00069 >> s=index_set(1); t=s.copy(); print(t)
00070 {1}
00071 """
00072 return index_set(self)
00073
00074 def __cinit__(self, other = 0):
00075 """
00076 Construct an object of type index_set.
00077
00078 >> print(index_set(1))
00079 {1}
00080 >> print(index_set({1,2}))
00081 {1,2}
00082 >> print(index_set(index_set({1,2})))
00083 {1,2}
00084 >> print(index_set({1,2}))
00085 {1,2}
00086 >> print(index_set({1,2,1}))
00087 {1,2}
00088 >> print(index_set("{1,2,1}"))
00089 {1,2}
00090 >> print(index_set(""))
00091 {}
00092 """
00093 error_msg_prefix = "Cannot initialize index_set object from"
00094 if isinstance(other, index_set):
00095 self.instance = new IndexSet((<index_set>other).unwrap())
00096 elif isinstance(other, numbers.Integral):
00097 self.instance = new IndexSet(<int>other)
00098 elif isinstance(other, (set, frozenset)):
00099 try:
00100 self.instance = new IndexSet()
00101 for idx in other:
00102 self[idx] = True
00103 except IndexError:
00104 raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
00105 except (RuntimeError, TypeError):
00106 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
00107 elif isinstance(other, str):
00108 try:
00109 bother = other.encode("UTF-8")
00110 self.instance = new IndexSet(<char *>bother)
00111 except RuntimeError:
00112 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")

```

```

00113 else:
00114 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00115
00116 def __dealloc__(self):
00117 """
00118 Clean up by deallocating the instance of C++ class IndexSet.
00119 """
00120 del self.instance
00121
00122 def __richcmp__(lhs, rhs, int op):
00123 """
00124 Compare two objects of class index_set.
00125
00126 >> index_set(1) == index_set({1})
00127 True
00128 >> index_set({1}) != index_set({1})
00129 False
00130 >> index_set({1}) != index_set({2})
00131 True
00132 >> index_set({1}) == index_set({2})
00133 False
00134 >> index_set({1}) < index_set({2})
00135 True
00136 >> index_set({1}) <= index_set({2})
00137 True
00138 >> index_set({1}) > index_set({2})
00139 False
00140 >> index_set({1}) >= index_set({2})
00141 False
00142 """
00143 if (lhs is None) or (rhs is None):
00144 eq = bool(lhs is rhs)
00145 if op == 2: # ==
00146 return eq
00147 elif op == 3: # !=
00148 return not eq
00149 else:
00150 if op == 0: # <
00151 return False
00152 elif op == 1: # <=
00153 return eq
00154 elif op == 4: # >
00155 return False
00156 elif op == 5: # >=
00157 return eq
00158 else:
00159 return NotImplemented
00160 else:
00161 eq = bool(toIndexSet(lhs) == toIndexSet(rhs))
00162 if op == 2: # ==
00163 return eq
00164 elif op == 3: # !=
00165 return not eq
00166 else:
00167 lt = bool(toIndexSet(lhs) < toIndexSet(rhs))
00168 if op == 0: # <
00169 return lt
00170 elif op == 1: # <=
00171 return lt or eq
00172 elif op == 4: # >
00173 return not (lt or eq)
00174 elif op == 5: # >=
00175 return not lt
00176 else:
00177 return NotImplemented
00178
00179 def __setitem__(self, idx, val):
00180 """
00181 Set the value of an index_set object at index idx to value val.
00182
00183 >> s=index_set({1}); s[2] = True; print(s)
00184 {1,2}
00185 >> s=index_set({1,2}); s[1] = False; print(s)
00186 {2}
00187 """
00188 self.instance.set(idx, val)
00189 return
00190
00191 def __getitem__(self, idx):
00192 """
00193 Get the value of an index_set object at an index.
00194
00195 >> index_set({1})[1]
00196 True
00197 >> index_set({1})[2]
00198 False
00199 >> index_set({2})[-1]

```

```

00200 False
00201 >> index_set({2})[1]
00202 False
00203 >> index_set({2})[2]
00204 True
00205 >> index_set({2})[33]
00206 False
00207 """
00208 return self.instance.getitem(idx)
00209
00210 def __contains__(self, idx):
00211 """
00212 Check that an index_set object contains the index idx: idx in self.
00213
00214 >> 1 in index_set({1})
00215 True
00216 >> 2 in index_set({1})
00217 False
00218 >> -1 in index_set({2})
00219 False
00220 >> 1 in index_set({2})
00221 False
00222 >> 2 in index_set({2})
00223 True
00224 >> 33 in index_set({2})
00225 False
00226 """
00227 return self.instance.getitem(idx)
00228
00229 def __iter__(self):
00230 """
00231 Iterate over the indices of an index_set.
00232
00233 >> for i in index_set({-3,4,7}):print(i, end=",")
00234 -3,4,7,
00235 """
00236 for idx in range(self.min(), self.max()+1):
00237 if idx in self:
00238 yield idx
00239
00240 def __invert__(self):
00241 """
00242 Set complement: not.
00243
00244 >>
00245 print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))
00246 {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}
00247 """
00248 return index_set().wrap(self.instance.invert())
00249
00250 def __xor__(lhs, rhs):
00251 """
00252 Symmetric set difference: exclusive or.
00253
00254 >> print(index_set({1}) ^ index_set({2}))
00255 {1,2}
00256 >> print(index_set({1,2}) ^ index_set({2}))
00257 {1}
00258 """
00259 return index_set().wrap(toIndexSet(lhs) ^ toIndexSet(rhs))
00260
00261 def __ixor__(self, rhs):
00262 """
00263 Symmetric set difference: exclusive or.
00264
00265 >> x = index_set({1}); x ^= index_set({2}); print(x)
00266 {1,2}
00267 >> x = index_set({1,2}); x ^= index_set({2}); print(x)
00268 {1}
00269 """
00270 return self.wrap(self.unwrap() ^ toIndexSet(rhs))
00271
00272 def __and__(lhs, rhs):
00273 """
00274 Set intersection: and.
00275
00276 >> print(index_set({1}) & index_set({2}))
00277 {}
00278 >> print(index_set({1,2}) & index_set({2}))
00279 {2}
00280 """
00281 return index_set().wrap(toIndexSet(lhs) & toIndexSet(rhs))
00282
00283 def __iand__(self, rhs):
00284 """
00285 Set intersection: and.

```

```

00285
00286 >> x = index_set({1}); x &= index_set({2}); print(x)
00287 {}
00288 >> x = index_set({1,2}); x &= index_set({2}); print(x)
00289 {2}
00290 """
00291 return self.wrap(self.unwrap() & toIndexSet(rhs))
00292
00293 def __or__(lhs, rhs):
00294 """
00295 Set union: or.
00296
00297 >> print(index_set({1}) | index_set({2}))
00298 {1,2}
00299 >> print(index_set({1,2}) | index_set({2}))
00300 {1,2}
00301 """
00302 return index_set().wrap(toIndexSet(lhs) | toIndexSet(rhs))
00303
00304 def __ior__(self, rhs):
00305 """
00306 Set union: or.
00307
00308 >> x = index_set({1}); x |= index_set({2}); print(x)
00309 {1,2}
00310 >> x = index_set({1,2}); x |= index_set({2}); print(x)
00311 {1,2}
00312 """
00313 return self.wrap(self.unwrap() | toIndexSet(rhs))
00314
00315 def count(self):
00316 """
00317 Cardinality: Number of indices included in set.
00318
00319 >> index_set({-1,1,2}).count()
00320 3
00321 """
00322 return self.instance.count()
00323
00324 def count_neg(self):
00325 """
00326 Number of negative indices included in set.
00327
00328 >> index_set({-1,1,2}).count_neg()
00329 1
00330 """
00331 return self.instance.count_neg()
00332
00333 def count_pos(self):
00334 """
00335 Number of positive indices included in set.
00336
00337 >> index_set({-1,1,2}).count_pos()
00338 2
00339 """
00340 return self.instance.count_pos()
00341
00342 def min(self):
00343 """
00344 Minimum member.
00345
00346 >> index_set({-1,1,2}).min()
00347 -1
00348 """
00349 return self.instance.min()
00350
00351 def max(self):
00352 """
00353 Maximum member.
00354
00355 >> index_set({-1,1,2}).max()
00356 2
00357 """
00358 return self.instance.max()
00359
00360 def hash_fn(self):
00361 """
00362 Hash function.
00363 """
00364 return self.instance.hash_fn()
00365
00366 def sign_of_mult(self, rhs):
00367 """
00368 Sign of geometric product of two Clifford basis elements.
00369
00370 >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
00371 1

```



```

00372 """
00373 return self.instance.sign_of_mult(toIndexSet(rhs))
00374
00375 def sign_of_square(self):
00376 """
00377 Sign of geometric square of a Clifford basis element.
00378
00379 >> s = index_set({1,2}); s.sign_of_square()
00380 -1
00381 """
00382 return self.instance.sign_of_square()
00383
00384 def __repr__(self):
00385 """
00386 The "official" string representation of self.
00387
00388 >> index_set({1,2}).__repr__()
00389 'index_set({1,2})'
00390 >> repr(index_set({1,2}))
00391 'index_set({1,2})'
00392 """
00393 return index_set_to_repr(self.unwrap()).decode()
00394
00395 def __str__(self):
00396 """
00397 The "informal" string representation of self.
00398
00399 >> index_set({1,2}).__str__()
00400 '{1,2}'
00401 >> str(index_set({1,2}))
00402 '{1,2}'
00403 """
00404 return index_set_to_str(self.unwrap()).decode()
00405
00406 def index_set_hidden_doctests():
00407 """
00408 Tests for functions that Doctest cannot see.
00409
00410 For index_set.__cinit__: Construct index_set.
00411
00412 >> print(index_set(1))
00413 {1}
00414 >> print(index_set({1,2}))
00415 {1,2}
00416 >> print(index_set(index_set({1,2})))
00417 {1,2}
00418 >> print(index_set({1,2}))
00419 {1,2}
00420 >> print(index_set({1,2,1}))
00421 {1,2}
00422 >> print(index_set({1,2,1}))
00423 {1,2}
00424 >> print(index_set(""))
00425 {}
00426 >> print(index_set("{}"))
00427 Traceback (most recent call last):
00428 ...
00429 ValueError: Cannot initialize index_set object from invalid string '{}'.
00430 >> print(index_set("{1}"))
00431 Traceback (most recent call last):
00432 ...
00433 ValueError: Cannot initialize index_set object from invalid string '{1}'.
00434 >> print(index_set("{1,2,100}"))
00435 Traceback (most recent call last):
00436 ...
00437 ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
00438 >> print(index_set({1,2,100}))
00439 Traceback (most recent call last):
00440 ...
00441 IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
00442 >> print(index_set([1,2]))
00443 Traceback (most recent call last):
00444 ...
00445 TypeError: Cannot initialize index_set object from <class 'list'>.
00446
00447 For index_set.__richcmp__: Compare two objects of class index_set.
00448
00449 >> index_set(1) == index_set({1})
00450 True
00451 >> index_set({1}) != index_set({1})
00452 False
00453 >> index_set({1}) != index_set({2})
00454 True
00455 >> index_set({1}) == index_set({2})
00456 False
00457 >> index_set({1}) < index_set({2})
00458 True

```

```

00459 >> index_set({1}) <= index_set({2})
00460 True
00461 >> index_set({1}) > index_set({2})
00462 False
00463 >> index_set({1}) >= index_set({2})
00464 False
00465 >> None == index_set({1,2})
00466 False
00467 >> None != index_set({1,2})
00468 True
00469 >> None < index_set({1,2})
00470 False
00471 >> None <= index_set({1,2})
00472 False
00473 >> None > index_set({1,2})
00474 False
00475 >> None >= index_set({1,2})
00476 False
00477 >> index_set({1,2}) == None
00478 False
00479 >> index_set({1,2}) != None
00480 True
00481 >> index_set({1,2}) < None
00482 False
00483 >> index_set({1,2}) <= None
00484 False
00485 >> index_set({1,2}) > None
00486 False
00487 >> index_set({1,2}) >= None
00488 False
00489 """
00490 return
00491
00492 cpdef inline compare(lhs,rhs):
00493 """
00494 "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
00495 -1 if a<b, +1 if a>b, 0 if a==b.
00496
00497 >> compare(index_set({1,2}),index_set({-1,3}))
00498 -1
00499 >> compare(index_set({-1,4}),index_set({-1,3}))
00500 1
00501 """
00502 return glucat.compare(toIndexSet(lhs), toIndexSet(rhs))
00503
00504 cpdef inline min_neg(obj):
00505 """
00506 Minimum negative index, or 0 if none.
00507
00508 >> min_neg(index_set({1,2}))
00509 0
00510 """
00511 return glucat.min_neg(toIndexSet(obj))
00512
00513 cpdef inline max_pos(obj):
00514 """
00515 Maximum positive index, or 0 if none.
00516
00517 >> max_pos(index_set({1,2}))
00518 2
00519 """
00520 return glucat.max_pos(toIndexSet(obj))
00521
00522 cdef inline vector[scalar_t] list_to_vector(lst):
00523 """
00524 Create a C++ std::vector[scalar_t] from an iterable Python object.
00525 """
00526 cdef vector[scalar_t] v
00527 for s in lst:
00528 v.push_back(<scalar_t>s)
00529 return v
00530
00531 # Forward reference.
00532 cdef class clifford
00533
00534 cdef inline Clifford toClifford(obj):
00535 return clifford(obj).instance[0]
00536
00537 cdef class clifford:
00538 """
00539 Python class clifford wraps C++ class Clifford.
00540 """
00541 cdef Clifford *instance # Wrapped instance of C++ class Clifford.
00542
00543 cdef inline wrap(clifford self, Clifford other):
00544 """
00545 Wrap an instance of the C++ class Clifford.

```

```

00546 """
00547 self.instance[0] = other
00548 return self
00549
00550 cdef inline Clifford unwrap(clifford self):
00551 """
00552 Return the wrapped C++ Clifford instance.
00553 """
00554 return self.instance[0]
00555
00556 cpdef copy(clifford self):
00557 """
00558 Copy this clifford object.
00559
00560 >> x=clifford("1{2}"); y=x.copy(); print(y)
00561 {2}
00562 """
00563 return clifford(self)
00564
00565 def __cinit__(self, other = 0, ixt = None):
00566 """
00567 Construct an object of type clifford.
00568
00569 >> print(clifford(2))
00570 2
00571 >> print(clifford(2.0))
00572 2
00573 >> print(clifford(1.0e-1))
00574 0.1
00575 >> print(clifford("2"))
00576 2
00577 >> print(clifford("2{1,2,3}"))
00578 2{1,2,3}
00579 >> print(clifford(clifford("2{1,2,3}")))
00580 2{1,2,3}
00581 >> print(clifford("-{1}"))
00582 -{1}
00583 >> print(clifford(2, index_set({1,2})))
00584 2{1,2}
00585 >> print(clifford([2,3], index_set({1,2})))
00586 2{1}+3{2}
00587 """
00588 error_msg_prefix = "Cannot initialize clifford object from"
00589 if ixt is None:
00590 try:
00591 if isinstance(other, clifford):
00592 self.instance = new Clifford((<clifford>other).unwrap())
00593 elif isinstance(other, index_set):
00594 self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
00595 elif isinstance(other, numbers.Real):
00596 self.instance = new Clifford(<scalar_t>other)
00597 elif isinstance(other, str):
00598 try:
00599 bother = other.encode("UTF-8")
00600 self.instance = new Clifford(<char *>bother)
00601 except RuntimeError:
00602 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00603 else:
00604 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00605 except RuntimeError as err:
00606 raise ValueError(error_msg_prefix + " " + str(type(other))
00607 + " value " + repr(other) + ":"
00608 + "\n\t" + str(err))
00609 elif isinstance(ixt, index_set):
00610 if isinstance(other, numbers.Real):
00611 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
00612 elif isinstance(other, collections.abc.Sequence):
00613 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
00614 else:
00615 raise TypeError(error_msg_prefix + " (" + str(type(other))
00616 + ", " + repr(ixt) + ").")
00617 else:
00618 raise TypeError(error_msg_prefix + " (" + str(type(other))
00619 + ", " + str(type(ixt)) + ").")
00620
00621 def __dealloc__(self):
00622 """
00623 Clean up by deallocating the instance of C++ class Clifford.
00624 """
00625 del self.instance
00626
00627 def __contains__(self, x):
00628 """
00629 Not applicable.
00630
00631 >> x=clifford(index_set({-3,4,7})); -3 in x
00632 Traceback (most recent call last):

```

```

00633 ...
00634 TypeError: Not applicable.
00635 """
00636 raise TypeError("Not applicable.")
00637
00638 def __iter__(self):
00639 """
00640 Not applicable.
00641
00642 >> for a in clifford(index_set({-3,4,7})):print(a, end=",")
00643 Traceback (most recent call last):
00644 ...
00645 TypeError: Not applicable.
00646 """
00647 raise TypeError("Not applicable.")
00648
00649 def reframe(self, ixt):
00650 """
00651 Put self into a larger frame, containing the union of self.frame() and index set ixt.
00652 This can be used to make multiplication faster, by multiplying within a common frame.
00653
00654 >> clifford("2+3{1}").reframe(index_set({1,2,3}))
00655 clifford("2+3{1}")
00656 >> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() ==
(s|t);
00657 True
00658 """
00659 error_msg_prefix = "Cannot reframe"
00660 if isinstance(ixt, index_set):
00661 try:
00662 result = clifford()
00663 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
00664 except RuntimeError as err:
00665 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame " +
00666 + str(ixt) + ":" +
00667 + "\n\t" + str(err))
00668 else:
00669 raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
00670 return result
00671
00672 def __richcmp__(lhs, rhs, int op):
00673 """
00674 Compare objects of type clifford.
00675
00676 >> clifford("{1}") == clifford("1{1}")
00677 True
00678 >> clifford("{1}") != clifford("1.0{1}")
00679 False
00680 >> clifford("{1}") != clifford("1.0")
00681 True
00682 >> clifford("{1,2}") == None
00683 False
00684 >> clifford("{1,2}") != None
00685 True
00686 >> None == clifford("{1,2}")
00687 False
00688 >> None != clifford("{1,2}")
00689 True
00690 """
00691 if op == 2: # ==
00692 if (lhs is None) or (rhs is None):
00693 return bool(lhs is rhs)
00694 else:
00695 return bool(toClifford(lhs) == toClifford(rhs))
00696 elif op == 3: # !=
00697 if (lhs is None) or (rhs is None):
00698 return not bool(lhs is rhs)
00699 else:
00700 return bool(toClifford(lhs) != toClifford(rhs))
00701 elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
00702 raise TypeError("This comparison operator is not implemented for "
00703 + str(type(lhs)) + ", " + str(type(rhs)) + ".")
00704 else:
00705 return NotImplemented
00706
00707 def __getitem__(self, ixt):
00708 """
00709 Subscripting: map from index set to scalar coordinate.
00710
00711 >> clifford("{1}")[index_set(1)]
00712 1.0
00713 >> clifford("{1}")[index_set({1})]
00714 1.0
00715 >> clifford("{1}")[index_set({1,2})]
00716 0.0
00717 >> clifford("2{1,2}")[index_set({1,2})]
00718 2.0

```

```

00719 """
00720 return self.instance.getitem(toIndexSet(ixt))
00721
00722 def __neg__(self):
00723 """
00724 Unary -.
00725
00726 >> print(-clifford("{1}"))
00727 -{1}
00728 """
00729 return clifford().wrap(self.instance.neg())
00730
00731 def __pos__(self):
00732 """
00733 Unary +.
00734
00735 >> print(+clifford("{1}"))
00736 {1}
00737 """
00738 return clifford(self)
00739
00740 def __add__(lhs, rhs):
00741 """
00742 Geometric sum.
00743
00744 >> print(clifford(1) + clifford("{2}"))
00745 1+{2}
00746 >> print(clifford("{1}") + clifford("{2}"))
00747 {1}+{2}
00748 """
00749 return clifford().wrap(toClifford(lhs) + toClifford(rhs))
00750
00751 def __radd__(rhs, lhs):
00752 """
00753 Geometric sum.
00754
00755 >> print(1 + clifford("{2}"))
00756 1+{2}
00757 """
00758 return clifford().wrap(toClifford(lhs) + toClifford(rhs))
00759
00760 def __iadd__(self, rhs):
00761 """
00762 Geometric sum.
00763
00764 >> x = clifford(1); x += clifford("{2}"); print(x)
00765 1+{2}
00766 """
00767 return self.wrap(self.unwrap() + toClifford(rhs))
00768
00769 def __sub__(lhs, rhs):
00770 """
00771 Geometric difference.
00772
00773 >> print(clifford(1) - clifford("{2}"))
00774 1-{2}
00775 >> print(clifford("{1}") - clifford("{2}"))
00776 {1}-{2}
00777 """
00778 return clifford().wrap(toClifford(lhs) - toClifford(rhs))
00779
00780 def __rsub__(rhs, lhs):
00781 """
00782 Geometric difference.
00783
00784 >> print(1 - clifford("{2}"))
00785 1-{2}
00786 """
00787 return clifford().wrap(toClifford(lhs) - toClifford(rhs))
00788
00789 def __isub__(self, rhs):
00790 """
00791 Geometric difference.
00792
00793 >> x = clifford(1); x -= clifford("{2}"); print(x)
00794 1-{2}
00795 """
00796 return self.wrap(self.unwrap() - toClifford(rhs))
00797
00798 def __mul__(lhs, rhs):
00799 """
00800 Geometric product.
00801
00802 >> print(clifford("{1}") * clifford("{2}"))
00803 {1,2}
00804 >> print(clifford(2) * clifford("{2}"))
00805 2{2}

```

```

00806 >> print(clifford("{1}") * clifford("{1,2}"))
00807 {2}
00808 """
00809 return clifford().wrap(toClifford(lhs) * toClifford(rhs))
00810
00811 def __rmul__(rhs, lhs):
00812 """
00813 Geometric product.
00814
00815 >> print(2 * clifford("{2}"))
00816 2{2}
00817 """
00818 return clifford().wrap(toClifford(lhs) * toClifford(rhs))
00819
00820 def __imul__(self, rhs):
00821 """
00822 Geometric product.
00823
00824 >> x = clifford(2); x *= clifford("{2}"); print(x)
00825 2{2}
00826 >> x = clifford("{1}"); x *= clifford("{2}"); print(x)
00827 {1,2}
00828 >> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
00829 {2}
00830 """
00831 return self.wrap(self.unwrap() * toClifford(rhs))
00832
00833 def __mod__(lhs, rhs):
00834 """
00835 Contraction.
00836
00837 >> print(clifford("{1}") % clifford("{2}"))
00838 0
00839 >> print(clifford(2) % clifford("{2}"))
00840 2{2}
00841 >> print(clifford("{1}") % clifford("{1}"))
00842 1
00843 >> print(clifford("{1}") % clifford("{1,2}"))
00844 {2}
00845 """
00846 return clifford().wrap(toClifford(lhs) % toClifford(rhs))
00847
00848 def __rmod__(rhs, lhs):
00849 """
00850 Contraction.
00851
00852 >> print(2 % clifford("{2}"))
00853 2{2}
00854 """
00855 return clifford().wrap(toClifford(lhs) % toClifford(rhs))
00856
00857 def __imod__(self, rhs):
00858 """
00859 Contraction.
00860
00861 >> x = clifford("{1}"); x %= clifford("{2}"); print(x)
00862 0
00863 >> x = clifford(2); x %= clifford("{2}"); print(x)
00864 2{2}
00865 >> x = clifford("{1}"); x %= clifford("{1}"); print(x)
00866 1
00867 >> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
00868 {2}
00869 """
00870 return self.wrap(self.unwrap() % toClifford(rhs))
00871
00872 def __and__(lhs, rhs):
00873 """
00874 Inner product.
00875
00876 >> print(clifford("{1}") & clifford("{2}"))
00877 0
00878 >> print(clifford(2) & clifford("{2}"))
00879 0
00880 >> print(clifford("{1}") & clifford("{1}"))
00881 1
00882 >> print(clifford("{1}") & clifford("{1,2}"))
00883 {2}
00884 """
00885 return clifford().wrap(toClifford(lhs) & toClifford(rhs))
00886
00887 def __rand__(rhs, lhs):
00888 """
00889 Inner product.
00890
00891 >> print(2 & clifford("{2}"))
00892 0

```

```

00893 """
00894 return clifford().wrap(toClifford(lhs) & toClifford(rhs))
00895
00896 def __iand__(self, rhs):
00897 """
00898 Inner product.
00899
00900 >> x = clifford("{1}"); x &= clifford("{2}"); print(x)
00901 0
00902 >> x = clifford(2); x &= clifford("{2}"); print(x)
00903 0
00904 >> x = clifford("{1}"); x &= clifford("{1}"); print(x)
00905 1
00906 >> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
00907 {2}
00908 """
00909 return self.wrap(self.unwrap() & toClifford(rhs))
00910
00911 def __xor__(lhs, rhs):
00912 """
00913 Outer product.
00914
00915 >> print(clifford("{1}") ^ clifford("{2}"))
00916 {1,2}
00917 >> print(clifford(2) ^ clifford("{2}"))
00918 2{2}
00919 >> print(clifford("{1}") ^ clifford("{1}"))
00920 0
00921 >> print(clifford("{1}") ^ clifford("{1,2}"))
00922 0
00923 """
00924 return clifford().wrap(toClifford(lhs) ^ toClifford(rhs))
00925
00926 def __rxor__(rhs, lhs):
00927 """
00928 Outer product.
00929
00930 >> print(2 ^ clifford("{2}"))
00931 2{2}
00932 """
00933 return clifford().wrap(toClifford(lhs) ^ toClifford(rhs))
00934
00935 def __ixor__(self, rhs):
00936 """
00937 Outer product.
00938
00939 >> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
00940 {1,2}
00941 >> x = clifford(2); x ^= clifford("{2}"); print(x)
00942 2{2}
00943 >> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
00944 0
00945 >> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
00946 0
00947 """
00948 return self.wrap(self.unwrap() ^ toClifford(rhs))
00949
00950 def __truediv__(lhs, rhs):
00951 """
00952 Geometric quotient.
00953
00954 >> print(clifford("{1}") / clifford("{2}"))
00955 {1,2}
00956 >> print(clifford(2) / clifford("{2}"))
00957 2{2}
00958 >> print(clifford("{1}") / clifford("{1}"))
00959 1
00960 >> print(clifford("{1}") / clifford("{1,2}"))
00961 -{2}
00962 """
00963 return clifford().wrap(toClifford(lhs) / toClifford(rhs))
00964
00965 def __rtruediv__(rhs, lhs):
00966 """
00967 Geometric quotient.
00968
00969 >> print(2 / clifford("{2}"))
00970 2{2}
00971 """
00972 return clifford().wrap(toClifford(lhs) / toClifford(rhs))
00973
00974 def __idiv__(self, rhs):
00975 """
00976 Geometric quotient.
00977
00978 >> x = clifford("{1}"); x /= clifford("{2}"); print(x)
00979 {1,2}

```

```

00980 >> x = clifford(2); x /= clifford("{2}"); print(x)
00981 2{2}
00982 >> x = clifford("{1}"); x /= clifford("{1}"); print(x)
00983 1
00984 >> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
00985 -{2}
00986 """
00987 return self.wrap(self.unwrap() / toClifford(rhs))
00988
00989 def inv(self):
00990 """
00991 Geometric multiplicative inverse.
00992
00993 >> x = clifford("{1}"); print(x.inv())
00994 {1}
00995 >> x = clifford(2); print(x.inv())
00996 0.5
00997 >> x = clifford("{1,2}"); print(x.inv())
00998 -{1,2}
00999 """
01000 return clifford().wrap(self.instance.inv())
01001
01002 def __or__(lhs, rhs):
01003 """
01004 Transform left hand side, using right hand side as a transformation.
01005
01006 >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
01007 -{1}
01008 >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
01009 -{1}
01010 """
01011 return clifford().wrap(toClifford(lhs) | toClifford(rhs))
01012
01013 def __ior__(self, rhs):
01014 """
01015 Transform left hand side, using right hand side as a transformation.
01016
01017 >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
01018 -{1}
01019 >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
01020 -{1}
01021 """
01022 return self.wrap(self.unwrap() | toClifford(rhs))
01023
01024 def __pow__(self, m, dummy):
01025 """
01026 Power: self to the m.
01027
01028 >> x=clifford("{1}"); print(x ** 2)
01029 1
01030 >> x=clifford("2"); print(x ** 2)
01031 4
01032 >> x=clifford("2+{1}"); print(x ** 0)
01033 1
01034 >> x=clifford("2+{1}"); print(x ** 1)
01035 2+{1}
01036 >> x=clifford("2+{1}"); print(x ** 2)
01037 5+4{1}
01038 >> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
01039 1
01040 """
01041 return pow(self, m)
01042
01043 def pow(self, m):
01044 """
01045 Power: self to the m.
01046
01047 >> x=clifford("{1}"); print(x.pow(2))
01048 1
01049 >> x=clifford("2"); print(x.pow(2))
01050 4
01051 >> x=clifford("2+{1}"); print(x.pow(0))
01052 1
01053 >> x=clifford("2+{1}"); print(x.pow(1))
01054 2+{1}
01055 >> x=clifford("2+{1}"); print(x.pow(2))
01056 5+4{1}
01057 >> print(clifford("1+{1}+{1,2}").pow(3))
01058 1+3{1}+3{1,2}
01059 >> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
01060 1
01061 """
01062 if isinstance(m, numbers.Integral):
01063 return clifford().wrap(self.instance.pow(m))
01064 else:
01065 return exp(m * log(self))
01066

```



```

01067 def outer_pow(self, m):
01068 """
01069 Outer product power.
01070
01071 >> x=clifford("2+{1}"); print(x.outer_pow(0))
01072 1
01073 >> x=clifford("2+{1}"); print(x.outer_pow(1))
01074 2+{1}
01075 >> x=clifford("2+{1}"); print(x.outer_pow(2))
01076 4+4{1}
01077 >> print(clifford("1+{1}+{1,2}").outer_pow(3))
01078 1+3{1}+3{1,2}
01079
01080 """
01081 return clifford().wrap(self.instance.outer_pow(m))
01082
01083 def __call__(self, grade):
01084 """
01085 Pure grade-vector part.
01086
01087 >> print(clifford("{1}") (1))
01088 {1}
01089 >> print(clifford("{1}") (0))
01090 0
01091 >> print(clifford("1+{1}+{1,2}") (0))
01092 1
01093 >> print(clifford("1+{1}+{1,2}") (1))
01094 {1}
01095 >> print(clifford("1+{1}+{1,2}") (2))
01096 {1,2}
01097 >> print(clifford("1+{1}+{1,2}") (3))
01098 0
01099 """
01100 return clifford().wrap(self.instance.call(grade))
01101
01102 def scalar(self):
01103 """
01104 Scalar part.
01105
01106 >> clifford("1+{1}+{1,2}").scalar()
01107 1.0
01108 >> clifford("{1,2}").scalar()
01109 0.0
01110 """
01111 return self.instance.scalar()
01112
01113 def pure(self):
01114 """
01115 Pure part.
01116
01117 >> print(clifford("1+{1}+{1,2}").pure())
01118 {1}+{1,2}
01119 >> print(clifford("{1,2}").pure())
01120 {1,2}
01121 """
01122 return clifford().wrap(self.instance.pure())
01123
01124 def even(self):
01125 """
01126 Even part of multivector, sum of even grade terms.
01127
01128 >> print(clifford("1+{1}+{1,2}").even())
01129 1+{1,2}
01130 """
01131 return clifford().wrap(self.instance.even())
01132
01133 def odd(self):
01134 """
01135 Odd part of multivector, sum of odd grade terms.
01136
01137 >> print(clifford("1+{1}+{1,2}").odd())
01138 {1}
01139 """
01140 return clifford().wrap(self.instance.odd())
01141
01142 def vector_part(self, frm = None):
01143 """
01144 Vector part of multivector, as a Python list, with respect to frm.
01145
01146 >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
01147 [2.0, 3.0]
01148 >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set((-1,1,2))))
01149 [0.0, 2.0, 3.0]
01150 """
01151 error_msg_prefix = "Cannot take vector part of "
01152 cdef vector[scalar_t] vec
01153 cdef int n

```

```

01154 cdef int i
01155 try:
01156 if frm is None:
01157 vec = self.instance.vector_part()
01158 else:
01159 vec = self.instance.vector_part((<index_set>frm).unwrap())
01160 n = vec.size()
01161 lst = [0.0]*n
01162 for i in xrange(n):
01163 lst[i] = vec[i]
01164 return lst
01165 except RuntimeError as err:
01166 raise ValueError(error_msg_prefix + str(self) + " using invalid "
01167 + repr(frm) + " as frame:\n\t"
01168 + str(err))
01169
01170 def involute(self):
01171 """
01172 Main involution, each {i} is replaced by -{i} in each term,
01173 eg. clifford("{1}") -> -clifford("{1}").
01174
01175 >> print(clifford("{1}").involute())
01176 -{1}
01177 >> print((clifford("{2}") * clifford("{1}")).involute())
01178 -{1,2}
01179 >> print((clifford("{1}") * clifford("{2}")).involute())
01180 {1,2}
01181 >> print(clifford("1+{1}+{1,2}").involute())
01182 1-{1}+{1,2}
01183 """
01184 return clifford().wrap(self.instance.involute())
01185
01186 def reverse(self):
01187 """
01188 Reversion, eg. clifford("{1})*clifford("{2}") -> clifford("{2})*clifford("{1}").
01189
01190 >> print(clifford("{1}").reverse())
01191 {1}
01192 >> print((clifford("{2}") * clifford("{1}")).reverse())
01193 {1,2}
01194 >> print((clifford("{1}") * clifford("{2}")).reverse())
01195 -{1,2}
01196 >> print(clifford("1+{1}+{1,2}").reverse())
01197 1+{1}-{1,2}
01198 """
01199 return clifford().wrap(self.instance.reverse())
01200
01201 def conj(self):
01202 """
01203 Conjugation, reverse o involute == involute o reverse.
01204
01205 >> print((clifford("{1}")).conj())
01206 -{1}
01207 >> print((clifford("{2}") * clifford("{1}")).conj())
01208 {1,2}
01209 >> print((clifford("{1}") * clifford("{2}")).conj())
01210 -{1,2}
01211 >> print(clifford("1+{1}+{1,2}").conj())
01212 1-{1}-{1,2}
01213 """
01214 return clifford().wrap(self.instance.conj())
01215
01216 def quad(self):
01217 """
01218 Quadratic form == (rev(x)*x)(0).
01219
01220 >> print(clifford("1+{1}+{1,2}").quad())
01221 3.0
01222 >> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
01223 2.0
01224 """
01225 return self.instance.quad()
01226
01227 def norm(self):
01228 """
01229 Norm == sum of squares of coordinates.
01230
01231 >> clifford("1+{1}+{1,2}").norm()
01232 3.0
01233 >> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
01234 4.0
01235 """
01236 return self.instance.norm()
01237
01238 def abs(self):
01239 """
01240 Absolute value: square root of norm.

```

```

01241
01242 >> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
01243 2.0
01244 """
01245 return glucat.abs(self.unwrap())
01246
01247 def max_abs(self):
01248 """
01249 Maximum of absolute values of components of multivector: multivector infinity norm.
01250
01251 >> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
01252 1.0
01253 >> clifford("3+2{1}+{1,2}").max_abs()
01254 3.0
01255 """
01256 return self.instance.max_abs()
01257
01258 def truncated(self, limit):
01259 """
01260 Remove all terms of self with relative size smaller than limit.
01261
01262 >> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
01263 clifford("100000000")
01264 >> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
01265 clifford("10000+{1}")
01266 """
01267 return clifford().wrap(self.instance.truncated(limit))
01268
01269 def isinf(self):
01270 """
01271 Check if a multivector contains any infinite values.
01272
01273 >> clifford().isinf()
01274 False
01275 """
01276 return self.instance.isnan()
01277
01278 def isnan(self):
01279 """
01280 Check if a multivector contains any IEEE NaN values.
01281
01282 >> clifford().isnan()
01283 False
01284 """
01285 return self.instance.isnan()
01286
01287 def frame(self):
01288 """
01289 Subalgebra generated by all generators of terms of given multivector.
01290
01291 >> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
01292 {-2,-1,1,2,7}
01293 >> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
01294 <class 'PyClical.index_set'>
01295 """
01296 return index_set().wrap(self.instance.frame())
01297
01298 def __repr__(self):
01299 """
01300 The "official" string representation of self.
01301
01302 >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
01303 'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
01304 """
01305 return clifford_to_repr(self.unwrap()).decode()
01306
01307 def __str__(self):
01308 """
01309 The "informal" string representation of self.
01310
01311 >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
01312 '1+3{-1}+2{1,2}+4{-2,7}'
01313 """
01314 return clifford_to_str(self.unwrap()).decode()
01315
01316 def clifford_hidden_doctests():
01317 """
01318 Tests for functions that Doctest cannot see.
01319
01320 For clifford.__cinit__: Construct an object of type clifford.
01321
01322 >> print(clifford(2))
01323 2
01324 >> print(clifford(2.0))
01325 2
01326 >> print(clifford(1.0e-1))
01327 0.1

```

```

01328 >> print(clifford("2"))
01329 2
01330 >> print(clifford("2{1,2,3}"))
01331 2{1,2,3}
01332 >> print(clifford(clifford("2{1,2,3}")))
01333 2{1,2,3}
01334 >> print(clifford("-{1}"))
01335 -{1}
01336 >> print(clifford(2,index_set({1,2})))
01337 2{1,2}
01338 >> print(clifford([2,3],index_set({1,2})))
01339 2{1}+3{2}
01340 >> print(clifford([1,2]))
01341 Traceback (most recent call last):
01342 ...
01343 TypeError: Cannot initialize clifford object from <class 'list'>.
01344 >> print(clifford(None))
01345 Traceback (most recent call last):
01346 ...
01347 TypeError: Cannot initialize clifford object from <class 'NoneType'>.
01348 >> print(clifford(None,[1,2]))
01349 Traceback (most recent call last):
01350 ...
01351 TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
01352 >> print(clifford([1,2],[1,2]))
01353 Traceback (most recent call last):
01354 ...
01355 TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
01356 >> print(clifford(""))
01357 Traceback (most recent call last):
01358 ...
01359 ValueError: Cannot initialize clifford object from invalid string "".
01360 >> print(clifford("{}"))
01361 Traceback (most recent call last):
01362 ...
01363 ValueError: Cannot initialize clifford object from invalid string '{}'.
01364 >> print(clifford("{1}"))
01365 Traceback (most recent call last):
01366 ...
01367 ValueError: Cannot initialize clifford object from invalid string '{1'.
01368 >> print(clifford("{1}"))
01369 Traceback (most recent call last):
01370 ...
01371 ValueError: Cannot initialize clifford object from invalid string '+'.
01372 >> print(clifford("-"))
01373 Traceback (most recent call last):
01374 ...
01375 ValueError: Cannot initialize clifford object from invalid string '-'.
01376 >> print(clifford("{1}+"))
01377 Traceback (most recent call last):
01378 ...
01379 ValueError: Cannot initialize clifford object from invalid string '{1}+'.
01380
01381 For clifford.__richcmp__: Compare objects of type clifford.
01382
01383 >> clifford("{1}") == clifford("1{1}")
01384 True
01385 >> clifford("{1}") != clifford("1.0{1}")
01386 False
01387 >> clifford("{1}") != clifford("1.0")
01388 True
01389 >> clifford("{1,2}") == None
01390 False
01391 >> clifford("{1,2}") != None
01392 True
01393 >> None == clifford("{1,2}")
01394 False
01395 >> None != clifford("{1,2}")
01396 True
01397 """
01398 return
01399
01400 cpdef inline error_squared_tol(obj):
01401 """
01402 Quadratic norm error tolerance relative to a specific multivector.
01403
01404 >> print(error_squared_tol(clifford("{1}")) * 3.0 - error_squared_tol(clifford("1{1}-2{2}+3{3}")))
01405 0.0
01406 """
01407 return glucat.error_squared_tol(toClifford(obj))
01408
01409 cpdef inline error_squared(lhs, rhs, threshold):
01410 """
01411 Relative or absolute error using the quadratic norm.
01412
01413 >> err2=scalar_epsilon*scalar_epsilon
01414

```

```

01415 >> print(error_squared(clifford("{1}"), clifford("1{1}"), err2))
01416 0.0
01417 >> print(error_squared(clifford("1{1}-3{2}+4{3}"), clifford("{1}"), err2))
01418 25.0
01419 """
01420 return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
01421
01422 cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
01423 """
01424 Test for approximate equality of multivectors.
01425
01426 >> err2=scalar_epsilon*scalar_epsilon
01427
01428 >> print(approx_equal(clifford("{1}"), clifford("1{1}")))
01429 True
01430 >> print(approx_equal(clifford("1{1}-3{2}+4{3}"), clifford("{1}")))
01431 False
01432 >> print(approx_equal(clifford("1{1}-3{2}+4{3}+0.001"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01433 False
01434 >> print(approx_equal(clifford("1{1}-3{2}+4{3}+1.0e-30"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01435 True
01436 """
01437 threshold = error_squared_tol(rhs) if threshold is None else threshold
01438 tol = error_squared_tol(rhs) if tol is None else tol
01439 return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold, <scalar_t>tol)
01440
01441 cpdef inline inv(obj):
01442 """
01443 Geometric multiplicative inverse.
01444
01445 >> print(inv(clifford("{1}")))
01446 {1}
01447 >> print(inv(clifford("{-1}")))
01448 -{-1}
01449 >> print(inv(clifford("{-2,-1}")))
01450 -{-2,-1}
01451 >> print(inv(clifford("{-1}+{1}")))
01452 nan
01453 """
01454 return clifford(obj).inv()
01455
01456 cpdef inline scalar(obj):
01457 """
01458 Scalar part.
01459
01460 >> scalar(clifford("1+{1}+{1,2}"))
01461 1.0
01462 >> scalar(clifford("{1,2}"))
01463 0.0
01464 """
01465 return clifford(obj).scalar()
01466
01467 cpdef inline real(obj):
01468 """
01469 Real part: synonym for scalar part.
01470
01471 >> real(clifford("1+{1}+{1,2}"))
01472 1.0
01473 >> real(clifford("{1,2}"))
01474 0.0
01475 """
01476 return clifford(obj).scalar()
01477
01478 cpdef inline imag(obj):
01479 """
01480 Imaginary part: deprecated (always 0).
01481
01482 >> imag(clifford("1+{1}+{1,2}"))
01483 0.0
01484 >> imag(clifford("{1,2}"))
01485 0.0
01486 """
01487 return 0.0
01488
01489 cpdef inline pure(obj):
01490 """
01491 Pure part
01492
01493 >> print(pure(clifford("1+{1}+{1,2}")))
01494 {1}+{1,2}
01495 >> print(pure(clifford("{1,2}")))
01496 {1,2}
01497 """
01498 return clifford(obj).pure()
01499
01500 cpdef inline even(obj):
01501 """

```

```

01502 Even part of multivector, sum of even grade terms.
01503
01504 >> print(even(clifford("1+{1}+{1,2}")))
01505 1+{1,2}
01506 """
01507 return clifford(obj).even()
01508
01509 cpdef inline odd(obj):
01510 """
01511 Odd part of multivector, sum of odd grade terms.
01512
01513 >> print(odd(clifford("1+{1}+{1,2}")))
01514 {1}
01515 """
01516 return clifford(obj).odd()
01517
01518 cpdef inline involute(obj):
01519 """
01520 Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
01521
01522 >> print(involute(clifford("{1}")))
01523 -{1}
01524 >> print(involute(clifford("{2}") * clifford("{1}")))
01525 -{1,2}
01526 >> print(involute(clifford("{1}") * clifford("{2}")))
01527 {1,2}
01528 >> print(involute(clifford("1+{1}+{1,2}")))
01529 1-{1}+{1,2}
01530 """
01531 return clifford(obj).involute()
01532
01533 cpdef inline reverse(obj):
01534 """
01535 Reversion, eg. {1}*{2} -> {2}*{1}
01536
01537 >> print(reverse(clifford("{1}")))
01538 {1}
01539 >> print(reverse(clifford("{2}") * clifford("{1}")))
01540 {1,2}
01541 >> print(reverse(clifford("{1}") * clifford("{2}")))
01542 -{1,2}
01543 >> print(reverse(clifford("1+{1}+{1,2}")))
01544 1+{1}-{1,2}
01545 """
01546 return clifford(obj).reverse()
01547
01548 cpdef inline conj(obj):
01549 """
01550 Conjugation, reverse o involute == involute o reverse.
01551
01552 >> print(conj(clifford("{1}")))
01553 -{1}
01554 >> print(conj(clifford("{2}") * clifford("{1}")))
01555 {1,2}
01556 >> print(conj(clifford("{1}") * clifford("{2}")))
01557 -{1,2}
01558 >> print(conj(clifford("1+{1}+{1,2}")))
01559 1-{1}-{1,2}
01560 """
01561 return clifford(obj).conj()
01562
01563 cpdef inline quad(obj):
01564 """
01565 Quadratic form == (rev(x)*x)(0).
01566
01567 >> print(quad(clifford("1+{1}+{1,2}")))
01568 3.0
01569 >> print(quad(clifford("1+{-1}+{1,2}+{1,2,3}")))
01570 2.0
01571 """
01572 return clifford(obj).quad()
01573
01574 cpdef inline norm(obj):
01575 """
01576 norm == sum of squares of coordinates.
01577
01578 >> norm(clifford("1+{1}+{1,2}"))
01579 3.0
01580 >> norm(clifford("1+{-1}+{1,2}+{1,2,3}"))
01581 4.0
01582 """
01583 return clifford(obj).norm()
01584
01585 cpdef inline abs(obj):
01586 """
01587 Absolute value of multivector: multivector 2-norm.
01588

```

```

01589 >> abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01590 2.0
01591 """
01592 return glucat.abs(toClifford(obj))
01593
01594 cpdef inline max_abs(obj):
01595 """
01596 Maximum absolute value of coordinates multivector: multivector infinity-norm.
01597
01598 >> max_abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01599 1.0
01600 >> max_abs(clifford("3+2{1}+{1,2}"))
01601 3.0
01602 """
01603
01604 return glucat.max_abs(toClifford(obj))
01605
01606 cpdef inline pow(obj, m):
01607 """
01608 Integer power of multivector: obj to the m.
01609
01610 >> x=clifford("{1}"); print(pow(x,2))
01611 1
01612 >> x=clifford("2"); print(pow(x,2))
01613 4
01614 >> x=clifford("2+{1}"); print(pow(x,0))
01615 1
01616 >> x=clifford("2+{1}"); print(pow(x,1))
01617 2+{1}
01618 >> x=clifford("2+{1}"); print(pow(x,2))
01619 5+4{1}
01620 >> print(pow(clifford("1+{1}+{1,2}"),3))
01621 1+3{1}+3{1,2}
01622 >> i=clifford("{1,2}"); print(exp(pi/2) * pow(i, i))
01623 1
01624 """
01625 try:
01626 math.pow(obj, m)
01627 except:
01628 return clifford(obj).pow(m)
01629
01630 cpdef inline outer_pow(obj, m):
01631 """
01632 Outer product power of multivector.
01633
01634 >> print(outer_pow(clifford("1+{1}+{1,2}"),3))
01635 1+3{1}+3{1,2}
01636 """
01637 return clifford(obj).outer_pow(m)
01638
01639 cpdef inline complexifier(obj):
01640 """
01641 Square root of -1 which commutes with all members of the frame of the given multivector.
01642
01643 >> print(complexifier(clifford(index_set({1}))))
01644 {1,2,3}
01645 >> print(complexifier(clifford(index_set({-1}))))
01646 {-1}
01647 >> print(complexifier(index_set({1})))
01648 {1,2,3}
01649 >> print(complexifier(index_set({-1})))
01650 {-1}
01651 """
01652 return clifford().wrap(glucat.complexifier(toClifford(obj)))
01653
01654 cpdef inline sqrt(obj, i = None):
01655 """
01656 Square root of multivector with optional complexifier.
01657
01658 >> print(sqrt(-1))
01659 {-1}
01660 >> print(sqrt(clifford("2{-1}")))
01661 1+{-1}
01662 >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)
01663 {1,2,3}
01664 -1
01665 >> j=sqrt(-1,"{1,2,3}"); print(j); print(j*j)
01666 {1,2,3}
01667 -1
01668 """
01669 if not (i is None):
01670 return clifford().wrap(glucat.sqrt(toClifford(obj), toClifford(i)))
01671 else:
01672 try:
01673 return math.sqrt(obj)
01674 except:
01675 return clifford().wrap(glucat.sqrt(toClifford(obj)))

```

```

01676
01677 cpdef inline exp(obj):
01678 """
01679 Exponential of multivector.
01680
01681 >> x=clifford("{1,2}") * pi/4; print(exp(x))
01682 0.7071+0.7071{1,2}
01683 >> x=clifford("{1,2}") * pi/2; print(exp(x))
01684 {1,2}
01685 """
01686 try:
01687 return math.exp(obj)
01688 except:
01689 return clifford().wrap(glucat.exp(toClifford(obj)))
01690
01691 cpdef inline log(obj,i = None):
01692 """
01693 Natural logarithm of multivector with optional complexifier.
01694
01695 >> x=clifford("{-1}"); print((log(x,"{-1}") * 2/pi))
01696 {-1}
01697 >> x=clifford("{1,2}"); print((log(x,"{1,2,3}") * 2/pi))
01698 {1,2}
01699 >> x=clifford("{1,2}"); print((log(x) * 2/pi))
01700 {1,2}
01701 >> x=clifford("{1,2}"); print((log(x,"{1,2}") * 2/pi))
01702 Traceback (most recent call last):
01703 ...
01704 RuntimeError: check_complex(val, i): i is not a valid complexifier for val
01705 """
01706 if not (i is None):
01707 return clifford().wrap(glucat.log(toClifford(obj), toClifford(i)))
01708 else:
01709 try:
01710 return math.log(obj)
01711 except:
01712 return clifford().wrap(glucat.log(toClifford(obj)))
01713
01714 cpdef inline cos(obj,i = None):
01715 """
01716 Cosine of multivector with optional complexifier.
01717
01718 >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01719 {1,2}
01720 >> x=clifford("{1,2}"); print(cos(acos(x)))
01721 {1,2}
01722 """
01723 if not (i is None):
01724 return clifford().wrap(glucat.cos(toClifford(obj), toClifford(i)))
01725 else:
01726 try:
01727 return math.cos(obj)
01728 except:
01729 return clifford().wrap(glucat.cos(toClifford(obj)))
01730
01731 cpdef inline acos(obj,i = None):
01732 """
01733 Inverse cosine of multivector with optional complexifier.
01734
01735 >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01736 {1,2}
01737 >> x=clifford("{1,2}"); print(cos(acos(x),"{-1,1,2,3,4}"))
01738 {1,2}
01739 >> print(acos(0) / pi)
01740 0.5
01741 >> x=clifford("{1,2}"); print(cos(acos(x)))
01742 {1,2}
01743 """
01744 if not (i is None):
01745 return clifford().wrap(glucat.acos(toClifford(obj), toClifford(i)))
01746 else:
01747 try:
01748 return math.acos(obj)
01749 except:
01750 return clifford().wrap(glucat.acos(toClifford(obj)))
01751
01752 cpdef inline cosh(obj):
01753 """
01754 Hyperbolic cosine of multivector.
01755
01756 >> x=clifford("{1,2}") * pi; print(cosh(x))
01757 -1
01758 >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01759 {1,2,3}
01760 >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01761 {1,2}
01762 """

```



```

01763 try:
01764 return math.cosh(obj)
01765 except:
01766 return clifford().wrap(glucat.cosh(toClifford(obj)))
01767
01768 cpdef inline acosh(obj,i = None):
01769 """
01770 Inverse hyperbolic cosine of multivector with optional complexifier.
01771
01772 >> print(acosh(0,"{-2,-1,1}"))
01773 1.571{-2,-1,1}
01774 >> x=clifford("{1,2,3}"); print(cosh(acosh(x,"{-1,1,2,3,4}")))
01775 {1,2,3}
01776 >> print(acosh(0))
01777 1.571{-1}
01778 >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01779 {1,2,3}
01780 >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01781 {1,2}
01782 """
01783 if not (i is None):
01784 return clifford().wrap(glucat.acosh(toClifford(obj), toClifford(i)))
01785 else:
01786 try:
01787 return math.acosh(obj)
01788 except:
01789 return clifford().wrap(glucat.acosh(toClifford(obj)))
01790
01791 cpdef inline sin(obj,i = None):
01792 """
01793 Sine of multivector with optional complexifier.
01794
01795 >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01796 {-1}
01797 >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01798 {-1}
01799 >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01800 {1,2,3}
01801 """
01802 if not (i is None):
01803 return clifford().wrap(glucat.sin(toClifford(obj), toClifford(i)))
01804 else:
01805 try:
01806 return math.sin(obj)
01807 except:
01808 return clifford().wrap(glucat.sin(toClifford(obj)))
01809
01810 cpdef inline asin(obj,i = None):
01811 """
01812 Inverse sine of multivector with optional complexifier.
01813
01814 >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01815 {-1}
01816 >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01817 {-1}
01818 >> print(asin(1) / pi)
01819 0.5
01820 >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01821 {1,2,3}
01822 """
01823 if not (i is None):
01824 return clifford().wrap(glucat.asin(toClifford(obj), toClifford(i)))
01825 else:
01826 try:
01827 return math.asin(obj)
01828 except:
01829 return clifford().wrap(glucat.asin(toClifford(obj)))
01830
01831 cpdef inline sinh(obj):
01832 """
01833 Hyperbolic sine of multivector.
01834
01835 >> x=clifford("{1,2}") * pi/2; print(sinh(x))
01836 {1,2}
01837 >> x=clifford("{1,2}") * pi/6; print(sinh(x))
01838 0.5{1,2}
01839 """
01840 try:
01841 return math.sinh(obj)
01842 except:
01843 return clifford().wrap(glucat.sinh(toClifford(obj)))
01844
01845 cpdef inline asinh(obj,i = None):
01846 """
01847 Inverse hyperbolic sine of multivector with optional complexifier.
01848
01849 >> x=clifford("{1,2}"); print(asinh(x,"{1,2,3}") * 2/pi)

```

```

01850 {1,2}
01851 >> x=clifford("{1,2}"); print(asinh(x) * 2/pi)
01852 {1,2}
01853 >> x=clifford("{1,2}") / 2; print(asinh(x) * 6/pi)
01854 {1,2}
01855 """
01856 if not (i is None):
01857 return clifford().wrap(glucat.asinh(toClifford(obj), toClifford(i)))
01858 else:
01859 try:
01860 return math.asinh(obj)
01861 except:
01862 return clifford().wrap(glucat.asinh(toClifford(obj)))
01863
01864 cpdef inline tan(obj,i = None):
01865 """
01866 Tangent of multivector with optional complexifier.
01867
01868 >> x=clifford("{1,2}"); print(tan(x,"{1,2,3}"))
01869 0.7616{1,2}
01870 >> x=clifford("{1,2}"); print(tan(x))
01871 0.7616{1,2}
01872 """
01873 if not (i is None):
01874 return clifford().wrap(glucat.tan(toClifford(obj), toClifford(i)))
01875 else:
01876 try:
01877 return math.tan(obj)
01878 except:
01879 return clifford().wrap(glucat.tan(toClifford(obj)))
01880
01881 cpdef inline atan(obj,i = None):
01882 """
01883 Inverse tangent of multivector with optional complexifier.
01884
01885 >> s=index_set({1,2,3}); x=clifford("{1}"); print(tan(atan(x,s),s))
01886 {1}
01887 >> x=clifford("{1}"); print(tan(atan(x)))
01888 {1}
01889 """
01890 if not (i is None):
01891 return clifford().wrap(glucat.atan(toClifford(obj), toClifford(i)))
01892 else:
01893 try:
01894 return math.atan(obj)
01895 except:
01896 return clifford().wrap(glucat.atan(toClifford(obj)))
01897
01898 cpdef inline tanh(obj):
01899 """
01900 Hyperbolic tangent of multivector.
01901
01902 >> x=clifford("{1,2}") * pi/4; print(tanh(x))
01903 {1,2}
01904 """
01905 try:
01906 return math.tanh(obj)
01907 except:
01908 return clifford().wrap(glucat.tanh(toClifford(obj)))
01909
01910 cpdef inline atanh(obj,i = None):
01911 """
01912 Inverse hyperbolic tangent of multivector with optional complexifier.
01913
01914 >> s=index_set({1,2,3}); x=clifford("{1,2}"); print(tanh(atanh(x,s)))
01915 {1,2}
01916 >> x=clifford("{1,2}"); print(tanh(atanh(x)))
01917 {1,2}
01918 """
01919 if not (i is None):
01920 return clifford().wrap(glucat.atanh(toClifford(obj), toClifford(i)))
01921 else:
01922 try:
01923 return math.atanh(obj)
01924 except:
01925 return clifford().wrap(glucat.atanh(toClifford(obj)))
01926
01927 cpdef inline random_clifford(index_set ixt, fill = 1.0):
01928 """
01929 Random multivector within a frame.
01930
01931 >> print(random_clifford(index_set({-3,-1,2})).frame())
01932 {-3,-1,2}
01933 """
01934 return clifford().wrap(clifford().instance.random(ixt.unwrap(), <scalar_t>fill))
01935
01936 cpdef inline cga3(obj):

```

```

01937 """
01938 Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
definition.
01939
01940 >> x=clifford("2{1}+9{2}+{3}"); print(cga3(x))
01941 87{-1}+4{1}+18{2}+2{3}+85{4}
01942 """
01943 return clifford().wrap(glucat.cga3(toClifford(obj)))
01944
01945 cpdef inline cga3std(obj):
01946 """
01947 Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
01948
01949 >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x)))
01950 87{-1}+4{1}+18{2}+2{3}+85{4}
01951 >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x))-cga3(x))
01952 0
01953 """
01954 return clifford().wrap(glucat.cga3std(toClifford(obj)))
01955
01956 cpdef inline agc3(obj):
01957 """
01958 Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
01959
01960 >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x)))
01961 2{1}+9{2}+{3}
01962 >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x))-x)
01963 0
01964 """
01965 return clifford().wrap(glucat.agc3(toClifford(obj)))
01966
01967 # Some abbreviations.
01968 scalar_epsilon = epsilon
01969
01970 pi = atan(clifford(1.0)) * 4.0
01971 tau = atan(clifford(1.0)) * 8.0
01972
01973 cl = clifford
01974 """
01975 Abbreviation for clifford.
01976
01977 >> print(cl(2))
01978 2
01979 >> print(cl(2.0))
01980 2
01981 >> print(cl(5.0e-1))
01982 0.5
01983 >> print(cl("2"))
01984 2
01985 >> print(cl("2{1,2,3}"))
01986 2{1,2,3}
01987 >> print(cl(cl("2{1,2,3}")))
01988 2{1,2,3}
01989 """
01990
01991 ist = index_set
01992 """
01993 Abbreviation for index_set.
01994
01995 >> print(ist("{1,2,3}"))
01996 {1,2,3}
01997 """
01998
01999 def e(obj):
02000 """
02001 Abbreviation for clifford(index_set(obj)).
02002
02003 >> print(e(1))
02004 {1}
02005 >> print(e(-1))
02006 {-1}
02007 >> print(e(0))
02008 1
02009 """
02010 return clifford(index_set(obj))
02011
02012 def istpq(p, q):
02013 """
02014 Abbreviation for index_set({-q,...p}).
02015
02016 >> print(istpq(2,3))
02017 {-3,-2,-1,1,2}
02018 """
02019 return index_set(set(range(-q,p+1)))
02020
02021 ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
02022 nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].

```

```

02023
02024 # Doctest interface.
02025 def _test():
02026 import PyClical, doctest
02027 return doctest.testmod(PyClical)
02028
02029 if __name__ == "__main__":
02030 _test()

```

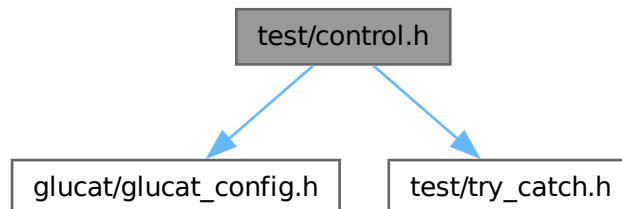
## 9.63 test/control.h File Reference

```

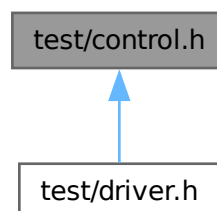
#include "glucat/glucat_config.h"
#include "test/try_catch.h"

```

Include dependency graph for control.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class [glucat::control\\_t](#)  
*Parameters to control tests.*

### Namespaces

- namespace [glucat](#)

## 9.64 control.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CONTROL_H
00002 #define _GLUCAT_CONTROL_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 control.h : Define and set parameters to control tests
00006 -----
00007 begin : 2010-04-21
00008 copyright : (C) 2010-2016 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033 #include "glucat/glucat_config.h"
00034 #include "test/try_catch.h"
00035
00036 namespace glucat
00037 {
00038 class control_t
00039 {
00040 private:
00041 bool m_valid;
00042 bool valid() const
00043 { return m_valid; }
00044
00045 bool m_catch_exceptions;
00046 bool catch_exceptions() const
00047 { return m_catch_exceptions; }
00048
00049 static bool m_verbose_output;
00050
00051 control_t(int argc, char ** argv);
00052 // Enforce singleton
00053 // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00054 control_t() = default;
00055 ~control_t() = default;
00056 control_t(const control_t&) = delete;
00057 control_t& operator= (const control_t&) = delete;
00058
00059 friend class friend_for_private_destructor;
00060 public:
00061 static const control_t& control(int argc, char ** argv)
00062 { static const control_t c(argc, argv); return c; }
00063
00064 int call(intfn f) const;
00065 int call(intintfn f, int arg) const;
00066
00067 static bool verbose()
00068 { return m_verbose_output; }
00069 };
00070
00071 bool control_t::m_verbose_output = false;
00072
00073 control_t::
00074 control_t(int argc, char ** argv)
00075 : m_valid(true), m_catch_exceptions(true)
00076 {
00077 bool print_help = false;
00078 const std::string& arg_0_str = argv[0];
00079 const std::string program_name = arg_0_str.substr(arg_0_str.find_last_of('/')+1);
00080 for (int arg_ndx = 1; arg_ndx < argc; ++arg_ndx)
00081 {
00082 const std::string& arg_str = argv[arg_ndx];

```

```

00098 bool valid = false;
00099 if (arg_str.substr(0,2) == "--")
00100 {
00101 valid = true;
00102 const std::string& arg_name = arg_str.substr(2);
00103 if (arg_name == "help")
00104 {
00105 this->m_valid = false;
00106 print_help = true;
00107 }
00108 else if (arg_name == "verbose")
00109 this->m_verbose_output = true;
00110 else if (arg_name == "no-catch")
00111 this->m_catch_exceptions = false;
00112 else
00113 valid = false;
00114 }
00115 if (!valid)
00116 {
00117 std::cout << "Invalid argument: " << arg_str << std::endl;
00118 this->m_valid = false;
00119 print_help = true;
00120 }
00121 }
00122 if (print_help)
00123 {
00124 std::cout << program_name << " for " << GLUCAT_PACKAGE_NAME << " version " << GLUCAT_VERSION << ":" <<
std::endl;
00125 std::cout << "Usage: " << program_name << " [option ...]" << std::endl;
00126 std::cout << "Options:" << std::endl;
00127 std::cout << " --help : Print this summary." << std::endl;
00128 std::cout << " --no-catch : Do not catch exceptions." << std::endl;
00129 std::cout << " --verbose : Produce more detailed test output." << std::endl;
00130 }
00131 }
00132
00133 inline
00134 int
00135 control_t::
00136 call(intfn f) const
00137 {
00138 if (valid())
00139 return (catch_exceptions())
00140 ? try_catch(f)
00141 : (*f)();
00142 else
00143 return 1;
00144 }
00145
00146 inline
00147 int
00148 control_t::
00149 call(intintfn f, int arg) const
00150 {
00151 if (valid())
00152 return (catch_exceptions())
00153 ? try_catch(f, arg)
00154 : (*f)(arg);
00155 else
00156 return 1;
00157 }
00158 }
00159 }
00160 }
00161 #endif // _GLUCAT_CONTROL_H

```

## 9.65 test/driver.h File Reference

```

#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include "test/tuning.h"
#include "test/try_catch.h"
#include "test/control.h"
#include <stdio>

```

Include dependency graph for driver.h:



## 9.66 driver.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_DRIVER_H
00002 #define GLUCAT_TEST_DRIVER_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 driver.h : Header for example and timing test driver
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2021 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/glucat.h"
00035 #include "glucat/glucat_imp.h"
00036 #include "test/tuning.h"
00037 #include "test/try_catch.h"
00038 #include "test/control.h"
00039 #include <cstdio>
00040
00041 #endif // GLUCAT_TEST_DRIVER_H

```

## 9.67 test/timing.h File Reference

### Namespaces

- namespace [glucat](#)
- namespace [glucat::timing](#)

### Functions

- static double [glucat::timing::elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

## Variables

- const double `glucat::timing::MS_PER_SEC` = 1000.0  
*Timing constant: milliseconds per second.*
- const double `glucat::timing::MS_PER_CLOCK` = `MS_PER_SEC` / `double(CLOCKS_PER_SEC)`  
*Timing constant: milliseconds per clock.*
- const int `glucat::timing::EXTRA_TRIALS` = 2  
*Timing constant: trial expansion factor.*

## 9.68 timing.h

[Go to the documentation of this file.](#)

```

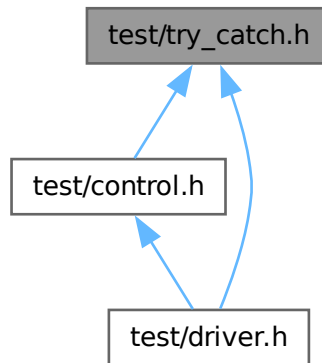
00001 #ifndef GLUCAT_TEST_TIMING_H
00002 #define GLUCAT_TEST_TIMING_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 timing.h : Common definitions for timing tests
00006 -----
00007 begin : Tue 2012-03-27
00008 copyright : (C) 2012 by Paul C. Leopardi
00009 *****/
00010
00011 This library is free software: you can redistribute it and/or modify
00012 it under the terms of the GNU Lesser General Public License as published
00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00036 namespace timing
00037 {
00038 const double MS_PER_SEC = 1000.0;
00039
00040 const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC);
00041
00042 const int EXTRA_TRIALS = 2;
00043
00044 inline
00045 static
00046 double
00047 elapsed(clock_t cpu_time)
00048 { return double(clock() - cpu_time) * MS_PER_CLOCK; }
00049 }
00050 }
00051 #endif // GLUCAT_TEST_TIMING_H

```



## 9.69 test/try\_catch.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- typedef int(\*) [glucat::intfn\(\)](#)  
*For exception catching: pointer to function returning int.*
- typedef int(\*) [glucat::intintfn\(int\)](#)  
*For exception catching: pointer to function of int returning int.*

### Functions

- int [glucat::try\\_catch](#) (intfn f)  
*Exception catching for functions returning int.*
- int [glucat::try\\_catch](#) (intintfn f, int arg)  
*Exception catching for functions of int returning int.*

## 9.70 try\_catch.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_TRY_CATCH_H
00002 #define _GLUCAT_TRY_CATCH_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 try_catch.h : Catch exceptions
00006 -----
00007 begin : Sun 2001-12-20
00008 copyright : (C) 2001-2010 by Paul C. Leopardi

```

```

00009 *****
00010
00011 This library is free software: you can redistribute it and/or modify
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00013 by the Free Software Foundation, either version 3 of the License, or
00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
00018 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019 GNU Lesser General Public License for more details.
00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 namespace glucat
00035 {
00037 typedef int (*intfn)();
00038
00040 typedef int (*intintfn)(int);
00041
00043 int try_catch(intfn f);
00044
00046 int try_catch(intintfn f, int arg);
00047
00049 int try_catch(intfn f)
00050 {
00051 int result = 0;
00052 try
00053 { result = (*f)(); }
00054 catch (const glucat_error& e)
00055 { e.print_error_msg(); }
00056 catch (const std::bad_alloc& e)
00057 { std::cerr << "bad_alloc" << std::endl; }
00058 catch (...)
00059 { std::cerr << "unexpected exception" << std::endl; }
00060 return result;
00061 }
00062
00064 int try_catch(intintfn f, int arg)
00065 {
00066 int result = 0;
00067 try
00068 { result = (*f)(arg); }
00069 catch (const glucat_error& e)
00070 { e.print_error_msg(); }
00071 catch (const std::bad_alloc& e)
00072 { std::cerr << "bad_alloc" << std::endl; }
00073 catch (...)
00074 { std::cerr << "unexpected exception" << std::endl; }
00075 return result;
00076 }
00077 }
00078 #endif // _GLUCAT_TRY_CATCH_H

```

## 9.71 test/undefine.h File Reference

## 9.72 undefine.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_UNDEFINE_H
00002 #define GLUCAT_TEST_UNDEFINE_H
00003 /*****
00004 GluCat : Generic library of universal Clifford algebra templates
00005 undefine.h : Undefine preprocessor macro names that control test tuning
00006 -----
00007 begin : Sun 2001-12-09
00008 copyright : (C) 2001-2016 by Paul C. Leopardi

```

```
00009 *****
00010
00011 This library is free software: you can redistribute it and/or modify
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00014 (at your option) any later version.
00015
00016 This library is distributed in the hope that it will be useful,
00017 but WITHOUT ANY WARRANTY; without even the implied warranty of
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00020
00021 You should have received a copy of the GNU Lesser General Public License
00022 along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 // Undefine tuning policy constants
00035 #undef _GLUCAT_TEST_TUNING_SLOW
00036 #undef _GLUCAT_TEST_TUNING_NAIVE
00037 #undef _GLUCAT_TEST_TUNING_FAST
00038 #undef _GLUCAT_TEST_TUNING_PROMOTED
00039 #undef _GLUCAT_TEST_TUNING_DEMOTED
00040
00041 #endif // GLUCAT_TEST_UNDEFINE_H
```



# Index

- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS`
  - `clifford_algebra.h`, [265](#)
- `_GLUCAT_CTAssert`
  - `global.h`, [336](#)
  - `glucat`, [28](#)
  - `tuning.h`, [434](#)
- `_GLUCAT_HASH_N`
  - `framed_multi_imp.h`, [307](#)
- `_GLUCAT_HASH_SIZE_T`
  - `framed_multi_imp.h`, [307](#)
- `_GLUCAT_ISINF`
  - `portability.h`, [413](#)
- `_GLUCAT_ISNAN`
  - `portability.h`, [413](#)
- `__add__`
  - `PyClical.clifford`, [98](#)
- `__and__`
  - `PyClical.clifford`, [98](#)
  - `PyClical.index_set`, [184](#)
- `__call__`
  - `PyClical.clifford`, [99](#)
- `__cinit__`
  - `PyClical.clifford`, [99](#)
  - `PyClical.index_set`, [184](#)
- `__contains__`
  - `PyClical.clifford`, [100](#)
  - `PyClical.index_set`, [184](#)
- `__dealloc__`
  - `PyClical.clifford`, [100](#)
  - `PyClical.index_set`, [185](#)
- `__getitem__`
  - `PyClical.clifford`, [100](#)
  - `PyClical.index_set`, [185](#)
- `__iadd__`
  - `PyClical.clifford`, [101](#)
- `__iand__`
  - `PyClical.clifford`, [101](#)
  - `PyClical.index_set`, [185](#)
- `__idiv__`
  - `PyClical.clifford`, [101](#)
- `__imod__`
  - `PyClical.clifford`, [102](#)
- `__imul__`
  - `PyClical.clifford`, [102](#)
- `__invert__`
  - `PyClical.index_set`, [186](#)
- `__ior__`
  - `PyClical.clifford`, [102](#)
  - `PyClical.index_set`, [186](#)
- `__isub__`
  - `PyClical.clifford`, [103](#)
- `__iter__`
  - `PyClical.clifford`, [103](#)
  - `PyClical.index_set`, [186](#)
- `__ixor__`
  - `PyClical.clifford`, [103](#)
  - `PyClical.index_set`, [186](#)
- `__mod__`
  - `PyClical.clifford`, [103](#)
- `__mul__`
  - `PyClical.clifford`, [104](#)
- `__neg__`
  - `PyClical.clifford`, [104](#)
- `__or__`
  - `PyClical.clifford`, [104](#)
  - `PyClical.index_set`, [187](#)
- `__pos__`
  - `PyClical.clifford`, [105](#)
- `__pow__`
  - `PyClical.clifford`, [105](#)
- `__radd__`
  - `PyClical.clifford`, [105](#)
- `__rand__`
  - `PyClical.clifford`, [106](#)
- `__repr__`
  - `PyClical.clifford`, [106](#)
  - `PyClical.index_set`, [187](#)
- `__richcmp__`
  - `PyClical.clifford`, [106](#)
  - `PyClical.index_set`, [187](#)
- `__rmod__`
  - `PyClical.clifford`, [107](#)
- `__rmul__`
  - `PyClical.clifford`, [107](#)
- `__rsub__`
  - `PyClical.clifford`, [107](#)
- `__rtruediv__`
  - `PyClical.clifford`, [108](#)
- `__rxor__`
  - `PyClical.clifford`, [108](#)
- `__setitem__`
  - `PyClical.index_set`, [188](#)
- `__str__`
  - `PyClical.clifford`, [108](#)
  - `PyClical.index_set`, [188](#)
- `__sub__`
  - `PyClical.clifford`, [108](#)
- `__truediv__`

- PyClical.clifford, 109
- \_\_version\_\_
  - PyClical, 89
- \_\_xor\_\_
  - PyClical.clifford, 109
  - PyClical.index\_set, 188
- \_test
  - PyClical, 86
- ~basis\_table
  - glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, 94
- ~clifford\_algebra
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 120
- ~control\_t
  - glucat::control\_t, 133
- ~framed\_multi
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 150
- ~generator\_table
  - glucat::gen::generator\_table< Matrix\_T >, 161
- ~glucat\_error
  - glucat::glucat\_error, 166
- ~matrix\_multi
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 200
- ~random\_generator
  - glucat::random\_generator< Scalar\_T >, 243
- ~reference
  - glucat::index\_set< LO, HI >::reference, 247
- ~var\_term
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term, 255
- abs
  - glucat, 28
  - glucat::numeric\_traits< Scalar\_T >, 214
  - PyClical.clifford, 109
- acos
  - glucat, 29
  - glucat::numeric\_traits< Scalar\_T >, 214
- acosh
  - glucat, 29, 30
- agc3
  - cga3, 13
- approx\_equal
  - glucat, 30
- are\_same
  - glucat::compare\_types< LHS\_T, RHS\_T >, 130
  - glucat::compare\_types< T, T >, 131
- array
  - pade::pade\_log\_denom< dd\_real >, 225
  - pade::pade\_log\_denom< float >, 226
  - pade::pade\_log\_denom< long double >, 226
  - pade::pade\_log\_denom< qd\_real >, 227
  - pade::pade\_log\_denom< Scalar\_T >, 224
  - pade::pade\_log\_number< dd\_real >, 229
  - pade::pade\_log\_number< float >, 230
  - pade::pade\_log\_number< long double >, 231
  - pade::pade\_log\_number< qd\_real >, 231
  - pade::pade\_log\_number< Scalar\_T >, 228
  - pade::pade\_sqrt\_denom< dd\_real >, 233
  - pade::pade\_sqrt\_denom< float >, 234
  - pade::pade\_sqrt\_denom< long double >, 235
  - pade::pade\_sqrt\_denom< qd\_real >, 236
  - pade::pade\_sqrt\_denom< Scalar\_T >, 232
  - pade::pade\_sqrt\_number< dd\_real >, 237
  - pade::pade\_sqrt\_number< float >, 238
  - pade::pade\_sqrt\_number< long double >, 239
  - pade::pade\_sqrt\_number< qd\_real >, 240
  - pade::pade\_sqrt\_number< Scalar\_T >, 237
- asin
  - glucat, 31
  - glucat::numeric\_traits< Scalar\_T >, 214
- asinh
  - glucat, 31, 32
- atan
  - glucat, 32
  - glucat::numeric\_traits< Scalar\_T >, 214
- atanh
  - glucat, 33
- basis
  - glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, 95
- basis\_element
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 205
- basis\_matrix\_t
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 198
- basis\_table
  - glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, 94
- BITS\_PER\_SET\_VALUE
  - glucat, 65
- bitset\_t
  - glucat::index\_set< LO, HI >, 172
- BOOST\_STATIC\_ASSERT
  - glucat::index\_set< LO, HI >, 174
- call
  - glucat::control\_t, 133
- cascade\_log
  - glucat, 33
- catch\_exceptions
  - glucat::control\_t, 133
- centre\_pm4\_qp4
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 154
- centre\_pp4\_qm4
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 154
- centre\_qp1\_pm1
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 154
- cga3, 13
  - agc3, 13

- cga3, [13](#)
- cga3std, [14](#)
- cga3std
  - cga3, [14](#)
- check\_complex
  - glucat, [34](#)
- cl
  - PyClical, [89](#)
- classify\_eigenvalues
  - glucat::matrix, [71](#)
- classname
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [120](#)
  - glucat::error< Class\_T >, [141](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [155](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term, [255](#)
  - glucat::glucat\_error, [166](#)
  - glucat::index\_set< LO, HI >, [174](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [205](#)
- Clifford
  - PyClical.h, [442](#)
- clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >::default\_truncation
  - glucat, [65](#)
- clifford\_algebra.h
  - \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS, [265](#)
- clifford\_exp
  - glucat, [34](#)
- clifford\_hidden\_doctests
  - PyClical, [86](#)
- clifford\_to\_repr
  - PyClical.h, [443](#)
- clifford\_to\_str
  - PyClical.h, [443](#)
- compare
  - glucat, [34](#)
  - glucat::index\_set< LO, HI >, [181](#)
- complexifier
  - glucat, [35](#)
- conj
  - glucat, [35](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [120](#)
  - glucat::numeric\_traits< Scalar\_T >, [215](#)
  - PyClical.clifford, [110](#)
- const\_iterator
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [147](#)
- control
  - glucat::control\_t, [133](#)
- control\_t
  - glucat::control\_t, [132](#), [133](#)
- cos
  - glucat, [35](#), [36](#)
- glucat::numeric\_traits< Scalar\_T >, [215](#)
- cosh
  - glucat, [36](#)
  - glucat::numeric\_traits< Scalar\_T >, [215](#)
- count
  - glucat::index\_set< LO, HI >, [174](#)
  - PyClical.index\_set, [189](#)
- count\_neg
  - glucat::index\_set< LO, HI >, [174](#)
  - PyClical.index\_set, [189](#)
- count\_pos
  - glucat::index\_set< LO, HI >, [175](#)
  - PyClical.index\_set, [189](#)
- cr\_sqrt
  - glucat, [36](#)
- crd\_of\_mult
  - glucat, [37](#)
- db\_sqrt
  - glucat, [37](#)
- db\_step
  - glucat, [37](#)
- DEFAULT\_HI
  - glucat, [65](#)
- default\_truncation
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [129](#)
- denom
  - pade::pade\_log\_denom< dd\_real >, [225](#)
  - pade::pade\_log\_denom< float >, [226](#)
  - pade::pade\_log\_denom< long double >, [227](#)
  - pade::pade\_log\_denom< qd\_real >, [227](#)
  - pade::pade\_log\_denom< Scalar\_T >, [224](#)
  - pade::pade\_sqrt\_denom< dd\_real >, [233](#)
  - pade::pade\_sqrt\_denom< float >, [234](#)
  - pade::pade\_sqrt\_denom< long double >, [235](#)
  - pade::pade\_sqrt\_denom< qd\_real >, [236](#)
  - pade::pade\_sqrt\_denom< Scalar\_T >, [233](#)
- divide
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [155](#)
- e
  - PyClical, [87](#)
- eig\_case\_t
  - glucat::matrix, [71](#)
- eigenvalues
  - glucat::matrix, [71](#)
- elapsed
  - glucat::timing, [77](#)
- elliptic
  - glucat, [38](#)
- epsilon
  - PyClical.h, [444](#)
- error
  - glucat::error< Class\_T >, [140](#)
- error\_squared
  - glucat, [38](#)
- error\_squared\_tol

- glucat, [38](#)
- error\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [147](#)
  - glucat::index\_set< LO, HI >, [172](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [198](#)
- even
  - glucat, [39](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [121](#)
  - PyClical.clifford, [110](#)
- exp
  - glucat, [39](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [157](#)
  - glucat::numeric\_traits< Scalar\_T >, [215](#)
- EXTRA\_TRIALS
  - glucat::timing, [77](#)
- fast
  - glucat, [39](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [155](#)
- fast\_framed\_multi
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [155](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [205](#)
- fast\_matrix\_multi
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [156](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [205](#)
- fill
  - PyClical, [89](#)
- flip
  - glucat::index\_set< LO, HI >, [175](#)
  - glucat::index\_set< LO, HI >::reference, [247](#)
- fmod
  - glucat::numeric\_traits< Scalar\_T >, [215](#)
- fold
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [156](#)
  - glucat::index\_set< LO, HI >, [175](#)
- folded\_dim
  - glucat, [40](#)
- frame
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [121](#)
  - PyClical.clifford, [110](#)
- framed\_multi
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [150–153](#), [157](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [206](#)
- framed\_multi\_imp.h
  - \_GLUCAT\_HASH\_N, [307](#)
  - \_GLUCAT\_HASH\_SIZE\_T, [307](#)
- framed\_multi\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [147](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [198](#)
- framed\_pair\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [147](#)
- friend\_for\_private\_destructor
  - glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, [95](#)
  - glucat::control\_t, [134](#)
  - glucat::gen::generator\_table< Matrix\_T >, [164](#)
  - glucat::random\_generator< Scalar\_T >, [244](#)
- gen\_from\_pm1\_qm1
  - glucat::gen::generator\_table< Matrix\_T >, [162](#)
- gen\_from\_pm4\_qp4
  - glucat::gen::generator\_table< Matrix\_T >, [162](#)
- gen\_from\_pp4\_qm4
  - glucat::gen::generator\_table< Matrix\_T >, [162](#)
- gen\_from\_qp1\_pm1
  - glucat::gen::generator\_table< Matrix\_T >, [163](#)
- gen\_vector
  - glucat::gen::generator\_table< Matrix\_T >, [163](#)
- generator
  - glucat::gen::generator\_table< Matrix\_T >, [163](#)
  - glucat::random\_generator< Scalar\_T >, [243](#)
- generator\_table
  - glucat::gen::generator\_table< Matrix\_T >, [161](#), [162](#)
- global.h
  - \_GLUCAT\_CTAssert, [336](#)
- glucat, [14](#)
  - \_GLUCAT\_CTAssert, [28](#)
  - abs, [28](#)
  - acos, [29](#)
  - acosh, [29](#), [30](#)
  - approx\_equal, [30](#)
  - asin, [31](#)
  - asinh, [31](#), [32](#)
  - atan, [32](#)
  - atanh, [33](#)
  - BITS\_PER\_SET\_VALUE, [65](#)
  - cascade\_log, [33](#)
  - check\_complex, [34](#)
  - clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >::default\_truncation, [65](#)
  - clifford\_exp, [34](#)
  - compare, [34](#)
  - complexifier, [35](#)
  - conj, [35](#)
  - cos, [35](#), [36](#)
  - cosh, [36](#)
  - cr\_sqrt, [36](#)
  - crd\_of\_mult, [37](#)
  - db\_sqrt, [37](#)
  - db\_step, [37](#)
  - DEFAULT\_HI, [65](#)



- elliptic, 38
- error\_squared, 38
- error\_squared\_tol, 38
- even, 39
- exp, 39
- fast, 39
- folded\_dim, 40
- imag, 40
- index\_t, 26
- intfn, 26
- intintfn, 26
- inv, 40
- inverse\_gray, 41
- inverse\_reversed\_gray, 41
- involute, 41
- l\_ln2, 65
- l\_pi, 65
- log, 41, 42
- log2, 42
- matrix\_log, 43
- matrix\_sqrt, 43
- max\_abs, 43
- max\_pos, 44
- min\_neg, 44
- MS\_PER\_S, 65
- norm, 44
- odd, 44
- offset\_level, 45
- operator!=, 45
- operator<<, 52
- operator>>, 53
- operator+, 49
- operator-, 50
- operator/, 50, 51
- operator%, 46
- operator&, 46, 47
- operator\*, 47–49
- operator<sup>^</sup>, 53, 54
- operator|, 54, 55
- outer\_pow, 55
- pade\_approx, 56
- pade\_log, 56
- pos\_mod, 56
- pow, 57
- pure, 57
- quad, 58
- real, 58
- reframe, 58
- reverse, 59
- scalar, 59
- set\_value\_t, 27
- sign\_of\_square, 59
- sin, 60
- sinh, 60
- sqrt, 60, 61
- star, 61, 62
- tan, 62
- tanh, 63
- to\_demote, 63
- to\_promote, 63
- try\_catch, 64
- tuning\_fast, 27
- Tuning\_Fast\_Basis\_Max\_Count, 66
- Tuning\_Fast\_CR\_Sqrt\_Max\_Steps, 66
- Tuning\_Fast\_DB\_Sqrt\_Max\_Steps, 66
- Tuning\_Fast\_Div\_Max\_Steps, 66
- Tuning\_Fast\_Fast\_Size\_Threshold, 66
- Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold, 66
- Tuning\_Fast\_Log\_Max\_Inner\_Steps, 66
- Tuning\_Fast\_Log\_Max\_Outer\_Steps, 67
- Tuning\_Fast\_Mult\_Matrix\_Threshold, 67
- Tuning\_Fast\_Products\_Size\_Threshold, 67
- Tuning\_Int\_Digits, 67
- Tuning\_Max\_Threshold, 67
- tuning\_naive, 27
- Tuning\_Naive\_Basis\_Max\_Count, 67
- Tuning\_Naive\_Fast\_Size\_Threshold, 67
- Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold, 67
- Tuning\_Naive\_Mult\_Matrix\_Threshold, 68
- tuning\_slow, 27
- Tuning\_Slow\_Basis\_Max\_Count, 68
- Tuning\_Slow\_Fast\_Size\_Threshold, 68
- Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold, 68
- Tuning\_Slow\_Mult\_Matrix\_Threshold, 68
- Tuning\_Slow\_Products\_Size\_Threshold, 68
- vector\_part, 64
- glucat Directory Reference, 11
- glucat/clifford\_algebra.h, 257, 265
- glucat/clifford\_algebra\_imp.h, 274, 282
- glucat/errors.h, 294, 295
- glucat/errors\_imp.h, 296, 297
- glucat/framed\_multi.h, 298, 301
- glucat/framed\_multi\_imp.h, 304, 308
- glucat/generation.h, 328, 329
- glucat/generation\_imp.h, 330, 331
- glucat/global.h, 334, 336
- glucat/glucat.h, 337, 338
- glucat/glucat\_config.h, 340, 343
- glucat/glucat\_imp.h, 345
- glucat/index\_set.h, 346, 348
- glucat/index\_set\_imp.h, 350, 352
- glucat/long\_double.h, 363, 365
- glucat/matrix.h, 365, 368
- glucat/matrix\_imp.h, 369, 371
- glucat/matrix\_multi.h, 378, 380
- glucat/matrix\_multi\_imp.h, 384, 388
- glucat/portability.h, 412, 414
- glucat/promotion.h, 415, 416
- glucat/qd.h, 419, 420
- glucat/random.h, 423, 424
- glucat/scalar.h, 425, 427
- glucat/scalar\_imp.h, 430, 431
- glucat/tuning.h, 433, 435
- glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, 93
  - ~basis\_table, 94
  - basis, 95

- basis\_table, 94
- friend\_for\_private\_destructor, 95
- operator=, 95
- glucat::bool\_to\_type< truth\_value >, 95
- value, 96
- glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 117
- ~clifford\_algebra, 120
- classname, 120
- conj, 120
- default\_truncation, 129
- even, 121
- frame, 121
- grade, 121
- index\_set\_t, 119
- inv, 121
- involute, 122
- isinf, 122
- isnan, 122
- max\_abs, 122
- multivector\_t, 119
- norm, 123
- odd, 123
- operator(), 123
- operator+=, 124
- operator-, 124
- operator=, 124, 125
- operator/=: 125
- operator==, 125
- operator%=: 123
- operator&=: 123
- operator[], 125
- operator\*=, 124
- operator^=: 126
- operator|=: 126
- outer\_pow, 126
- pair\_t, 120
- pow, 126
- pure, 126
- quad, 127
- reverse, 127
- scalar, 127
- scalar\_t, 120
- truncated, 127
- v\_hi, 129
- v\_lo, 129
- vector\_part, 128
- vector\_t, 120
- write, 128
- glucat::compare\_types< LHS\_T, RHS\_T >, 129
- are\_same, 130
- glucat::compare\_types< T, T >, 130
- are\_same, 131
- glucat::control\_t, 131
- ~control\_t, 133
- call, 133
- catch\_exceptions, 133
- control, 133
- control\_t, 132, 133
- friend\_for\_private\_destructor, 134
- m\_catch\_exceptions, 135
- m\_valid, 135
- m\_verbose\_output, 135
- operator=, 134
- valid, 134
- verbose, 134
- glucat::CTAssertion< bool >, 135
- glucat::CTAssertion< true >, 136
- glucat::error< Class\_T >, 139
- classname, 141
- error, 140
- heading, 141
- print\_error\_msg, 141
- glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 142
- ~framed\_multi, 150
- centre\_pm4\_qp4, 154
- centre\_pp4\_qm4, 154
- centre\_qp1\_pm1, 154
- classname, 155
- const\_iterator, 147
- divide, 155
- error\_t, 147
- exp, 157
- fast, 155
- fast\_framed\_multi, 155
- fast\_matrix\_multi, 156
- fold, 156
- framed\_multi, 150–153, 157
- framed\_multi\_t, 147
- framed\_pair\_t, 147
- index\_set\_t, 148
- iterator, 148
- map\_t, 148
- matrix\_multi, 158
- matrix\_multi\_t, 148
- matrix\_t, 148
- multivector\_t, 148
- nbr\_terms, 156
- operator<<, 159
- operator>>, 159
- operator+=, 156
- operator/, 158
- operator%, 158
- operator&, 158
- operator\*, 158
- operator^, 159
- operator|, 159
- random, 157
- scalar\_t, 149
- size\_type, 149
- sorted\_map\_t, 149
- star, 159
- term\_t, 149
- tune\_p, 149
- unfold, 157
- var\_term\_t, 149

- vector\_t, 150
- glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P  
     >::hash\_size\_t, 167
- hash\_size\_t, 168
- n, 168
- operator(), 168
- glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P  
     >::var\_term, 253
- ~var\_term, 255
- classname, 255
- operator\*=, 255
- var\_pair\_t, 255
- var\_term, 255
- glucat::gen, 69
- offset\_to\_super, 69
- signature\_t, 69
- glucat::gen::generator\_table< Matrix\_T >, 160
- ~generator\_table, 161
- friend\_for\_private\_destructor, 164
- gen\_from\_pm1\_qm1, 162
- gen\_from\_pm4\_qp4, 162
- gen\_from\_pp4\_qm4, 162
- gen\_from\_qp1\_pm1, 163
- gen\_vector, 163
- generator, 163
- generator\_table, 161, 162
- operator(), 163
- operator=, 164
- glucat::glucat\_error, 165
- ~glucat\_error, 166
- classname, 166
- glucat\_error, 166
- heading, 166
- name, 167
- print\_error\_msg, 166
- glucat::index\_set< LO, HI >, 169
- bitset\_t, 172
- BOOST\_STATIC\_ASSERT, 174
- classname, 174
- compare, 181
- count, 174
- count\_neg, 174
- count\_pos, 175
- error\_t, 172
- flip, 175
- fold, 175
- hash\_fn, 176
- index\_pair\_t, 172
- index\_set, 173, 174
- index\_set\_t, 172
- is\_contiguous, 176
- lex\_less\_than, 176
- max, 176
- min, 176
- operator!=, 177
- operator<, 177
- operator==, 177
- operator&, 181
- operator&=, 177
- operator[], 178
- operator~, 178
- operator^, 181
- operator^=, 178
- operator|, 181
- operator|=, 178
- reference, 182
- reset, 179
- set, 179
- sign\_of\_mult, 180
- sign\_of\_square, 180
- test, 180
- unfold, 180
- v\_hi, 182
- v\_lo, 182
- value\_of\_fold, 181
- glucat::index\_set< LO, HI >::reference, 245
- ~reference, 247
- flip, 247
- index\_set, 249
- m\_idx, 249
- m\_pst, 249
- operator bool, 247
- operator=, 247, 248
- operator==, 248
- operator~, 248
- reference, 246
- glucat::index\_set\_hash< LO, HI >, 192
- index\_set\_t, 192
- operator(), 193
- glucat::matrix, 69
- classify\_eigenvalues, 71
- eig\_case\_t, 71
- eigenvalues, 71
- inner, 72
- isinf, 72
- isnan, 72
- kron, 72
- mono\_kron, 73
- mono\_prod, 73
- nnz, 73
- nork, 74
- nork\_range, 74
- norm\_frob2, 74
- prod, 75
- signed\_perm\_nork, 75
- sparse\_prod, 75
- to\_blaze, 75
- trace, 76
- unit, 76
- glucat::matrix::eig\_genus< Matrix\_T >, 137
- m\_eig\_case, 138
- m\_is\_singular, 138
- m\_safe\_arg, 138
- Scalar\_T, 137
- glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 193
- ~matrix\_multi, 200

- basis\_element, [205](#)
- basis\_matrix\_t, [198](#)
- classname, [205](#)
- error\_t, [198](#)
- fast\_framed\_multi, [205](#)
- fast\_matrix\_multi, [205](#)
- framed\_multi, [206](#)
- framed\_multi\_t, [198](#)
- index\_set\_t, [198](#)
- m\_frame, [209](#)
- m\_matrix, [209](#)
- matrix\_index\_t, [199](#)
- matrix\_log, [206](#)
- matrix\_multi, [200–204](#), [207](#)
- matrix\_multi\_t, [199](#)
- matrix\_sqrt, [207](#)
- matrix\_t, [199](#)
- multivector\_t, [199](#)
- operator<<, [208](#)
- operator>>, [208](#)
- operator+&, [205](#)
- operator/, [208](#)
- operator=, [206](#)
- operator%, [207](#)
- operator&, [207](#)
- operator\*, [207](#)
- operator^, [208](#)
- operator|, [208](#)
- orientation\_t, [199](#)
- random, [206](#)
- reframe, [208](#)
- scalar\_t, [199](#)
- star, [209](#)
- term\_t, [200](#)
- tune\_p, [200](#)
- vector\_t, [200](#)
- glucat::numeric\_traits< Scalar\_T >, [212](#)
  - abs, [214](#)
  - acos, [214](#)
  - asin, [214](#)
  - atan, [214](#)
  - conj, [215](#)
  - cos, [215](#)
  - cosh, [215](#)
  - exp, [215](#)
  - fmod, [215](#)
  - imag, [216](#)
  - isInf, [216](#)
  - isNaN, [217](#)
  - isNaN\_or\_isInf, [217](#)
  - ln\_2, [218](#)
  - log, [218](#)
  - log2, [218](#)
  - NaN, [219](#)
  - pi, [219](#)
  - pow, [219](#)
  - real, [220](#)
  - sin, [220](#)
  - sinh, [220](#)
  - sqrt, [220](#)
  - tan, [220](#)
  - tanh, [221](#)
  - to\_double, [221](#)
  - to\_int, [221](#)
  - to\_scalar\_t, [221–223](#)
- glucat::numeric\_traits< Scalar\_T >::demoted, [136](#)
- type, [136](#)
- glucat::numeric\_traits< Scalar\_T >::promoted, [240](#)
- type, [241](#)
- glucat::random\_generator< Scalar\_T >, [241](#)
  - ~random\_generator, [243](#)
  - friend\_for\_private\_destructor, [244](#)
  - generator, [243](#)
  - normal, [243](#)
  - normal\_dist, [244](#)
  - operator=, [243](#)
  - random\_generator, [242](#)
  - seed, [244](#)
  - uint\_gen, [244](#)
  - uniform, [243](#)
  - uniform\_dist, [244](#)
- glucat::sorted\_range< Map\_T, Sorted\_Map\_T >, [249](#)
  - map\_t, [250](#)
  - sorted\_begin, [251](#)
  - sorted\_end, [251](#)
  - sorted\_iterator, [250](#)
  - sorted\_map\_t, [250](#)
  - sorted\_range, [251](#)
- glucat::sorted\_range< Sorted\_Map\_T, Sorted\_Map\_T >, [251](#)
  - map\_t, [252](#)
  - sorted\_begin, [253](#)
  - sorted\_end, [253](#)
  - sorted\_iterator, [252](#)
  - sorted\_map\_t, [252](#)
  - sorted\_range, [252](#)
- glucat::timing, [76](#)
  - elapsed, [77](#)
  - EXTRA\_TRIALS, [77](#)
  - MS\_PER\_CLOCK, [77](#)
  - MS\_PER\_SEC, [77](#)
- glucat\_config.h
  - GLUCAT\_HAVE\_CXX11, [340](#)
  - GLUCAT\_HAVE\_INTTYPES\_H, [340](#)
  - GLUCAT\_HAVE\_STDINT\_H, [341](#)
  - GLUCAT\_HAVE\_STDIO\_H, [341](#)
  - GLUCAT\_HAVE\_STDLIB\_H, [341](#)
  - GLUCAT\_HAVE\_STRING\_H, [341](#)
  - GLUCAT\_HAVE\_STRINGS\_H, [341](#)
  - GLUCAT\_HAVE\_SYS\_STAT\_H, [341](#)
  - GLUCAT\_HAVE\_SYS\_TYPES\_H, [341](#)
  - GLUCAT\_HAVE\_UNISTD\_H, [341](#)
  - GLUCAT\_PACKAGE, [342](#)
  - GLUCAT\_PACKAGE\_BUGREPORT, [342](#)
  - GLUCAT\_PACKAGE\_NAME, [342](#)
  - GLUCAT\_PACKAGE\_STRING, [342](#)

- GLUCAT\_PACKAGE\_TARNAME, 342
- GLUCAT\_PACKAGE\_URL, 342
- GLUCAT\_PACKAGE\_VERSION, 342
- GLUCAT\_STDC\_HEADERS, 343
- GLUCAT\_VERSION, 343
- glucat\_error
  - glucat::glucat\_error, 166
- GLUCAT\_HAVE\_CXX11
  - glucat\_config.h, 340
- GLUCAT\_HAVE\_INTTYPES\_H
  - glucat\_config.h, 340
- GLUCAT\_HAVE\_STDINT\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_STDIO\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_STDLIB\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_STRING\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_STRINGS\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_SYS\_STAT\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_SYS\_TYPES\_H
  - glucat\_config.h, 341
- GLUCAT\_HAVE\_UNISTD\_H
  - glucat\_config.h, 341
- GLUCAT\_PACKAGE
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_BUGREPORT
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_NAME
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_STRING
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_TARNAME
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_URL
  - glucat\_config.h, 342
- GLUCAT\_PACKAGE\_VERSION
  - glucat\_config.h, 342
- glucat\_package\_version
  - PyClical.h, 444
- GLUCAT\_STDC\_HEADERS
  - glucat\_config.h, 343
- GLUCAT\_VERSION
  - glucat\_config.h, 343
- grade
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 121
- hash\_fn
  - glucat::index\_set< LO, HI >, 176
  - PyClical.index\_set, 190
- hash\_size\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t, 168
- heading
  - glucat::error< Class\_T >, 141
- glucat::glucat\_error, 166
- hi\_ndx
  - PyClical.h, 444
- i
  - PyClical, 90
- imag
  - glucat, 40
  - glucat::numeric\_traits< Scalar\_T >, 216
- index\_pair\_t
  - glucat::index\_set< LO, HI >, 172
- index\_set
  - glucat::index\_set< LO, HI >, 173, 174
  - glucat::index\_set< LO, HI >::reference, 249
- index\_set\_hidden\_doctests
  - PyClical, 88
- index\_set\_t
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 119
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 148
  - glucat::index\_set< LO, HI >, 172
  - glucat::index\_set\_hash< LO, HI >, 192
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 198
- index\_set\_to\_repr
  - PyClical.h, 443
- index\_set\_to\_str
  - PyClical.h, 443
- index\_t
  - glucat, 26
- IndexSet
  - PyClical.h, 442
- inner
  - glucat::matrix, 72
- instance
  - PyClical.clifford, 116
  - PyClical.index\_set, 192
- intfn
  - glucat, 26
- intintfn
  - glucat, 26
- inv
  - glucat, 40
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 121
  - PyClical.clifford, 111
- inverse\_gray
  - glucat, 41
- inverse\_reversed\_gray
  - glucat, 41
- involute
  - glucat, 41
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 122
  - PyClical.clifford, 111
- is\_contiguous
  - glucat::index\_set< LO, HI >, 176
- isInf

- glucat::numeric\_traits< Scalar\_T >, 216
- isinf
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 122
  - glucat::matrix, 72
  - PyClical.clifford, 111
- isNaN
  - glucat::numeric\_traits< Scalar\_T >, 217
- isnan
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 122
  - glucat::matrix, 72
  - PyClical.clifford, 112
- isNaN\_or\_isInf
  - glucat::numeric\_traits< Scalar\_T >, 217
- ist
  - PyClical, 90
- istpq
  - PyClical, 89
- iterator
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 148
- ixt
  - PyClical, 90
- kron
  - glucat::matrix, 72
- l\_in2
  - glucat, 65
- l\_pi
  - glucat, 65
- lex\_less\_than
  - glucat::index\_set< LO, HI >, 176
- lhs
  - PyClical, 90
- ln\_2
  - glucat::numeric\_traits< Scalar\_T >, 218
- lo\_ndx
  - PyClical.h, 444
- log
  - glucat, 41, 42
  - glucat::numeric\_traits< Scalar\_T >, 218
- log2
  - glucat, 42
  - glucat::numeric\_traits< Scalar\_T >, 218
- m\_catch\_exceptions
  - glucat::control\_t, 135
- m\_eig\_case
  - glucat::matrix::eig\_genus< Matrix\_T >, 138
- m\_frame
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 209
- m\_idx
  - glucat::index\_set< LO, HI >::reference, 249
- m\_is\_singular
  - glucat::matrix::eig\_genus< Matrix\_T >, 138
- m\_matrix
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 209
- m\_pst
  - glucat::index\_set< LO, HI >::reference, 249
- m\_safe\_arg
  - glucat::matrix::eig\_genus< Matrix\_T >, 138
- m\_valid
  - glucat::control\_t, 135
- m\_verbose\_output
  - glucat::control\_t, 135
- map\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 148
  - glucat::sorted\_range< Map\_T, Sorted\_Map\_T >, 250
  - glucat::sorted\_range< Sorted\_Map\_T, Sorted\_Map\_T >, 252
- matrix\_index\_t
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 199
- matrix\_log
  - glucat, 43
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 206
- matrix\_multi
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 158
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 200–204, 207
- matrix\_multi\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 148
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 199
- matrix\_sqrt
  - glucat, 43
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 207
- matrix\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 148
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 199
- max
  - glucat::index\_set< LO, HI >, 176
  - PyClical.index\_set, 190
- max\_abs
  - glucat, 43
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 122
  - PyClical.clifford, 112
- max\_pos
  - glucat, 44
- min
  - glucat::index\_set< LO, HI >, 176
  - PyClical.index\_set, 190
- min\_neg
  - glucat, 44

- mono\_kron
  - glucat::matrix, [73](#)
- mono\_prod
  - glucat::matrix, [73](#)
- MS\_PER\_CLOCK
  - glucat::timing, [77](#)
- MS\_PER\_S
  - glucat, [65](#)
- MS\_PER\_SEC
  - glucat::timing, [77](#)
- multivector\_t
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [119](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [148](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [199](#)
- n
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t, [168](#)
- name
  - glucat::glucat\_error, [167](#)
- NaN
  - glucat::numeric\_traits< Scalar\_T >, [219](#)
- nbar3
  - PyClical, [90](#)
- nbr\_terms
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [156](#)
- ninf3
  - PyClical, [90](#)
- nnz
  - glucat::matrix, [73](#)
- None
  - PyClical, [90](#)
- nork
  - glucat::matrix, [74](#)
- nork\_range
  - glucat::matrix, [74](#)
- norm
  - glucat, [44](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [123](#)
  - PyClical.clifford, [112](#)
- norm\_frob2
  - glucat::matrix, [74](#)
- normal
  - glucat::random\_generator< Scalar\_T >, [243](#)
- normal\_dist
  - glucat::random\_generator< Scalar\_T >, [244](#)
- numer
  - pade::pade\_log\_numer< dd\_real >, [229](#)
  - pade::pade\_log\_numer< float >, [230](#)
  - pade::pade\_log\_numer< long double >, [231](#)
  - pade::pade\_log\_numer< qd\_real >, [232](#)
  - pade::pade\_log\_numer< Scalar\_T >, [228](#)
  - pade::pade\_sqrt\_numer< dd\_real >, [238](#)
  - pade::pade\_sqrt\_numer< float >, [238](#)
  - pade::pade\_sqrt\_numer< long double >, [239](#)
  - pade::pade\_sqrt\_numer< qd\_real >, [240](#)
  - pade::pade\_sqrt\_numer< Scalar\_T >, [237](#)
- obj
  - PyClical, [90](#)
- odd
  - glucat, [44](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [123](#)
  - PyClical.clifford, [113](#)
- offset\_level
  - glucat, [45](#)
- offset\_to\_super
  - glucat::gen, [69](#)
- operator bool
  - glucat::index\_set< LO, HI >::reference, [247](#)
- operator!=
  - glucat, [45](#)
  - glucat::index\_set< LO, HI >, [177](#)
- operator<
  - glucat::index\_set< LO, HI >, [177](#)
- operator<<
  - glucat, [52](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [159](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [208](#)
- operator>>
  - glucat, [53](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [159](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [208](#)
- operator()
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [123](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t, [168](#)
  - glucat::gen::generator\_table< Matrix\_T >, [163](#)
  - glucat::index\_set\_hash< LO, HI >, [193](#)
- operator+
  - glucat, [49](#)
- operator+=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [124](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [156](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [205](#)
- operator-
  - glucat, [50](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [124](#)
- operator-=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [124](#), [125](#)
- operator/
  - glucat, [50](#), [51](#)

- glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 158
- glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 208
- operator/=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 125
- operator=
  - glucat::basis\_table< Scalar\_T, LO, HI, Matrix\_T >, 95
  - glucat::control\_t, 134
  - glucat::gen::generator\_table< Matrix\_T >, 164
  - glucat::index\_set< LO, HI >::reference, 247, 248
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 206
  - glucat::random\_generator< Scalar\_T >, 243
- operator==
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 125
  - glucat::index\_set< LO, HI >, 177
  - glucat::index\_set< LO, HI >::reference, 248
- operator%
  - glucat, 46
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 158
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 207
- operator%=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 123
- operator&
  - glucat, 46, 47
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 158
  - glucat::index\_set< LO, HI >, 181
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 207
- operator&=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 123
  - glucat::index\_set< LO, HI >, 177
- operator[]
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 125
  - glucat::index\_set< LO, HI >, 178
- operator\*
  - glucat, 47–49
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 158
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 207
- operator\*=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 124
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term, 255
- operator~
  - glucat::index\_set< LO, HI >, 178
- glucat::index\_set< LO, HI >::reference, 248
- operator^
  - glucat, 53, 54
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 159
  - glucat::index\_set< LO, HI >, 181
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 208
- operator^=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 126
  - glucat::index\_set< LO, HI >, 178
- operator|
  - glucat, 54, 55
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 159
  - glucat::index\_set< LO, HI >, 181
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 208
- operator|=
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 126
  - glucat::index\_set< LO, HI >, 178
- orientation\_t
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 199
- outer\_pow
  - glucat, 55
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 126
  - PyClical.clifford, 113
- pade, 78
  - pade\_log\_denom< dd\_real >::denom, 79
  - pade\_log\_denom< float >::denom, 79
  - pade\_log\_denom< longdouble >::denom, 79
  - pade\_log\_denom< qd\_real >::denom, 79
  - pade\_log\_denom< Scalar\_T >::denom, 80
  - pade\_log\_numer< dd\_real >::numer, 80
  - pade\_log\_numer< float >::numer, 81
  - pade\_log\_numer< longdouble >::numer, 81
  - pade\_log\_numer< qd\_real >::numer, 81
  - pade\_log\_numer< Scalar\_T >::numer, 82
  - pade\_sqrt\_denom< dd\_real >::denom, 82
  - pade\_sqrt\_denom< float >::denom, 82
  - pade\_sqrt\_denom< longdouble >::denom, 83
  - pade\_sqrt\_denom< qd\_real >::denom, 83
  - pade\_sqrt\_denom< Scalar\_T >::denom, 83
  - pade\_sqrt\_numer< dd\_real >::numer, 84
  - pade\_sqrt\_numer< float >::numer, 84
  - pade\_sqrt\_numer< longdouble >::numer, 84
  - pade\_sqrt\_numer< qd\_real >::numer, 84
  - pade\_sqrt\_numer< Scalar\_T >::numer, 85
- pade::pade\_log\_denom< dd\_real >, 224
  - array, 225
  - denom, 225
- pade::pade\_log\_denom< float >, 225
  - array, 226
  - denom, 226



- pade::pade\_log\_denom< long double >, 226
  - array, 226
  - denom, 227
- pade::pade\_log\_denom< qd\_real >, 227
  - array, 227
  - denom, 227
- pade::pade\_log\_denom< Scalar\_T >, 223
  - array, 224
  - denom, 224
- pade::pade\_log\_numer< dd\_real >, 229
  - array, 229
  - numer, 229
- pade::pade\_log\_numer< float >, 229
  - array, 230
  - numer, 230
- pade::pade\_log\_numer< long double >, 230
  - array, 231
  - numer, 231
- pade::pade\_log\_numer< qd\_real >, 231
  - array, 231
  - numer, 232
- pade::pade\_log\_numer< Scalar\_T >, 228
  - array, 228
  - numer, 228
- pade::pade\_sqrt\_denom< dd\_real >, 233
  - array, 233
  - denom, 233
- pade::pade\_sqrt\_denom< float >, 234
  - array, 234
  - denom, 234
- pade::pade\_sqrt\_denom< long double >, 234
  - array, 235
  - denom, 235
- pade::pade\_sqrt\_denom< qd\_real >, 235
  - array, 236
  - denom, 236
- pade::pade\_sqrt\_denom< Scalar\_T >, 232
  - array, 232
  - denom, 233
- pade::pade\_sqrt\_numer< dd\_real >, 237
  - array, 237
  - numer, 238
- pade::pade\_sqrt\_numer< float >, 238
  - array, 238
  - numer, 238
- pade::pade\_sqrt\_numer< long double >, 239
  - array, 239
  - numer, 239
- pade::pade\_sqrt\_numer< qd\_real >, 239
  - array, 240
  - numer, 240
- pade::pade\_sqrt\_numer< Scalar\_T >, 236
  - array, 237
  - numer, 237
- pade\_approx
  - glucat, 56
- pade\_log
  - glucat, 56
- pade\_log\_denom< dd\_real >::denom
  - pade, 79
- pade\_log\_denom< float >::denom
  - pade, 79
- pade\_log\_denom< longdouble >::denom
  - pade, 79
- pade\_log\_denom< qd\_real >::denom
  - pade, 79
- pade\_log\_denom< Scalar\_T >::denom
  - pade, 80
- pade\_log\_numer< dd\_real >::numer
  - pade, 80
- pade\_log\_numer< float >::numer
  - pade, 81
- pade\_log\_numer< longdouble >::numer
  - pade, 81
- pade\_log\_numer< qd\_real >::numer
  - pade, 81
- pade\_log\_numer< Scalar\_T >::numer
  - pade, 82
- pade\_sqrt\_denom< dd\_real >::denom
  - pade, 82
- pade\_sqrt\_denom< float >::denom
  - pade, 82
- pade\_sqrt\_denom< longdouble >::denom
  - pade, 83
- pade\_sqrt\_denom< qd\_real >::denom
  - pade, 83
- pade\_sqrt\_denom< Scalar\_T >::denom
  - pade, 83
- pade\_sqrt\_numer< dd\_real >::numer
  - pade, 84
- pade\_sqrt\_numer< float >::numer
  - pade, 84
- pade\_sqrt\_numer< longdouble >::numer
  - pade, 84
- pade\_sqrt\_numer< qd\_real >::numer
  - pade, 84
- pade\_sqrt\_numer< Scalar\_T >::numer
  - pade, 85
- pair\_t
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 120
- pi
  - glucat::numeric\_traits< Scalar\_T >, 219
  - PyClical, 91
- portability.h
  - \_GLUCAT\_ISINF, 413
  - \_GLUCAT\_ISNAN, 413
  - UBLAS\_ABS, 414
  - UBLAS\_SQRT, 414
- pos\_mod
  - glucat, 56
- pow
  - glucat, 57
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 126
  - glucat::numeric\_traits< Scalar\_T >, 219

- PyClical.clifford, 113
- print\_error\_msg
  - glucat::error< Class\_T >, 141
  - glucat::glucat\_error, 166
- prod
  - glucat::matrix, 75
- pure
  - glucat, 57
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 126
  - PyClical.clifford, 114
- PyClical, 85
  - \_\_version\_\_, 89
  - \_test, 86
  - cl, 89
  - clifford\_hidden\_doctests, 86
  - e, 87
  - fill, 89
  - i, 90
  - index\_set\_hidden\_doctests, 88
  - ist, 90
  - istpq, 89
  - ixt, 90
  - lhs, 90
  - nbar3, 90
  - ninf3, 90
  - None, 90
  - obj, 90
  - pi, 91
  - rhs, 91
  - scalar\_epsilon, 91
  - tau, 91
  - threshold, 91
  - tol, 91
- pyclical Directory Reference, 12
- PyClical.clifford, 96
  - \_\_add\_\_, 98
  - \_\_and\_\_, 98
  - \_\_call\_\_, 99
  - \_\_cinit\_\_, 99
  - \_\_contains\_\_, 100
  - \_\_dealloc\_\_, 100
  - \_\_getitem\_\_, 100
  - \_\_iadd\_\_, 101
  - \_\_iand\_\_, 101
  - \_\_idiv\_\_, 101
  - \_\_imod\_\_, 102
  - \_\_imul\_\_, 102
  - \_\_ior\_\_, 102
  - \_\_isub\_\_, 103
  - \_\_iter\_\_, 103
  - \_\_ixor\_\_, 103
  - \_\_mod\_\_, 103
  - \_\_mul\_\_, 104
  - \_\_neg\_\_, 104
  - \_\_or\_\_, 104
  - \_\_pos\_\_, 105
  - \_\_pow\_\_, 105
  - \_\_radd\_\_, 105
  - \_\_rand\_\_, 106
  - \_\_repr\_\_, 106
  - \_\_richcmp\_\_, 106
  - \_\_rmod\_\_, 107
  - \_\_rmul\_\_, 107
  - \_\_rsub\_\_, 107
  - \_\_rtruediv\_\_, 108
  - \_\_rxor\_\_, 108
  - \_\_str\_\_, 108
  - \_\_sub\_\_, 108
  - \_\_truediv\_\_, 109
  - \_\_xor\_\_, 109
  - abs, 109
  - conj, 110
  - even, 110
  - frame, 110
  - instance, 116
  - inv, 111
  - involute, 111
  - isinf, 111
  - isnan, 112
  - max\_abs, 112
  - norm, 112
  - odd, 113
  - outer\_pow, 113
  - pow, 113
  - pure, 114
  - quad, 114
  - reframe, 114
  - reverse, 115
  - scalar, 115
  - truncated, 115
  - vector\_part, 116
- PyClical.h
  - Clifford, 442
  - clifford\_to\_repr, 443
  - clifford\_to\_str, 443
  - epsilon, 444
  - glucat\_package\_version, 444
  - hi\_ndx, 444
  - index\_set\_to\_repr, 443
  - index\_set\_to\_str, 443
  - IndexSet, 442
  - lo\_ndx, 444
  - PyFloat\_FromDouble, 444
  - scalar\_t, 442
  - String, 443
- PyClical.index\_set, 183
  - \_\_and\_\_, 184
  - \_\_cinit\_\_, 184
  - \_\_contains\_\_, 184
  - \_\_dealloc\_\_, 185
  - \_\_getitem\_\_, 185
  - \_\_iand\_\_, 185
  - \_\_invert\_\_, 186
  - \_\_ior\_\_, 186
  - \_\_iter\_\_, 186

- `__ixor__`, 186
- `__or__`, 187
- `__repr__`, 187
- `__richcmp__`, 187
- `__setitem__`, 188
- `__str__`, 188
- `__xor__`, 188
- `count`, 189
- `count_neg`, 189
- `count_pos`, 189
- `hash_fn`, 190
- `instance`, 192
- `max`, 190
- `min`, 190
- `sign_of_mult`, 191
- `sign_of_square`, 191
- `pyclical/glucat.pxd`, 439
- `pyclical/PyClical.h`, 441, 445
- `pyclical/PyClical.pxd`, 447
- `pyclical/PyClical.pyx`, 447, 448
- `PyFloat_FromDouble`
  - `PyClical.h`, 444
- `quad`
  - `glucat`, 58
  - `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`, 127
  - `PyClical.clifford`, 114
- `random`
  - `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`, 157
  - `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`, 206
- `random_generator`
  - `glucat::random_generator< Scalar_T >`, 242
- `real`
  - `glucat`, 58
  - `glucat::numeric_traits< Scalar_T >`, 220
- `reference`
  - `glucat::index_set< LO, HI >`, 182
  - `glucat::index_set< LO, HI >::reference`, 246
- `reframe`
  - `glucat`, 58
  - `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`, 208
  - `PyClical.clifford`, 114
- `reset`
  - `glucat::index_set< LO, HI >`, 179
- `reverse`
  - `glucat`, 59
  - `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`, 127
  - `PyClical.clifford`, 115
- `rhs`
  - `PyClical`, 91
- `scalar`
  - `glucat`, 59
- `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`, 127
- `PyClical.clifford`, 115
- `scalar_epsilon`
  - `PyClical`, 91
- `Scalar_T`
  - `glucat::matrix::eig_genus< Matrix_T >`, 137
- `scalar_t`
  - `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`, 120
  - `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`, 149
  - `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`, 199
  - `PyClical.h`, 442
- `seed`
  - `glucat::random_generator< Scalar_T >`, 244
- `set`
  - `glucat::index_set< LO, HI >`, 179
- `set_value_t`
  - `glucat`, 27
- `sign_of_mult`
  - `glucat::index_set< LO, HI >`, 180
  - `PyClical.index_set`, 191
- `sign_of_square`
  - `glucat`, 59
  - `glucat::index_set< LO, HI >`, 180
  - `PyClical.index_set`, 191
- `signature_t`
  - `glucat::gen`, 69
- `signed_perm_nork`
  - `glucat::matrix`, 75
- `sin`
  - `glucat`, 60
  - `glucat::numeric_traits< Scalar_T >`, 220
- `sinh`
  - `glucat`, 60
  - `glucat::numeric_traits< Scalar_T >`, 220
- `size_type`
  - `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`, 149
- `sorted_begin`
  - `glucat::sorted_range< Map_T, Sorted_Map_T >`, 251
  - `glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >`, 253
- `sorted_end`
  - `glucat::sorted_range< Map_T, Sorted_Map_T >`, 251
  - `glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >`, 253
- `sorted_iterator`
  - `glucat::sorted_range< Map_T, Sorted_Map_T >`, 250
  - `glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >`, 252
- `sorted_map_t`
  - `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`,

- 149
- glucat::sorted\_range< Map\_T, Sorted\_Map\_T >, 250
- glucat::sorted\_range< Sorted\_Map\_T, Sorted\_Map\_T >, 252
- sorted\_range
  - glucat::sorted\_range< Map\_T, Sorted\_Map\_T >, 251
  - glucat::sorted\_range< Sorted\_Map\_T, Sorted\_Map\_T >, 252
- sparse\_prod
  - glucat::matrix, 75
- sqrt
  - glucat, 60, 61
  - glucat::numeric\_traits< Scalar\_T >, 220
- star
  - glucat, 61, 62
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 159
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 209
- std, 91
- std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >, 210
- std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > >, 211
- String
  - PyClical.h, 443
- tan
  - glucat, 62
  - glucat::numeric\_traits< Scalar\_T >, 220
- tanh
  - glucat, 63
  - glucat::numeric\_traits< Scalar\_T >, 221
- tau
  - PyClical, 91
- term\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 149
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 200
- test
  - glucat::index\_set< LO, HI >, 180
- test Directory Reference, 12
- test/control.h, 472, 473
- test/driver.h, 474, 475
- test/timing.h, 475, 476
- test/try\_catch.h, 477
- test/tuning.h, 437, 438
- test/undefine.h, 478
- threshold
  - PyClical, 91
- to\_blaze
  - glucat::matrix, 75
- to\_demote
  - glucat, 63
- to\_double
  - glucat::numeric\_traits< Scalar\_T >, 221
- to\_int
  - glucat::numeric\_traits< Scalar\_T >, 221
- to\_promote
  - glucat, 63
- to\_scalar\_t
  - glucat::numeric\_traits< Scalar\_T >, 221–223
- tol
  - PyClical, 91
- Trace
  - glucat::matrix, 76
- truncated
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, 127
  - PyClical.clifford, 115
- try\_catch
  - glucat, 64
- tune\_p
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, 149
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, 200
- tuning.h
  - \_GLUCAT\_CTAssert, 434
- tuning\_fast
  - glucat, 27
- Tuning\_Fast\_Basis\_Max\_Count
  - glucat, 66
- Tuning\_Fast\_CR\_Sqrt\_Max\_Steps
  - glucat, 66
- Tuning\_Fast\_DB\_Sqrt\_Max\_Steps
  - glucat, 66
- Tuning\_Fast\_Div\_Max\_Steps
  - glucat, 66
- Tuning\_Fast\_Fast\_Size\_Threshold
  - glucat, 66
- Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold
  - glucat, 66
- Tuning\_Fast\_Log\_Max\_Inner\_Steps
  - glucat, 66
- Tuning\_Fast\_Log\_Max\_Outer\_Steps
  - glucat, 67
- Tuning\_Fast\_Mult\_Matrix\_Threshold
  - glucat, 67
- Tuning\_Fast\_Products\_Size\_Threshold
  - glucat, 67
- Tuning\_Int\_Digits
  - glucat, 67
- Tuning\_Max\_Threshold
  - glucat, 67
- tuning\_naive
  - glucat, 27
- Tuning\_Naive\_Basis\_Max\_Count
  - glucat, 67
- Tuning\_Naive\_Fast\_Size\_Threshold
  - glucat, 67
- Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold
  - glucat, 67
- Tuning\_Naive\_Mult\_Matrix\_Threshold

- glucat, [68](#)
- tuning\_slow
  - glucat, [27](#)
- Tuning\_Slow\_Basis\_Max\_Count
  - glucat, [68](#)
- Tuning\_Slow\_Fast\_Size\_Threshold
  - glucat, [68](#)
- Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold
  - glucat, [68](#)
- Tuning\_Slow\_Mult\_Matrix\_Threshold
  - glucat, [68](#)
- Tuning\_Slow\_Products\_Size\_Threshold
  - glucat, [68](#)
- type
  - glucat::numeric\_traits< Scalar\_T >::demoted, [136](#)
  - glucat::numeric\_traits< Scalar\_T >::promoted, [241](#)
- UBLAS\_ABS
  - portability.h, [414](#)
- UBLAS\_SQRT
  - portability.h, [414](#)
- uint\_gen
  - glucat::random\_generator< Scalar\_T >, [244](#)
- unfold
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [157](#)
  - glucat::index\_set< LO, HI >, [180](#)
- uniform
  - glucat::random\_generator< Scalar\_T >, [243](#)
- uniform\_dist
  - glucat::random\_generator< Scalar\_T >, [244](#)
- unit
  - glucat::matrix, [76](#)
- v\_hi
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [129](#)
  - glucat::index\_set< LO, HI >, [182](#)
- v\_lo
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [129](#)
  - glucat::index\_set< LO, HI >, [182](#)
- valid
  - glucat::control\_t, [134](#)
- value
  - glucat::bool\_to\_type< truth\_value >, [96](#)
- value\_of\_fold
  - glucat::index\_set< LO, HI >, [181](#)
- var\_pair\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term, [255](#)
- var\_term
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term, [255](#)
- var\_term\_t
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [149](#)
- vector\_part
  - glucat, [64](#)
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [128](#)
  - PyClical.clifford, [116](#)
- vector\_t
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [120](#)
  - glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >, [150](#)
  - glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >, [200](#)
- verbose
  - glucat::control\_t, [134](#)
- write
  - glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >, [128](#)